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Comparing the Performance of Alternative Generalised Autoregressive Conditional Heteroskedasticity Models in Modelling Nigeria Crude Oil Production Volatility Series

***A. E. Usoro, C. E. Awakessien and C. O. Omekara**

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Abstract There is no gainsaying the fact that crude oil production remains a major factor to the Nigeria economic growth given its significant contribution to the nation's gross domestic product. Preponderance of the researches in the oil sector dwell more on oil prices, with less focus on the volatility of crude oil production. What cannot be overemphasized in oil sector is the production volatility effect which is mostly caused by unstable production quantity due to certain nation's economic, social, political factors. In this paper, volatility of crude oil production was considered, and different Generalised Autoregressive Conditional Heteroskedasticity (GARCH) models were fitted to Nigeria crude oil production volatility series. Data for the work were monthly crude oil quantity data from January 2010 to August 2019 (NNPC ASB) from which the crude oil production volatility was measured. The suggested GARCH models included GARCH (0,1), GARCH (0,2), GARCH (1,1), GARCH (1,2), GARCH (2,1) and GARCH (2,2). Using Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Schwarz's Information Criterion (SIC), GARCH (1,2) and GARCH(2,1) competed favourably. The MSE of forecast revealed GARCH (2,1) to perform better for the forecast of crude oil production volatility. Further findings will reveal other alternative models as the crude oil production pattern changes in the future.

Key Words: *GARCH (p, q), GARCH (o, q), volatility measure.*

***A. E. Usoro**
Department of Statistics, Akwa Ibom State University,
Mkpat Enin, Akwa Ibom State, Nigeria
Email: anthonyusoro@aksu.edu.ng

C. E. Awakessien
Department of Statistics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria

C. O. Omekara
Department of Statistics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria

1.0 Introduction

In univariate time series, a time dependent variable, say X_t may be characterised by linear, nonlinear or mixed process. Autoregresive (AR), Moving Average (MA) and Autoregressive Moving Average (ARMA) models are popular and commonly used in fitting stationary linear time series; (Box and Jenkins, 1976). The problem in the applications of AR, MA and ARMA is that they are not appropriate models for time series process described by large changes in variance at various time periods; (Franses, 1998). This is because the aforementioned models account for linear processes and cannot capture high variance properties of a series indexed at time t. The consistent change in variance over time may be increasing or decreasing, and this systematic change is called heteroskedasticity or volatility. Autoregressive Conditional Heteroskedasticity (ARCH) is a method that explicitly models the change in variance over time in a time series; (Engle, 1982). The ARCH model is considered suitable when the variance of the process or error variance in a time series follows an autoregressive (AR) process. If the variance is accounted for by autoregressive moving average (ARMA) process, the model is generalised autoregressive conditional heteroskedasticity (GARCH); (Bollerslev, 1986). The GARCH model is an extension of the ARCH model which considers autoregressive and moving average components of the process variance and error respectively. Originally, Autoregressive Conditional Heteroskedasticity is written in the form ARCH (q). The "q" is the order of the model, which indicates the number of lag errors as the predictor variables of the ARCH model. With the Generalised Autoregressive Conditional Heteroskedasticity, GARCH (p,q), "p" assumes the order of the autoregressive component (the number of lags of the variance to be included as predictors), while "q" assumes the order of moving average component (the number of lags of the residual error that predict the volatility; (Gujarati and Porter, 1997).

The ARCH and GARCH are volatility models are found suitable in modelling financial and economic time series characterised by time-varying dispersions from their mean values. These are evident in modelling volatilities of some macro-economic variables such as inflation, crude oil price, exchange rate, stock exchange, consumer price index, etc; (Bollerslev, 1986). Different kinds of GARCH model have been used to fit volatility series of economic and financial data. One of the macroeconomic variables that have triggered many researchers to investigate due to its dynamic nature is inflation. Different GARCH models have been adopted to study inflation using consumer price index volatility; (Babatunde and Sani, 2012), (Ismail and Oluwasegun, 2017). Babatunde and Sani revealed that GARCH (1,1) was adequate for food CPI, while the asymmetric TGARCH (1,1) provided an appropriate paradigm for the dynamics of headline and core CPI. Other volatility models on exchange rate and Nigerian Stock Index include (Bala and Asemota, 2013) and (Yaya, 2013). All share index of Nigeria, Kenya, United State, Germany, South Africa and China have been studied, data spanning from February 14, 200 to February 14, 2013 using TGARCH and EGARCH; (Stephen et al, 2015). Also, on exchange rate and Nigerian Stock market includes (Reuben et al, 2016), (Yaya and Shittu, 2014), (Isenah et al, 2013), (Yaya et al, 2016) and (David, 2018). Foreign Direct Investment and Foreign Portfolio Investment have been examined with EGARCH model; (Philip and Adeleke, 2017). On the investigation of trading volume volatility in Nigeria's banking sector with GARCH (1,1) and BL-GARCH (1,1); (Onyeka-Ubaka et al, 2018). Comparatively, BL-GARCH (1,1) was found more suitable in fitting trading volume volatility in the banking sector.

In this paper, we consider different GARCH (p,q) models for the crude oil volatility series with the aim to identify a suitable model for estimation and forecast of the crude oil volatility series.

2.0 Materials and Methods

In this section, we consider different kinds of GARCH models with proposals of some for the crude oil volatility data.

2.1 ARCH and GARCH models

ARCH (q) model; (Engle, 1982) is given as

$$\sigma_t^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \cdots + \alpha_q\epsilon_{t-q}^2 = \sum_{i=1}^{q} \alpha_i\epsilon_{t-i}^2 \quad (1)$$

where, $\alpha_0 > 0 \; and \; \alpha_i \geq 0, i > 0$. GARCH model; (Bollerslev, 1986) is given as

$$\sigma_t^2 = \omega + \alpha_1\sigma_{t-1}^2 + \cdots + \alpha_p\sigma_{t-p}^2 + \alpha_1\epsilon_{t-1}^2 + \cdots + \alpha_q\epsilon_{t-q}^2 = \omega + \sum_{i=1}^{p}\alpha_i\sigma_{t-i}^2 + \sum_{j=1}^{q}\beta_j\epsilon_{t-j}^2 \quad (2)$$

where, σ_t^2 is conditional variance of the GARCH model, ϵ_t^2 is the squared error term. Similar to ARMA model, α_i and β_j are parameters of the lagged variance and squared error terms respectively.

The ARCH (q) model; (Engle, 1982) could also be expressed as GARCH (0,q) as a component of GARCH (p,q) model. Therefore, different orders of ARCH (0,q) and GARCH (p,q) models will be fitted for comparative performances using crude oil volatility data.

Here, we make review of the existing classes of generalised autoregressive conditional heteroskedasticity models as follows;

i. NGARCH: Nonlinear Asymetric GARCH (1,1) is a model of the form,

$$\sigma_t^2 = \omega + \alpha(\epsilon_{t-1} - \theta\sigma_{t-1})^2 + \sigma_{t-1}^2 \quad (3)$$

where $\alpha \geq 0, \beta \geq 0, \omega > 0 \; and \; \alpha(1+\theta^2) + \beta < 1$, which ensures the non-negativity and stationarity of the variance process; (Engle and Ng, 1993).

ii. IGARCH: Integrated Generalised Autoregressive Conditional Heteroskedasticity is a restricted GARCH model, where the parameters of the two components are summed up to one with a unit root, and is expressed as,

$$\sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{q}\beta_j = 1 \quad (4)$$

iii. EGARCH: Exponential Generalised Autoregressive Conditional Heteroskedasticity; (Nelson, 1991). The EGARCH (p,q) is

$$log\sigma_t^2 = \omega + \sum_{k=1}^{q}\beta_{kg}(Z_{t-k}) + \sum_{k=1}^{p}\alpha_k log\sigma_{t-k}^2 \quad (5)$$

where $g(Z_t) = \theta Z_t + \mu\langle|Z_t| - E(|Z_t|)\rangle, \sigma_t^2$ is the conditional variance, $\omega, \beta, \alpha, \theta$ and μ are coefficients. Z_t may be standard normal variable or come from a generalised error distribution. The formulation for $g(Z_t)$ allows the sign and the magnitude of Z_t to have separate effects on the volatility.

iv. QGARCH: Quadratic Generalised Autoregressive Conditional Heteroskedasticity; (Sentana, 1995) is used to model asymmetric effects of positive and negative shocks. A simple form of it is QGARCH (1,1), expressed as,

$$\sigma_t^2 = k + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \emptyset\epsilon_{t-1} \tag{6}$$

where $\epsilon_t = \sigma_t z_t$, and z_t is iid.

v. GJR-GARCH: Glosten-Jagannathan-Runkle Generalised Autoregressive Conditional Heteroskedasticity; (Glosten et al, 1993) models asymmetry in the ARCH process. The model is of the form,

$$\sigma_t^2 = k + \delta\sigma_{t-1}^2 + \alpha\sigma_{t-1}^2 + \emptyset\epsilon_{t-1}^2 I_{t-1} \tag{7}$$

where $\epsilon_t = \sigma_t z_t$, and z_t is iid, $I_{t-1} = 0$ if $\epsilon_{t-1} \geq 0$, and $I_{t-1} = 1$ if $\epsilon_{t-1} < 0$.

vi. TGARCH: Threshold Generalised Autoregressive Conditional Heteroskedasticity; (Zakoian, 1994)

$$\sigma_t = k + \delta\sigma_{t-1} + \alpha_1^+\epsilon_{t-1}^+ + \alpha_1^-\epsilon_{t-1}^- \tag{8}$$

Where $\epsilon_{t-1}^+ = \epsilon_{t-1}$ if $\epsilon_{t-1} > 0$, and $\epsilon_{t-1}^+ = 0$ if $\epsilon_{t-1} \leq 0$. Likewise, $\epsilon_{t-1}^- = \epsilon_{t-1}$ if $\epsilon_{t-1} \leq 0$, and $\epsilon_{t-1}^- = 0$ if $\epsilon_{t-1} > 0$

vii. FGARCH: Family Generalised Autoregressive Conditional Heteroskedasticity; (Hentschel, 1995) nests a variety of other symmetric and asymmetric GARCH models, including APARCH, GJR, AVGARCH, NGARCH, etc.

viii. COGARCH: Continuous-time GARCH model; (Claudia et al, 2004) has simple first order equation of the form,

$$\sigma_t^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 = \alpha_0 + \alpha_1\sigma_{t-1}^2 z_{t-1}^2 + \beta_1\sigma_{t-1}^2, \tag{9}$$

where $\epsilon_t = \sigma_t z_t$

ix. ZD-GARCH: Zero-Drift GARCH model; (Dong et al, 2018) lets the drift term $\omega = 0$ in the first order GARCH model. The model is presented thus,

$$\sigma_t^2 = \alpha_1\epsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \tag{10}$$

x. Spartial GARCH: The Spartial Generalised Autoregressive Conditional Heteroskedasticity; (Philipp et al, 2018). The spartial model is given by $\epsilon(s_i) = \sigma(s_i)z(s_i)$ and

$$\sigma(s_i)^2 = \alpha_i + \sum_{v=1}^{n} \rho\omega_{iv}\,\epsilon(s_v)^2 \tag{11}$$

where s_i denotes the i-th spartial location and ω_{iv} refers to the iv-th entry of the spartial weight matrix and $\omega_{ii} = 0$ for $i = 1, \ldots, n$.

xi. BL-GARCH: Bilinear Generalised Autoregressive Conditional Heteroskedasticity; (Storti and Vitale, 2003). The BL-GARCH is of the form,

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i\sigma_{t-i}^2 + \sum_{j=1}^{q} \beta_j\epsilon_{t-j}^2 + \sum_{k=1}^{r} \gamma_k\sigma_{t-k}\epsilon_{t-k} \tag{12}$$

where $\sigma_t^2, \epsilon_t^2, \alpha_i$ and β_j are as defined in equation "2", $\gamma_k$is the parameter of the nonlinear part of the model.

Given a time series process, Y_t, and $Y_t^* = \log of\ Y_t$, $dY_t^* = Y_t^* - Y_{t-1}^*$ and $X_t = dY_t^* - d\bar{Y}_t^*$. X_t is the return series. The square of the return series X_t^2 as the variance (σ_t^2) measures volatility of the series, (Gujarati and Porter, 2009). Engle (1982) expressed σ_t^2as the variance of the stochastic error term ϵ_t, and $\epsilon_t = \sigma_t z_t$, where $z_t \sim N(0,1)$. In this paper, we obtain variance of the original series as a measure of volatility. That is $\sigma_t^2 = E(Y_t - \mu_t)^2$. The order of the model is chosen from autocorrelation and partial autocorrelation functions.

2.2 Model specification

What is considered firstly in model specification is the lag length, and it is established in three different ways: (i) Estimate the best fitting AR (p) model, (ii) computation and plot of autocorrelation and partial autocorrelation functions and (iii) Ljung-Box Q-statistic which follows chi-square distribution with n degrees of freedom; (Engle, 1982). In this work, ACF and PACF are adopted for the choice of the lag length of the model.

The autocorrelation function of X_t^2 is

$$\rho_{y_t} = \frac{\sum_{t=1}^{n}(Y_t^2 - \mu_t)(Y_{t-1}^2 - \mu_t)}{\sum_{t=1}^{n}(Y_t^2 - \mu_t)^2} \tag{13}$$

where ρ_{x_t} is the acf, X_t^2 (σ_t^2) is the measure of volatility and its mean μ_t.

2.3 Model selection criteria

(i) Akaike Information Criterion (AIC):

$$AIC = ln\left(\frac{RSS}{n}\right) + \left(\frac{2k}{n}\right) \quad (14)$$

where, RSS = residual sum of squares, n = number of observations, k = number of parameters in the model.

(ii) Bayesian Information Criterion (BIC):

$$BIC = n \times ln\left(\frac{RSS}{n}\right) + K\{ln(n)\} \quad (15)$$

where, RSS, n and K are as defined above.

(iii) Schwarz's Information Criterion (SIC):

$$SIC = ln\left(\frac{RSS}{n}\right) + \left(\frac{k}{n}\right)\ln(n) \quad (16)$$

where, RSS, n and K are as defined above.

2.4 Error of forecast

Error of forecast is

$$\boldsymbol{\varepsilon_{t+k}} = \sigma^2_{t+k} - \hat{\sigma}^2_{t+k} \quad (17)$$

$$MSE(\boldsymbol{\varepsilon_{t+k}}) = E(\sigma^2_{t+k} - \hat{\sigma}^2_{t+k})^{\mathbf{2}} \quad (18)$$

3. 0 Results and Discussion

This section considers graphical presentation and parameter estimates of the proposed classes GARCH model. Data for the work are monthly crude oil quantity data from January 2010 to August 2019 (NNPC ASB).

3.1 Time Graph and Correlogram

Figs. 1 to 3 show the graph of non-stationary and stationary crude oil volatility data, autocorrection and partial autocorrection functions of the stationary series. The trend analysis shows that the original crude oil data are not stationary.

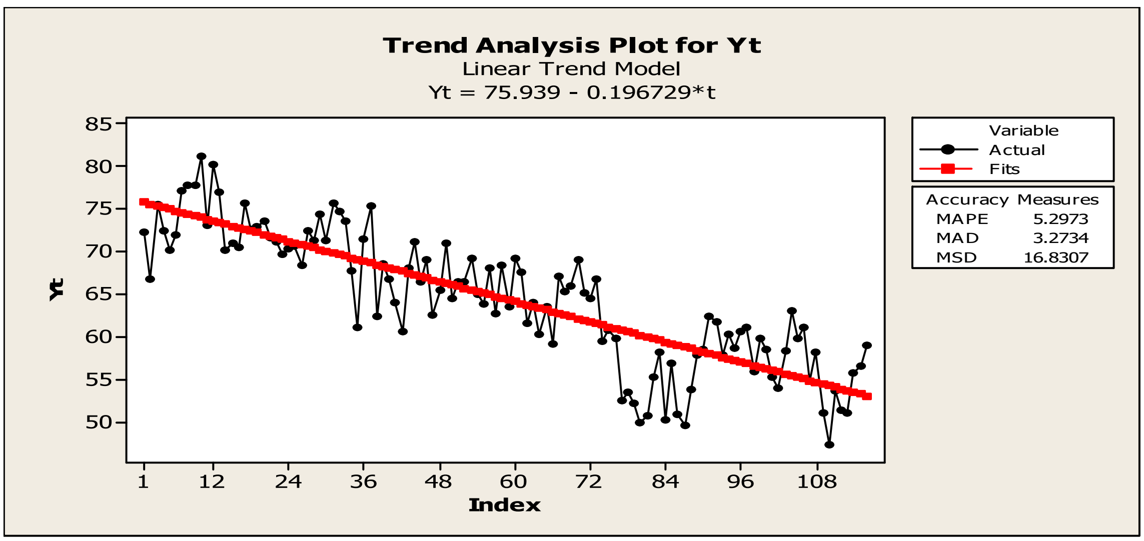

Fig. 1: Trend Analysis of the original series Y_t

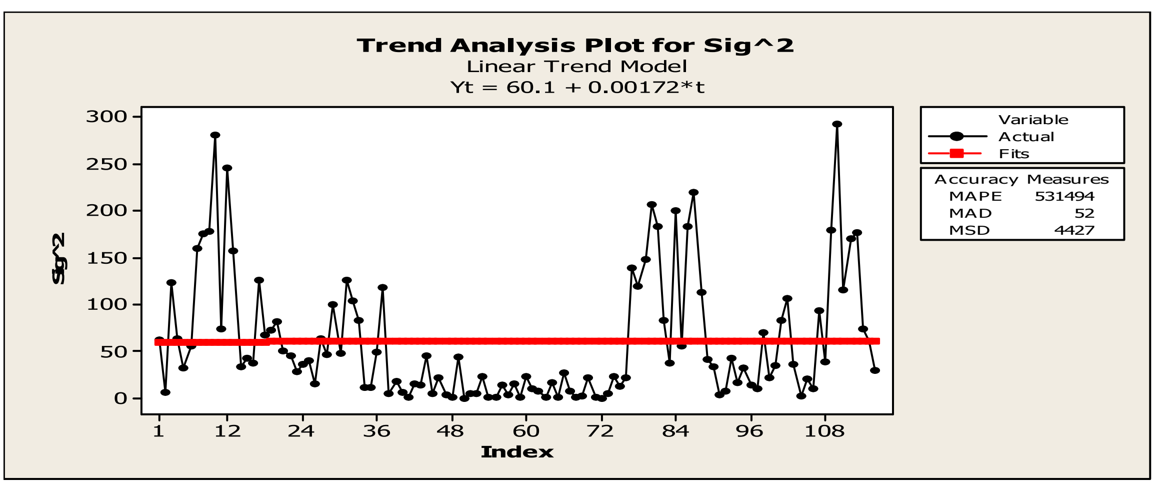

Fig. 2: Trend analysis of σ^2_t(Volatility Measure)

The above graph is the trend analysis of the stationary volatility measure of the crude oil quantity data. There is exhibition of wide swings at some periods, especially in 2010, 2012, 2016 and 2019, indicating significant variability in the series.

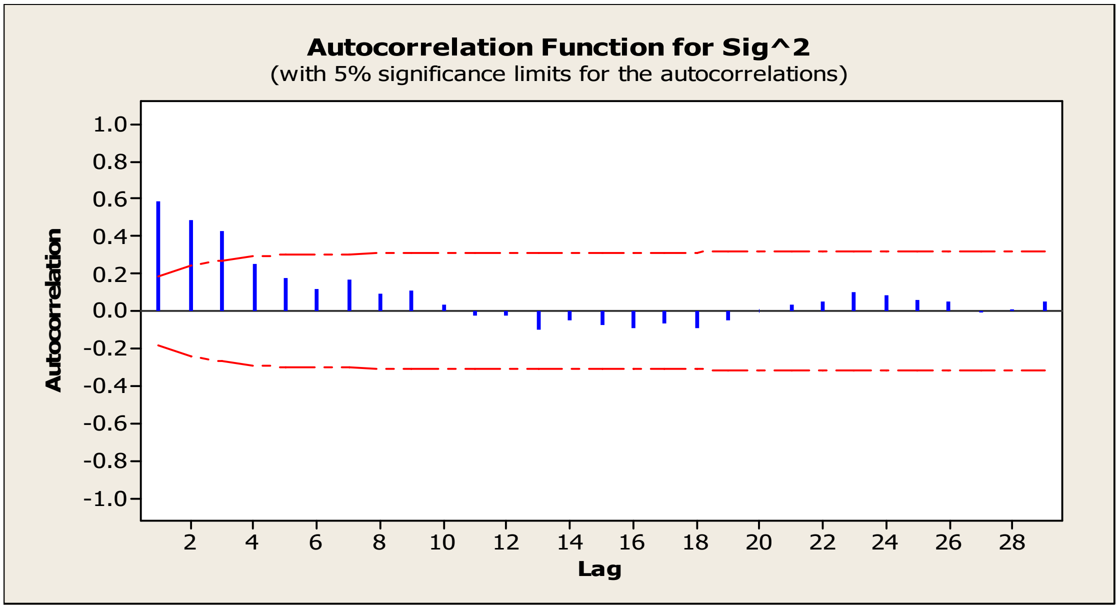

Fig. 3: Autocorrelation Function of the variance, σ_t^2(Volatility Measure)

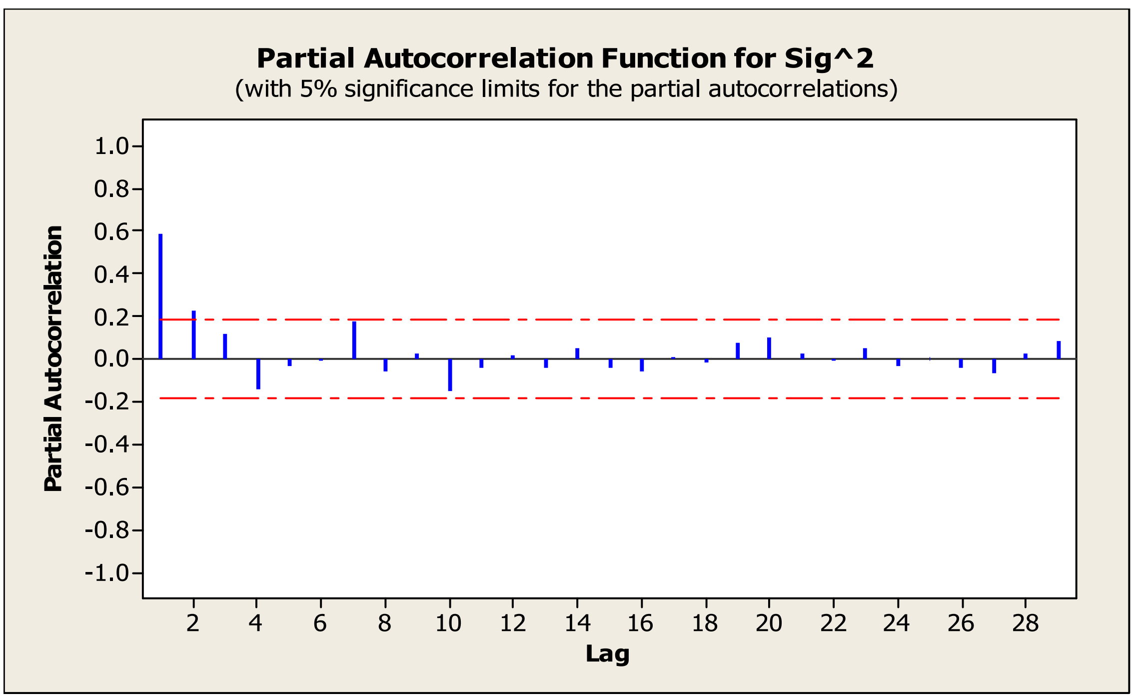

Fig. 4: Partial Autocorrelation Function of the variance, σ_t^2(Volatility Measure)

From Fig. 3 and 4, the lag length is 2, determining the order of the model. Conspicuously, there is exponential decay in the autocorrelation function and significant cut-off at the first 2 lags in the partial autocorrelation function. Hence, GARCH (0, 1), GARCH (0,2), GARCH (1,1), GARCH (1,2), GARCH (2,1) and GARCH (2,2) are suggested for the crude oil volatility series.

3.2 Model specification

(i) GARCH (p,q) Model

Given the GARCH model

$$\sigma_t^2 = \omega + \sum_{i=1}^{2} \alpha_i \sigma_{t-i}^2 + \sum_{i=1}^{2} \beta_j \epsilon_{t-j}^2 \qquad 19)$$

where σ_t^2 is the variance measure, α_i and β are the parameters of the lagged variance and squared error respectively, $\epsilon_t \sim N(0, \sigma_{\epsilon_t}^2)$, special orders of p and q are considered for estimation of parameters.

Parameter Estimates different GARCH (p,q) Model

Parameter estimates for the alternative GARCH model are presented below (i.e Table 1)

Table 1: Estimates with regression model using MINITAB

Predictor	Coeff	SE. Coeff	T	P
GARCH (0,1)				
Constant	45.647	6.575	6.94	0.000
ϵ_{t-1}^2	0.005229	0.001133	4.61	0.000
GARCH (0,2)				
Constant	38.254	6.656	5.75	0.000
ϵ_{t-1}^2	0.003861	0.001141	3.38	0.001
ϵ_{t-2}^2	0.004138	0.001140	3.63	0.000
GARCH (1,1)				
Constant	24.684	6.957	3.55	0.001
σ_{t-1}^2	0.6080	0.1094	5.56	0.000
ϵ_{t-1}^2	-0.000450	0.001435	-0.31	0.754
GARCH (1,2)				
Constant	22.295	6.876	3.24	0.002
σ_{t-1}^2	0.5409	0.1103	4.90	0.000
ϵ_{t-1}^2	-0.000776	0.001404	-0.55	0.582
ϵ_{t-2}^2	0.002806	0.001073	2.62	0.010
GARCH (2,1)				
Constant	19.524	7.198	2.71	0.008
σ_{t-1}^2	0.4750	0.1215	3.91	0.000
σ_{t-2}^2	0.22486	0.09262	2.43	0.017
ϵ_{t-1}^2	-0.000415	0.001409	-0.29	0.769
GARCH (2,2)				
Constant	20.256	7.191	2.82	0.006
σ_{t-1}^2	0.4912	0.1216	4.04	0.000
σ_{t-2}^2	0.1179	0.1213	0.97	0.333
ϵ_{t-1}^2	-0.000632	0.001413	-0.45	0.656
ϵ_{t-2}^2	0.001916	0.001410	1.36	0.177

3.3 Model Selection

This section presents the criteria for the selection of the more suitable model for forecast. The information is presented in Table 2.

From Table 2, the two competitive models selected for further comparison using error of forecast are GARCH (1,2) and GARCH (2,1).

Table2: Information criteria

S/N	Model Specification	AIC	BIC	SIC
1	GARCH (0,1)	8.2490	951.3904	8.2730
2	GARCH (0,2)	8.1595	935.6597	8.2075
3	GARCH (1,1)	8.0226	928.0937	8.0704
4	GARCH (1,2)	7.9792	917.841	8.0512
5	GARCH (2,1)	7.9875	918.7872	8.0512
6	GARCH (2,2)	7.9881	921.5854	8.0841

3.4 Forecast function and error of forecast

3.4.1: The estimated GARCH (1,2) model is

$$\hat{\sigma}_t^2 = 22.3 + 0.541\sigma_{t-1}^2 - 0.00078\epsilon_{t-1}^2 + 0.00281\epsilon_{t-2}^2 \qquad (20)$$

3.4.2: The estimated GARCH (2,1) model is

$$\hat{\sigma}_t^2 = 19.5 + 0.475\sigma_{t-1}^2 + 0.225\sigma_{t-2}^2 - 0.00041\epsilon_{t-1}^2 \qquad (21)$$

3.4.3: Forecast function for GARCH (1,2) is

$$\hat{\sigma}_{t+k}^2 = 22.3 + 0.541\sigma_{t-1+k}^2 - 0.00078\epsilon_{t-1+k}^2 + 0.00281\epsilon_{t-2+k}^2 \qquad (22)$$

3.4.4: Forecast function for GARCH (2,1) is

$$\hat{\sigma}_{t+k}^2 = 19.5 + 0.475\sigma_{t-1+k}^2 + 0.225\sigma_{t-2+k}^2 - 0.00041\epsilon_{t-1+k}^2 \qquad (23)$$

3.4.5: Error of Forecast

Error of forecast is

$$\boldsymbol{\varepsilon_{t+k}} = \sigma_{t+k}^2 - \hat{\sigma}_{t+k}^2 \qquad (24)$$

$$MSE(\boldsymbol{\varepsilon_{t+k}}) = E(\sigma_{t+k}^2 - \hat{\sigma}_{t+k}^2)^{\mathbf{2}} \qquad \mathbf{(}25)$$

$MSE(\varepsilon_{t+k})$ of GARCH (1,2) model is 5392

$MSE(\varepsilon_{t+k})$ of GARCH (2,1) model is 5369.

The analysis and estimates of the parameters of the six suggested models are in Table 1. The "t" and "p" values of the coefficients have revealed the parameters that are significant and those that are not. From the results, all the parameters of GARCH (0, 1) and GARCH (2,2) are significant. The contributions of ϵ_{t-1}^2 to GARCH (1,1), GARCH (1,2) and GARCH (2,1) and σ_{t-2}^2, ϵ_{t-1}^2

and ϵ_{t-2}^{2} to GARCH (2,2) are not significant as evident in the t and p values of the parameter estimates. With the outcome of the parameter estimates of the suggested models, different model selection criteria are used to detect the best of the GARCH models. The Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Schwarz's Information Criterion (SIC) adopted have revealed two competitive models, and these are GARCH (1,2) and GARCH (2,1). GARCH (1,2) model has 0.0083 and 0.9462 AIC and BIC values less than GARCH(2,1). The AIC and BIC values of the two models places GARCH (1,2) model superior to GARCH (2,1) model in modelling the crude oil production volatility data. Looking at the mean square error of forecast from the two models, GARCH (2,1) has a value 23 mean square error less than GARCH (1,2), placing the model on a higher comparative advantage in forecast than the other. The two models GARCH (1,2) and GARCH (2,1) are comparatively good, but the latter is recommended for better forecast of the Nigerian crude oil production volatility series. Further findings may reveal other alternative models as the crude oil production pattern changes in the future.

4.0 Conclusion

Incontrovertibly, crude oil production quantity has triggered serious concern as much as price, given the fact that the two economic variables constitute a determinant factor to the amount of revenue derived from oil sector in any oil producing nation. In as much as crude oil production quantity has significant effect on the revenue, routine research investigations on the production quantity are advocated. Due to certain political, economic, social, insecurity factors, crude production sometimes dwindles in quantity, therefore, accounts for volatility in the production quantity. This explains the need to develop series of time series models to capture the asymmetry of the crude oil production pattern. The popular GARCH (p,q) model has been explored with different orders of "p" and "q", producing alternative GARCH models for the crude o107-196il production volatility series. Information criteria adopted have placed GARCH (1,2) and GARCH (2,1) models on comparative advantage. The superiority of GARCH (2,1) over GARCH (1,2) is on the mean square error of forecast. Notwithstanding the findings in this study, further researches about crude oil production quantity volatility and its effect on the nation's economy are very pertinent.

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Appendix 1: Nigeria monthly crude oil production in millions of barrels

	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
JAN	72.29	77	70.71	75.3	71.05	67.63	66.71	56.95	61.19	51.03
FEB	66.78	70.23	68.38	62.36	64.5	61.68	59.58	50.9	56.03	47.32
MARCH	75.57	70.93	72.4	68.56	66.48	64.04	60.87	49.57	59.75	53.69
APRIL	72.42	70.49	71.28	66.82	66.48	60.39	59.8	53.79	58.56	51.39
MAY	70.15	75.66	74.43	64.01	69.25	63.5	52.63	57.96	55.35	51.12
JUNE	71.92	72.63	71.3	60.56	65.06	59.19	53.49	58.6	54.08	55.86
JULY	77.07	72.94	75.66	68.07	63.82	67.04	52.26	62.46	58.42	56.64
AUG	77.7	73.49	74.65	71.12	68.1	65.39	50.04	61.82	63.05	59.06
SEP	77.81	71.56	73.56	66.52	62.69	65.93	50.86	57.92	59.86	
OCT	81.2	71.18	67.78	69.08	68.32	69.08	55.3	60.34	61.2	
NOV	73	69.73	61.17	62.65	63.6	65.14	58.28	58.75	54.76	
DEC	80.13	70.41	71.45	65.43	69.21	64.44	50.25	60.66	58.2	