Comparing the Performance of Alternative Generalised Autoregressive Conditional Heteroskedasticity Models in Modelling Nigeria Crude Oil Production Volatility Series

*A. E. Usoro, C. E. Awakessien and C. O. Omekara

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Abstract There is no gainsaying the fact that crude oil production remains a major factor to the Nigeria economic growth given its significant contribution to the nation's gross domestic product. Preponderance of the researches in the oil sector dwell more on oil prices, with less focus on 1.0 the volatility of crude oil production. What cannot be overemphasized in oil sector is the production volatility effect which is mostly caused by unstable production quantity due to certain nation's economic, social, political factors. In this paper, volatility of crude oil production was Autoregressive Conditional Heteroskedasticity (GARCH) models were fitted to Nigeria crude oil production volatility series. Data for the work were monthly crude oil quantity data from January 2010 to August 2019 (NNPC ASB) from which the crude oil production volatility was measured. The suggested GARCH models included GARCH (0,1), GARCH (0,2), GARCH (1,1), GARCH $(1,2)$, GARCH $(2,1)$ and GARCH $(2,2)$. Using Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Schwarz's Information Criterion (SIC), GARCH (1,2) and GARCH(2,1) competed favourably. The MSE of forecast revealed GARCH (2,1) to perform better for the forecast of crude oil production volatility. Further findings will reveal other alternative models as the crude oil production pattern changes in the future.

Key Words: $GARCH$ (p, q), $GARCH$ (o, q), conditional volatility measure.

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Introduction

considered, and different Generalised time series; (Box and Jenkins, 1976). The problem In univariate time series, a time dependent variable, say X_t may be characterised by linear, nonlinear or mixed process. Autoregresive (AR), Moving Average (MA) and Autoregressive Moving Average (ARMA) models are popular and commonly used in fitting stationary linear in the applications of AR, MA and ARMA is that they are not appropriate models for time series process described by large changes in variance at various time periods; (Franses, 1998). This is because the aforementioned models account for linear processes and cannot capture high variance properties of a series indexed at time t. The consistent change in variance over time may be increasing or decreasing, and this systematic change is called heteroskedasticity or volatility. Autoregressive Conditional Heteroskedasticity (ARCH) is a method that explicitly models the change in variance over time in a time series; (Engle, 1982). The ARCH model is considered suitable when the variance of the process or error variance in a time series follows an autoregressive (AR) process. If the variance is accounted for by autoregressive moving average (ARMA) process, the model is generalised autoregressive heteroskedasticity (GARCH); (Bollerslev, 1986). The GARCH model is an extension of the ARCH model which considers autoregressive and moving average components of the process variance and error respectively. Autoregressive Conditional Heteroskedasticity is written in the form ARCH (q). The "q" is the order of the model, which indicates the number of lag errors as the predictor variables of the ARCH model. With the Autoregressive Conditional Heteroskedasticity, GARCH (p,q), "p" assumes

the order of the autoregressive component (the number of lags of the variance to be included as predictors), while "q" assumes the order of moving average component (the number of lags of the residual error that predict the volatility; (Gujarati and Porter, 1997).

The ARCH and GARCH are volatility models are found suitable in modelling financial and economic time series characterised by timevarying dispersions from their mean values. These are evident in modelling volatilities of some macro-economic variables such as inflation, crude oil price, exchange rate, stock exchange, consumer price index, etc; (Bollerslev, 1986). Different kinds of GARCH model have been used to fit volatility series of economic and financial data. One of the macroeconomic variables that have triggered many researchers to investigate due to its dynamic nature is inflation. Different GARCH models have been adopted to study inflation using consumer price index volatility; (Babatunde and Sani, 2012), (Ismail and Oluwasegun, 2017). Babatunde and Sani revealed that GARCH (1,1) was adequate for food CPI, while the asymmetric TGARCH (1,1) provided an appropriate paradigm for the dynamics of headline and core CPI. Other volatility models on 2.1 exchange rate and Nigerian Stock Index include nominum time series characterised by time-
(isenan ef al, 2015), (Yaya et al, 2019), the comparison from the evident in modelling volatilities of some Portfolio Investment have been examples evident in modelling volatilit senes characterised by time-

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modelling vola are evident in modeling Volatilities of some Pertroico Investment have been examined with the construction of the investigation of training volatiles pure and some parts, such as inflation, crude EGARCII model; (Philip an active consider and smitted in the EVARCH mode: $\sinh(\theta)$ and Active EVARCH modes.

The method of CARCH model have been used BL-GARCH (1,1); (Onyeka-Ubadi

ifferent kinds of CARCH model have been used BL-GARCH (1,1); (Onye is variables such as miniato, crude EXACH models; (Finip and Aneclese, 2017). On

the index, etc; (Bollerslev, 1986). Nigeria's banking sector with GARCH (1,1) and

of GARCH model have been used BL-GARCH (1,1); (Onyeka-Ub as mination, crude EUARCH model; (Trining and Actecke, 2017). On
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delinate been used BL

 $\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \dots + \alpha_q \epsilon_{t-q}^2 =$ $2 \left(\frac{1}{2} \right)$ $i=1$

where, σ_t^2 is conditional variance of the GARCH i. NGARCH: No model, ϵ_t^2 is the squared error term. Similar to is a model of the ARMA model, α_i and β_j are parameters of the $\sigma_t^2 = \omega + \alpha(\epsilon_t)$. lagged variance and squared error terms where $\alpha \ge 0, \beta \ge 0, \omega > 0$ and $\alpha(1 + \theta^2)$ + respectively. $\beta < 1$, which ensures the non-negativity and respectively. cchange rate and Nigerian Stock Index include
 RCH (q) model; (Engle, 1982) is g
 $t_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 = \sum_{k=1}^q \alpha_i \epsilon_{t-k}^2$

here, $\alpha_0 > 0$ and $\alpha_i \ge 0, i > 0$. GARCH model; (Bollerslev, 1986) is giv

The ARCH (q) model; (Engle, 1982) could also be expressed as GARCH (0,q) as a component of GARCH (p,q) model. Therefore, different orders ii. IGARCH: of ARCH (0,q) and GARCH (p,q) models will be fitted for comparative performances using crude oil volatility data.

Here, we make review of the existing classes of generalised autoregressive conditional heteroskedasticity models as follows;

$$
\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j = 1
$$
 (4)

(Bala and Asemota, 2013) and (Yaya, 2013). All share index of Nigeria, Kenya, United State, Germany, South Africa and China have been studied, data spanning from February 14, 200 to February 14, 2013 using TGARCH and EGARCH; (Stephen et al, 2015). Also, on exchange rate and Nigerian Stock market includes (Reuben et al, 2016), (Yaya and Shittu, 2014), (Isenah et al, 2013), (Yaya et al, 2016) and (David, 2018). Foreign Direct Investment and Foreign Portfolio Investment have been examined with EGARCH model; (Philip and Adeleke, 2017). On the investigation of trading volume volatility in Nigeria's banking sector with GARCH (1,1) and BL-GARCH (1,1); (Onyeka-Ubaka et al, 2018). Comparatively, BL-GARCH (1,1) was found more suitable in fitting trading volume volatility in the banking sector. tersea by time-
(isenan et al., 2013), (Yaya et al., 2019) and (1240a,
nean values. These 2018). Foreign Direct Investment and Foreign
atilities of some Portfolio Investment have been examined with
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m, erude EVARCH (1), (Onycka-Ubaka-Ual, the investigation of trading volume volatility in

1986). Nigeria's banking sector with GARCH (1,1) and

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given it is banking sector with GARCH (1,1) and

defined in the sector with GARCH (1,1) and

reparatively, BL-GARCH (1,1) (was found

In this paper, we consider different GARCH (p,q) models for the crude oil volatility series with the aim to identify a suitable model for estimation and forecast of the crude oil volatility series.

Materials and Methods

In this section, we consider different kinds of GARCH models with proposals of some for the crude oil volatility data.

ARCH and **GARCH** models

ARCH (q) model; (Engle, 1982) is given as $q \hspace{1.5cm}$

$$
(1) \quad
$$

 $\sigma_t^2 = \omega + \alpha_1 \sigma_{t-1}^2 + \cdots + \alpha_p \sigma_{t-p}^2 + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 = \omega + \sum_{i=1}^p \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^q \beta_j \epsilon_{t-j}^2$ (2) i. NGARCH: Nonlinear Asymetric GARCH (1,1) is a model of the form,

 $\sigma_t^2 = \omega + \alpha (\epsilon_{t-1} - \theta \sigma_{t-1})^2 + \sigma_{t-1}^2$ orient (*x_i*, to conserve the bank of the simulatively, BL-GARCH (1,1) was found
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odels for the crude oil volatility series $\binom{2}{3}$ (3) where we consider in fitting trading volume volatility
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How the consider different GARCH (p,q)
models for the crude oil volatility series with the $) +$ in the banking sector.

In this paper, we consider different GARCH (p,q)

models for the crude oil volatility series with the

models for the crude oil volatility series.

2.0 Materials and Methods

1. B. section, we cons stationarity of the variance process; (Engle and Ng, 1993). GARCH model; (Bollerslev, 1986) is given as
 $\alpha_1 \epsilon_{\ell-1}^2 + \cdots + \alpha_q \epsilon_{\ell-q}^2 = \omega + \sum_{i=1}^p \beta_i \epsilon_{i-1}^2$ (2)
 $\alpha_1 \epsilon_{\ell-1}^2 + \cdots + \alpha_q \epsilon_{\ell-q}^2 = \omega + \sum_{i=1}^p \beta_i \epsilon_{i-1}^2$ (3)

of the GARCH i. NGARCH: Nonlinear Asymetric GARC

Integrated Generalised Autoregressive Conditional Heteroskedasticity is a restricted GARCH model, where the parameters of the two components are summed up to one with a unit root, and is expressed as,

iii. EGARCH: Exponential Generalised Autoregressive Conditional Heteroskedasticity; (Nelson, 1991). The EGARCH (p,q) is

range rate and Nigerian Stock Index include ARCH (q) model; (Engle, 1982) is given as

\n
$$
= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 = \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2
$$
\n(1)

\nre, $\alpha_0 > 0$ and $\alpha_i \geq 0, i > 0$. GARCH model; (Bollerslev, 1986) is given as

\n
$$
= \omega + \alpha_1 \sigma_{t-1}^2 + \cdots + \alpha_p \sigma_{t-p}^2 + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 = \omega + \sum_{i=1}^p \alpha_i \sigma_{t-i}^2 + \sum_{i=1}^q \beta_j \epsilon_{t-j}^2
$$
\n(2)

\nre, σ_t^2 is conditional variance of the GARCH

\ni. NGARCH: Nonlinear Asymetric GARCH (1, 1, 1)el, ϵ_t^2 is the squared error term. Similar to is a model of the form,

\nMA model, α_i and β_j are parameters of the $\sigma_t^2 = \omega + \alpha(\epsilon_{t-1} - \theta \sigma_{t-1})^2 + \sigma_{t-1}^2$

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where $g(Z_t) = \theta Z_t + \mu \langle |Z_t| - E(|Z_t|) \rangle, \sigma_t^2$ is $\sigma_t^2 = k + \delta \sigma_{t-}^2$ **cation in Physical Sciences 2019, 4(2): 87-94**
 $) = \theta Z_t + \mu(|Z_t| - E(|Z_t|))$, σ_t^2 is $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \Phi \epsilon_{t-1}^2 l_{t-1}$ (7)

hal variance, $\omega, \beta, \alpha, \theta$ and μ are where $\epsilon_t = \sigma_t z_t$, and z_t is iid, $l_{$ **Communication in Physical Sciences 2019, 4(2): 87-94** http://journalcps.com/

where $g(Z_t) = \theta Z_t + \mu(|Z_t| - E(|Z_t|)), \sigma_t^2$ is $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \Phi \epsilon_{t-1}^2 I_{t-1}$ (7)

the conditional variance, $\omega, \beta, \alpha, \theta$ and coefficients. Z_t may be standard normal variable 0 if $\epsilon_{t-1} \ge 0$, as or come from a generalised error distribution. The vi. TGARCH: formulation for $g(Z_t)$ allows the sign and the Autoregressive magnitude of Z_t to have separate effects on the (Zakoian, 1994) volatility. **Communication in Physical Sciences 2019, 4(2): 87-94** http://iour

here $g(Z_t) = \theta Z_t + \mu(|Z_t| - E(|Z_t|)), \sigma_t^2$ is $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \vartheta \epsilon_{t-1}^2$

eefficients. Z_t may be standard normal variable 0 if $\epsilon_{t-1} \$ **Communication in Physical Sciences 2019, 4(2): 87-94**

where $g(Z_t) = \theta Z_t + \mu(|Z_t| - E(|Z_t|))) \sigma_t^2$ is $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \theta \epsilon_{t-1}^2 t_t$

the conditional variance, $\omega, \beta, \alpha, \theta$ and μ are where $\epsilon_t = \sigma_t z_t$, and $g(Z_t) = \theta Z_t + \mu(|Z_t| - E(|Z_t|)), \sigma_t^2$ is $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \sigma \sigma_{t-1}^2 + \theta \epsilon_{t-1}^2 l_{t-1}$

anditional variance, ω, β, α, θ and μ are where $\epsilon_t = \sigma_t z_t$, and z_t is iid, $l_{t-1} =$

emst. Z_t may be standard normal variable 0 where $g(Z_t) = \theta Z_t + \mu(|Z_t| - E(|Z_t|))$, σ_t^2 is $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \phi \epsilon_{t-1}^2 L_t$
the coefficiental variance, $\omega_t \beta$, $\alpha_t \theta$ and μ are where $\epsilon_t = \sigma_z z_t$, and z_t is iid, $|t_{t-1} = 1$ if $\epsilon_{t-1} \leq 0$, and

Autoregressive Conditional Heteroskedasticity; $\epsilon_{t-1} \le 0$. Likewise, $\epsilon_{t-1} = \epsilon_{t-1}$ if $\epsilon_{t-1} \le 0$, and (Sentana, 1995) is used to model asymmetric $\epsilon_{t-1} = 0$ if $\epsilon_{t-1} > 0$ effects of positive and negative shocks. A simple vii. FGARCH: form of it is QGARCH (1,1), expressed as,

$$
\sigma_t^2 = k + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \varnothing \epsilon_{t-1}
$$
\n
$$
\text{where } \epsilon_t = \sigma_t z_t, \text{ and } z_t \text{ is iid.}
$$
\n(6)

v. GJR-GARCH: Glosten-Jagannathan-Runkle Generalised Autoregressive Conditional Heteroskedasticity; (Glosten et al, 1993) models asymmetry in the ARCH process. The model is of the form,

$$
\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 z_{t-1}^2 + \beta_1 \sigma_{t-1}^2,
$$
\n(9)

ix. ZD-GARCH: Zero-Drift GARCH model; first order GARCH model. The model is presented thus,

$$
\sigma_t^2 = \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (10)
$$

$$
\sigma(s_i)^2 = \alpha_i + \sum_{v=1}^n \rho \omega_{iv} \epsilon(s_v)^2
$$

where s_i denotes the *i*-th spartial location and ω_{iv} xi. BL-GARCH: Bilinear Ge refers to the iv -th entry of the spartial weight

$$
\begin{aligned}\n\frac{2}{t} \text{ is } \sigma_t^2 &= k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \Phi \epsilon_{t-1}^2 I_{t-1} \quad (7) \\
\text{where } \epsilon_t &= \sigma_t z_t, \text{ and } z_t \text{ is } \text{iid}, I_{t-1} = \\
\text{able } \quad 0 \text{ if } \epsilon_{t-1} > 0, \text{ and } I_{t-1} &= 1 \text{ if } \epsilon_{t-1} < 0.\n\end{aligned}
$$

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 $|-E(|Z_t|)$, σ_t^2 is $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \Phi \epsilon_{t-1}^2 I_{t-1}$ (7)
 ω, β, α, θ and μ are where $\epsilon_t = \sigma_t z_t$, and z_t is iid, $I_{t-1} =$

ard normal variable 0 if $\epsilon_{t-1} \ge 0$ **noss 2019, 4(2): 87-94.** http://journalcps.com/
 $|$)), σ_t^2 is $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \varphi \epsilon_{t-1}^2 l_{t-1}$ (7)

and μ are where $\epsilon_t = \sigma_t z_t$, and z_t is iid, $l_{t-1} = 1$ variable 0 if $\epsilon_{t-1} \ge 0$, and **i. 87-94.**
 $\frac{http://journalcps.com/}{t^2} = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \varphi \epsilon_{t-1}^2 I_{t-1}$ (7)

ere $\epsilon_t = \sigma_t z_t$, and z_t is iid, $I_{t-1} =$
 $f \epsilon_{t-1} \geq 0$, and $I_{t-1} = 1$ if $\epsilon_{t-1} < 0$.

TGARCH: Threshold Generalised

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 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{1}{2}$, $\frac{1}{2}$ + $\alpha \sigma_{t-1}^2 + \phi \epsilon_{t-1}^2$ (7)
 $\frac{1}{2}$, and $\frac{1}{2}$ is iid, $I_{t-1} =$

and $I_{t-1} = 1$ if $\epsilon_{t-1} < 0$.

H: Threshold Generalised

Conditional Het **http://journalcps.com/**
 $\frac{2}{t-1} + \Phi \epsilon_{t-1}^2 l_{t-1}$ (7)
 $\frac{1}{t}$ is iid, $l_{t-1} =$
 $= 1$ if $\epsilon_{t-1} < 0$.

hreshold Generalised

onal Heteroskedasticity;
 $\frac{1}{t-1} + \alpha_1^- \epsilon_{t-1}^-$ (8) **(2): 87-94**
 $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \phi \epsilon_{t-1}^2 I_{t-1}$ (7)

where $\epsilon_t = \sigma_t z_t$, and z_t is iid, $I_{t-1} = 0$ if $\epsilon_{t-1} \ge 0$, and $I_{t-1} = 1$ if $\epsilon_{t-1} < 0$.

vi. TGARCH: Threshold Generalised

Autoregressive **http://journalcps.com/**
 $\frac{1}{1} + \alpha \sigma_{t-1}^2 + \varphi \epsilon_{t-1}^2 I_{t-1}$ (7)
 and z_t is *iid*, $I_{t-1} =$
 $\frac{1}{1}$ $I_{t-1} = 1$ if $\epsilon_{t-1} < 0$.
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Conditional Heteroskedasticity;
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 $\frac{1}{2}$ + $\Phi \epsilon_{t-1}^2$ (7)
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 ϵ_{t-1} > 0, and ϵ_{t-1}^+ = 0 if (2): 87-94
 $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \varphi \epsilon_{t-1}^2 l_{t-1}$ (7)

where $\epsilon_t = \sigma_t z_t$, and z_t is iid, $l_{t-1} = 0$ if $\epsilon_{t-1} \ge 0$, and $l_{t-1} = 1$ if $\epsilon_{t-1} < 0$.

vi. TGARCH: Threshold Generalised

Autoregressive C Threshold Generalised Autoregressive Conditional Heteroskedasticity; (Zakoian, 1994)

volatility.

iv. QGARCH: Quadratic Generalised Where $\epsilon_{t-1}^+ = \epsilon_{t-1}$ if $\epsilon_{t-1} > 0$, and $\epsilon_{t-1}^+ = 0$ if **(2): 87-94** http://journalcps.com/
 $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \vartheta \epsilon_{t-1}^2 I_{t-1}$ (7)

where $\epsilon_t = \sigma_t z_t$, and z_t is iid, $I_{t-1} = 0$ if $\epsilon_{t-1} \ge 0$, and $I_{t-1} = 1$ if $\epsilon_{t-1} < 0$.

vi. TGARCH: Threshold Gene $\sigma_t = k + \delta \sigma_{t-1} + \alpha_1^+ \epsilon_{t-1}^+ + \alpha_1^- \epsilon_{t-1}^-$ 94
 $\frac{\text{http://journalcps.com/}}{+\delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \Phi \epsilon_{t-1}^2 I_{t-1}}$ (7)
 $= \sigma_t z_t$, and z_t is iid, $I_{t-1} = 1$
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GARCH: Threshold Generalised

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 $\frac{\text{http://journals.csc.}}{\text{2t}$ any be standard normal variable 0 if $\epsilon_{t-1} \ge 0$, and $t_{t-1} = 1$ if $\epsilon_{t-1} < 0$.
 $\frac{\text{a}}$ **n Physical Sciences 2019, 4(2): 87-94** http://journalcps.com/
 $+\mu(|Z_t| - E(|Z_t|))$, σ_t^2 is $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \vartheta \epsilon_{t-1}^2 I_{t-1}$ (7)

ance, $\omega, \beta, \alpha, \theta$ and μ are where $\epsilon_t = \sigma_t \epsilon_t$, and z_t is iid, **tion in Physical Sciences 2019, 4(2): 87-94** http://journalcps
 $= \theta Z_t + \mu(|Z_t| - E(|Z_t|)), \sigma_t^2$ is $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \vartheta \epsilon_{t-1}^2 I_{t-1}$

variance, $\omega, \beta, \alpha, \theta$ and μ are where $\epsilon_t = \sigma_t z_t$, and z_t is till **(2): 87-94** http://journalcps.com/
 $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \Phi \epsilon_{t-1}^2 l_{t-1}$ (7)

where $\epsilon_t = \sigma_t z_t$, and z_t is iid, $l_{t-1} = 0$

0 if $\epsilon_{t-1} \ge 0$, and $l_{t-1} = 1$ if $\epsilon_{t-1} < 0$.

vi. TGARCH: Threshold Ge **(2): 87-94** http://journalcps.com/
 $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \Phi \epsilon_{t-1}^2 I_{t-1}$ (7)

where $\epsilon_t = \sigma_t z_t$, and z_t is iid, $l_{t-1} =$

0 if $\epsilon_{t-1} \ge 0$, and $l_{t-1} = 1$ if $\epsilon_{t-1} < 0$.

vi. TGARCH: Threshold Gene Family Generalised Autoregressive Conditional Heteroskedasticity; (Hentschel, 1995) nests a variety of other symmetric and asymmetric GARCH models, GJR, AVGARCH, NGARCH, etc. + $\mu(|Z_t| - E(|Z_t|))$, σ_t^2 is $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \sigma_{t-1}^2 + \delta \epsilon_{t-1}^2 I_{t-1}$ (7)

iance, ω, β, α, θ and μ are where $\epsilon_t = \sigma_t z_t$, and z_t is iid, $l_{t-1} =$

be standard normal variable 0 if $\epsilon_{t-1} \ge 0$, and $l_{t-1} =$ $-E(1Z_t|))$, σ_t^2 is $\sigma_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \Phi \epsilon_{t-1}^2$ (7)
 $β, α, θ$ and $μ$ are where $\epsilon_t = \sigma_t z_t$, and z_t is iid, $l_{t-1} = 1$

1 normal variable 0 if $\epsilon_{t-1} \ge 0$, and $l_{t-1} = 1$ if $\epsilon_{t-1} < 0$.

dist $σ_t^2 = k + \delta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 + \phi \epsilon_{t-1}^2 l_{t-1}$ (7)

where $\epsilon_t = \sigma_t z_\ell$, and z_t is iid, $l_{t-1} = 1$

0 if $\epsilon_{t-1} \ge 0$, and $l_{t-1} = 1$ if $\epsilon_{t-1} < 0$.

vi. TGARCH: Threshold Generalised

Autoregressive Conditiona Exertifical continuous values of α , β , α , α and α is the drift term α is the drift term of the drift term of α is α is α for α f For our graph and the relation of $y(z_t)$ and the relation of \vec{z}_{t-1} and \vec{z}_{t-1} and \vec{z}_{t-1} and \vec{z}_{t-1} and \vec{z}_{t-1} and the \vec{z}_{t-1} and the \vec{z}_{t-1} and the \vec{z}_{t-1} and to model asymmetric *yci* anows are sign and in Characteristic Characteristic to have separate effects on the (Zakoian, 1994)

1: Quadratic Generalised Where $t_{c-1}^2 = \epsilon_{t-1}$ if $\epsilon_{t-1} > 0$, and $\epsilon_{t-1}^2 = 0$ is used to model asymmetric vi.

Autoregressive Conditional Heteroskedasticity;

(Zakoian, 1994)
 $\sigma_t = k + \delta \sigma_{t-1} + \alpha_t^+ \epsilon_{t-1}^+ + \alpha_\tau^- \epsilon_{t-1}^-$ (8)

Where $\epsilon_{t-1}^+ = \epsilon_{t-1}$ if $\epsilon_{t-1} > 0$, and $\epsilon_{t-1}^+ = 0$ if $\epsilon_{t-1} \le 0$. Likewise, ϵ_{t-1}^- (9) GARCH: Quadratic Generalised Where $\epsilon_{t-1}^2 + u_t \cdot \epsilon_{t-1}$ and ϵ_{t-1}^2 and ϵ_{t-1}^2 and ϵ_{t-1}^2 and ϵ_{t-1}^2 and ϵ_{t-1}^2 and $\epsilon_{t-1}^2 \le 0$. Likewise, $\epsilon_{t-1} = \epsilon_{t-1}$ if $\epsilon_{t-1} \le 0$, of positiv

viii. COGARCH: Continuous-time GARCH model; (Claudia et al, 2004) has simple first order equation of the form,

x. Spatial GARCH: The Spatial Generalised
Autoregressive Conditional Heteroskedasticity;
(Philipp et al, 2018). The spatial model is given
by
$$
\epsilon(\epsilon) = \tau(\epsilon) \tau(\epsilon)
$$
 and

(11)

xi. BL-GARCH: Bilinear Generalised Autoregressive Conditional Heteroskedasticity; (Storti and Vitale, 2003). The BL-GARCH is of the form,

Frestes to positive and negative success. A simple "III." F4 CAKCH: " family "General's
form of it is QGARCH (1,1), expressed as, Autherford: " family "General's

$$
σ_t^2 = k + αε_t^2 - 1 + βσ_{t-1}^2 + θε_{t-1}
$$
 (6) (Hentschel, 1995) nets a variety of other
where $ε_t = σ_t z_t$, and z_t is itid. [30sten-Jagannathan-Runkle including APARCH, GIR, AVGARCH,
Generalised Autoregressive Conditional NGARCH, GIR, AVGARCH,
Generalised Autoregressive Conditional NGARCH, GIR, AVGARCH,
Generalised Autoregressive Conditional NGARCH, GIR, AVGARCH,
Geenerlised
is, X.D-GARCH: Zero-Drift GARCH model is of model; (Clautia et al, 2004) has simple first order
the form,
 $σ_t^2 = α_0 + α_1 ε_{t-1}^2 + β_1 σ_{t-1}^2 = α_0 + α_1 σ_{t-1}^2 z_{t-1}^2 + β_1 σ_{t-1}^2$ (9)
where $ε_t = σ_t z_t$
(Dong et al, 2018) lets the drift term $ω = 0$ in the Autoregressive Conditional Heteroskedasticity;
first order GARCH model. The model is presented (Philip et al, 2018). The spatial model is given
thus,
 $σ_t^2 = α_1 ε_{t-1}^2 + β_1 σ_{t-1}^2$ (10)
 $σ(s_t)^2 = α_t + \sum_{\nu=1}^n ρ ω_{\nu\nu} ε(s_{\nu})^2$ (11)
 $σ(s_t)^2 = α_t + \sum_{\nu=1}^n ρ ω_{\nu\nu} ε(s_{\nu})^2$ (11)
where s_t denotes the *i*-th spartial location and $ω_{\nu}$ xi. BL-GARCH: Bilinear Generalised
refers to the *iv*-th entry of the spartial weight Autoregressive Conditional Heteroskedasticity;
matrix, matrix and $ω_{\nu i} = 0$ for $i = 1, ..., n$. (Storti and Vitate, 2003). The BL-GARCH is of
 $α_t^2 = ω + \sum_{\nu=1}^n α_{\nu} α_{\nu-1}^2 + \sum_{\nu=1}^n β_{\nu} ε_{\$

where σ_t^2 , ϵ_t^2 , α_i and β_j are as defined in equation "2", γ_k is the parameter of the nonlinear part of the model.

Given a time series process, Y_t , and model, (i)

 $\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 z_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

where $\epsilon_t = \sigma_t z_t$

x. ED-GARCH: Alternative Mondel, and the spacial CARCH The Spartial

Dong et al. 2018) lets the drift term ω = 0 in t $Y_t^* = \log of Y_t$, $dY_t^* = Y_t^* - Y_{t-1}^*$ and $X_t =$ where $\epsilon_t = \sigma_t z_t$

where $\epsilon_t = \sigma_t z_t$

ix. ZD-GARCH: Zero-Drift GARCH model; x. Spartial GARCH: The Spart

ix. ZD-GARCH: Zero-Drift GARCH model; x. Spartial GARCH: The Spart

first order GARCH model. The model is present X_t is the return series. The square of functions and (i the return series X_t^2 as the variance (σ_t^2) measures follows chi-square volatility of the series, (Gujarati and Porter, 2009). Engle (1982) expressed σ_t^2 as the variance of the PACF are adopted stochastic error term ϵ_t , and $\epsilon_t = \sigma_t z_t$, where $z_t \sim N(0,1)$. In this paper, we obtain variance of the original series as a measure of volatility. That is $\sigma_t^2 = E(Y_t - \mu_t)^2$. The order of the model is $\sigma_t^2 = \frac{\sum_{t=1}^{n} \mu_t}{\sigma_t^2}$ $\sigma(s_i)^2 = \alpha_i + \sum_{y=1}^{\infty} \rho \omega_{iv} \epsilon(s_y)^2$ (11)

re s_i denotes the *l*-th spartial location and ω_{iv} xi. BL-GARCH: Bilinear Genes to the *iv*-th entry of the spartial weight intergal (Storti and Vitale, 2003). The BL-GAR chosen from autocorrelation and partial autocorrelation functions.

2.2 Model specification

What is considered firstly in model specification is the lag length, and it is established in three different ways: (i) Estimate the best fitting AR (p)

+ $\alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 z_{t-1}^2 + \beta_1 \sigma_{t-1}^2$. (9)
 \vdots Zero-Drift GARCH model; x. Spartial GARCH: The Spartial Generalised

8) lets the drift term $\omega = 0$ in the Autoregressive Conditional He and $X_t =$ autocorrelation and partial autocorrelation $f(t)$ measures follows chi-square distribution with n degrees of σ_{k-1}^2 (10)
 $\rho\omega_{t\nu} \epsilon(s_i) = \sigma(s_i)z(s_i)$ and
 $\rho\omega_{t\nu} \epsilon(s_{\nu})^2$ (11)

spartial location and $\omega_{i\nu}$ xi. BL-GARCH: Bilinear Generalised

spartial location and $\omega_{i\nu}$ xi. BL-GARCH: Bilinear Generalised
 $= 1,...,n$, where of the model. model, (ii) computation and plot of functions and (iii) Ljung-Box Q-statistic which freedom; (Engle, 1982). In this work, ACF and PACF are adopted for the choice of the lag length (11)

GARCH: Bilinear Generalised

sive Conditional Heteroskedasticity;

Vitale, 2003). The BL-GARCH is of

(12)

parameter of the nonlinear part of the

ii) computation and plot of

tion and partial autocorrelation

und (11)

H: Bilinear Generalised

onditional Heteroskedasticity;

2003). The BL-GARCH is of

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ter of the nonlinear part of the

omputation and plot of

and partial autocorrelation

Ljung-Box Q-statistic which

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Heteroskedasticity;

ie BL-GARCH is of

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nonlinear part of the

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ial autocorrelation

x Q-statistic which

on with n degrees of

this work, ACF and

pice of the lag length

of X CH: Bilinear Generalised
Conditional Heteroskedasticity;
e, 2003). The BL-GARCH is of
(12)
meter of the nonlinear part of the
computation and plot of
and partial autocorrelation
ii) Ljung-Box Q-statistic which
is in Ligne Bilinear Generalised

onal Heteroskedasticity;

). The BL-GARCH is of

(12)

f the nonlinear part of the

tation and plot of

partial autocorrelation

ng-Box Q-statistic which

biliotion with n degrees of

. In this work,

The autocorrelation function of X_t^2 is $2i_s$

$$
\rho_{y_t} = \frac{\sum_{t=1}^{n} (Y_t^2 - \mu_t)(Y_{t-1}^2 - \mu_t)}{\sum_{t=1}^{n} (Y_t^2 - \mu_t)^2} \tag{13}
$$

where ρ_{x_t} is the acf, X_t^2 (σ_t^2) is the measure of volatility and its mean μ_t .

2.3 Model selection criteria

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2.3 Model selection criteria

(i) Akaike Information Criterion (AIC): $AIC = ln\left(\frac{RSS}{n}\right) + \left(\frac{2k}{n}\right)$ (14)

where, RSS = residual sum of squares, n = number $MSE(\epsilon_{$ where, $RSS = residual sum of squares, n = number$ of observations, $k =$ number of parameters in the model.

(ii) Bayesian Information Criterion (BIC):

$$
BIC = n \times ln\left(\frac{RSS}{n}\right) + K\{ln(n)\} \quad (15)
$$

where, RSS, n and K are as defined above. (iii) Schwarz's Information Criterion (SIC):

$$
SIC = \ln\left(\frac{RSS}{n}\right) + \left(\frac{k}{n}\right)\ln\left(n\right) \tag{16}
$$

where, RSS, n and K are as defined above. 2.4 Error of forecast

Error of forecast is

$$
\varepsilon_{t+k} = \sigma_{t+k}^2 - \hat{\sigma}_{t+k}^2 \tag{17}
$$

$$
AIC = ln\left(\frac{RSS}{n}\right) + \left(\frac{2k}{n}\right) \tag{14}
$$

$$
MSE(\varepsilon_{t+k}) = E(\sigma_{t+k}^2 - \hat{\sigma}_{t+k}^2)^2
$$
 (18)
3. 0 Results and Discussion

Model selection criteria
 Model selection criteria $n \int_{0}^{+\infty}$ (19) crude oil quantity data from January 2010 to **thysical Sciences 2019, 4(2): 87-94** 90
 Peria

Criterion (AIC): $AIC = ln\left(\frac{RSS}{n}\right) + \left(\frac{2k}{n}\right)$ (14)

14)

140

15) among the summer $MSE(\epsilon_{t+k}) = E(\sigma_{t+k}^2 - \hat{\sigma}_{t+k}^2)^2$ (18)

16) among the **3.0 Results and Discussion**
 2): 87-94 90
 $AIC = ln\left(\frac{RSS}{n}\right) + \left(\frac{2k}{n}\right)$ (14)
 $MSE(\epsilon_{t+k}) = E(\sigma_{t+k}^2 - \hat{\sigma}_{t+k}^2)^2$ (18)
 3. 0 Results and Discussion

This section considers graphical presentation and

parameter estimates of the proposed classes

G 90
 $\left(\frac{2k}{n}\right)$ (14)
 $\left(\frac{2}{n}\right)$ (14)
 d Discussion

ers graphical presentation and

es of the proposed classes

ata for the work are monthly

data from January 2010 to

C ASB). This section considers graphical presentation and parameter estimates of the proposed classes GARCH model. Data for the work are monthly August 2019 (NNPC ASB).

3.1 Time Graph and Correlogram

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Model selection criteria

kaike Information Criterion (AIC): $\Delta I C = \ln \left(\frac{RSS}{n}\right) + \left(\frac{2k}{n}\right)$

e, RSS = residual sum of squares, n = number $MSE(\epsilon_{t+k}) = E(\sigma_{t+k}^2 - \hat{\sigma}_{$ $\left(\frac{\pi}{n}\right)$ ln (n) (16) Figs. 1 to 3 show the graph of hon-stationary and stationary crude oil volatility data, autocorrection $\binom{k}{n}$ in (n) (16) Figs. 1 to 3 show the graph of non-stationary and containing and containery and containing in the current containery of the current containing in the current containing the current containing in the and partial autocorrection functions of the stationary series. The trend analysis shows that the original crude oil data are not stationary.

Fig. 1: Trend Analysis of the original series Y_t

Fig. 2: Trend analysis of σ_t^2 (Volatility Measure)

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The above graph is the trend analysis of the stationary volatility measure of the crude oil quantity data. There is exhibition of wide swings at some periods, especially in 2010, 2012, 2016 and 2019, indicating significant variability in the series.

Fig. 3: Autocorrelation Function of the variance, σ_t^2 (Volatility Measure)

Fig. 4: Partial Autocorrelation Function of the variance, σ_t^2 (Volatility Measure)

From Fig. 3 and 4, the lag length is 2, determining the order of the model. Conspicuously, there is exponential decay in the autocorrelation function and significant cut-off at the first 2 lags in the

partial autocorrelation function. Hence, GARCH (0, 1), GARCH (0,2), GARCH (1,1), GARCH (1,2), GARCH (2,1) and GARCH (2,2) are suggested for the crude oil volatility series.

3.2 Model specification (i) GARCH (p,q) Model

Given the GARCH model

$$
\sigma_t^2 = \omega + \sum_{i=1}^2 \alpha_i \sigma_{t-i}^2 + \sum_{i=1}^2 \beta_j \epsilon_{t-j}^2
$$
 19) Table2: Information criteria

Model specification
 Model specification
 SARCH (p,q) Model
 EXARCH (p,q) Model
 where σ_t^2 is the variance measure, α_i and β are the S/Model parameters of the lagged variance and squared error respectively, $\epsilon_t \sim N(0, \sigma_{\epsilon_t}^2)$, special orders of p and q are considered for estimation of parameters.

Parameter Estimates different GARCH (p,q) Model

Parameter estimates for the alternative GARCH model are presented below (i.e Table 1)

Table 1: Estimates with regression model using MINITAB

Predictor	Coeff	SE. Coeff	T	P
GARCH (0,1)				
Constant	45.647	6.575	6.94	0.000
ϵ_{t-1}^2	0.005229	0.001133	4.61	0.000
GARCH (0,2)				
Constant	38.254	6.656	5.75	0.000
ϵ_{t-1}^2	0.003861	0.001141	3.38	0.001
ϵ_{t-2}^2	0.004138	0.001140	3.63	0.000
GARCH (1,1)				
Constant	24.684	6.957	3.55	0.001
σ_{t-1}^2	0.6080	0.1094	5.56	0.000
ϵ_{t-1}^2	-0.000450	0.001435	-0.31	0.754
GARCH (1,2)				
Constant	22.295	6.876	3.24	0.002
σ_{t-1}^2	0.5409	0.1103	4.90	0.000
ϵ_{t-1}^2	-0.000776	0.001404	-0.55	0.582
ϵ_{t-2}^2	0.002806	0.001073	2.62	0.010
GARCH(2,1)				
Constant	19.524	7.198	2.71	0.008
σ_{t-1}^2	0.4750	0.1215	3.91	0.000
σ_{t-2}^2	0.22486	0.09262	2.43	0.017
ϵ_{t-1}^2	-0.000415	0.001409	-0.29	0.769
GARCH (2,2)				
Constant	20.256	7.191	2.82	0.006
σ_{t-1}^2	0.4912	0.1216	4.04	0.000
σ_{t-2}^2	0.1179	0.1213	0.97	0.333
ϵ_{t-1}^2	-0.000632	0.001413	-0.45	0.656
ϵ_{t-2}^2	0.001916	0.001410	1.36	0.177

3.3 Model Selection

This section presents the criteria for the selection of the more suitable model for forecast. The information is presented in Table 2.

From Table 2, the two competitive models selected for further comparison using error of forecast are GARCH (1,2) and GARCH (2,1).

3.4 Forecast function and error of forecast

3.4.1: The estimated GARCH (1,2) model is

 $\hat{\sigma}_t^2 = 22.3 + 0.541 \sigma_{t-1}^2 - 0.00078$ $\frac{2}{t-1} +$ $0.00281 \epsilon_{t-2}^2$ (20)

3.4.2: The estimated GARCH (2,1) model is

$$
\hat{\sigma}_t^2 = 19.5 + 0.475\sigma_{t-1}^2 + 0.225\sigma_{t-2}^2 - 0.00041\epsilon_{t-1}^2 \tag{21}
$$

3.4.3: Forecast function for GARCH (1,2) is

$$
\hat{\sigma}_{t+k}^2 = 22.3 + 0.541 \sigma_{t-1+k}^2 -
$$

0.00078 $\epsilon_{t-1+k}^2 + 0.00281 \epsilon_{t-2+k}^2$ (22)

3.4.4: Forecast function for GARCH (2,1) is

6 GARCH (2,2) 7.9881 921.5854 8.0841
\n3.4 Forecast function and error of forecast
\n3.4.1: The estimated GARCH (1,2) model is
\n
$$
\hat{\sigma}_t^2 = 22.3 + 0.541\sigma_{t-1}^2 - 0.00078\epsilon_{t-1}^2 +
$$

\n0.00281 ϵ_{t-2}^2 (20)
\n3.4.2: The estimated GARCH (2,1) model is
\n $\hat{\sigma}_t^2 = 19.5 + 0.475\sigma_{t-1}^2 + 0.225\sigma_{t-2}^2 -$
\n0.00041 ϵ_{t-1}^2 (21)
\n3.4.3: Forecast function for GARCH (1,2) is
\n $\hat{\sigma}_{t+k}^2 = 22.3 + 0.541\sigma_{t-1+k}^2 -$
\n0.00078 $\epsilon_{t-1+k}^2 + 0.00281\epsilon_{t-2+k}^2$ (22)
\n3.4.4: Forecast function for GARCH (2,1) is
\n $\hat{\sigma}_{t+k}^2 = 19.5 + 0.475\sigma_{t-1+k}^2 + 0.225\sigma_{t-2+k}^2 -$
\n0.00041 ϵ_{t-1+k}^2 (23)
\n3.4.5: Error of Forecast
\nError of forecast is
\n $\epsilon_{t+k} = \sigma_{t,k}^2 - \hat{\sigma}_{t,k}^2$ (24)

3.4.5: Error of Forecast Error of forecast is

$$
\varepsilon_{t+k} = \sigma_{t+k}^2 - \hat{\sigma}_{t+k}^2 \tag{24}
$$

$$
MSE(\varepsilon_{t+k}) = E(\sigma_{t+k}^2 - \hat{\sigma}_{t+k}^2)^2
$$
 (25)

0.00281 ϵ_{t-2}^2 (20)

3.4.2: The estimated GARCH (2,1) model is
 $\hat{\sigma}_t^2 = 19.5 + 0.475\sigma_{t-1}^2 + 0.225\sigma_{t-2}^2 -$

0.00041 ϵ_{t-1}^2 (21)

3.4.3: Forecast function for GARCH (1,2) is
 $\hat{\sigma}_{t+k}^2 = 22.3 + 0.541\sigma_{t-1+k}$ $\vec{t} = \vec{t} - \vec$ 3.4.2: The estimated GARCH (2,1) model is
 $\hat{\sigma}_t^2 = 19.5 + 0.475\sigma_{t-1}^2 + 0.225\sigma_{t-2}^2 - 0.00041\epsilon_{t-1}^2$ (21)

3.4.3: Forecast function for GARCH (1,2) is
 $\hat{\sigma}_{t+k}^2 = 22.3 + 0.541\sigma_{t-1+k}^2$ (22)

3.4.4: Forecast fu d GARCH (2,1) model is
 $i\sigma_{t-1}^2 + 0.225\sigma_{t-2}^2 -$

(21)

tion for GARCH (1,2) is
 $41\sigma_{t-1+k}^2 -$
 $0.00281\epsilon_{t-2+k}^2$ (22)

tion for GARCH (2,1) is
 $75\sigma_{t-1+k}^2 + 0.225\sigma_{t-2+k}^2 -$

(23)

ccast

cast

(24)

k

k (24 $MSE(\varepsilon_{t+k})$ of GARCH (1,2) model is 5392 $MSE(\varepsilon_{t+k})$ of GARCH (2,1) model is 5369. The analysis and estimates of the parameters of the six suggested models are in Table 1. The "t" and "p" values of the coefficients have revealed the parameters that are significant and those that are not. From the results, all the parameters of GARCH $(0, 1)$ and GARCH $(2, 2)$ are significant. The contributions of ϵ_{t-1}^2 to GARCH (1,1), GARCH (1,2) and GARCH (2,1) and σ_{t-2}^2 , ϵ_{t-1}^2 , ϵ_{t-1}^2

and ϵ_{t-2}^2 to GARCH (2,2) are not significant as 5.0 Referen evident in the t and p values of the parameter estimates. With the outcome of the parameter estimates of the suggested models, different model selection criteria are used to detect the best of the GARCH models. The Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Schwarz's Information Criterion (SIC) adopted have revealed two competitive models, and these are GARCH (1,2) and GARCH (2,1). GARCH (1,2) model has 0.0083 and 0.9462 AIC and BIC values less than GARCH(2,1). The AIC and BIC values of the two models places GARCH (1,2) model superior to GARCH (2,1) model in modelling the crude oil production volatility data. Looking at the mean square error of forecast from the two models, GARCH (2,1) has a value 23 mean square error less than GARCH (1,2), placing the model on a higher comparative advantage in forecast than the other. The two models GARCH (1,2) and GARCH (2,1) are comparatively good, but the latter is recommended for better forecast of the Nigerian crude oil production volatility series. Further findings may reveal other alternative models as the crude oil production pattern changes in the future.

4.0 Conclusion

Incontrovertibly, crude oil production quantity has triggered serious concern as much as price, given the fact that the two economic variables constitute a determinant factor to the amount of revenue derived from oil sector in any oil producing nation. In as much as crude oil production quantity has significant effect on the revenue, routine research investigations on the production quantity are advocated. Due to certain political, economic, social, insecurity factors, crude production sometimes dwindles in quantity, therefore, accounts for volatility in the production quantity. This explains the need to develop series of time series models to capture the asymmetry of the crude oil production pattern. The popular GARCH (p,q) model has been explored with different orders of "p" and "q", producing alternative GARCH models for the crude o107- 196il production volatility series. Information criteria adopted have placed GARCH (1,2) and GARCH $(2,1)$ models on comparative advantage. The superiority of GARCH (2,1) over GARCH (1,2) is on the mean square error of forecast. Notwithstanding the findings in this study, further researches about crude oil production quantity volatility and its effect on the nation's economy are very pertinent.

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Appendix 1: Nigeria monthly crude oil production in millions of barrels

