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Mathematical Modeling of an Oscillatory MHD Blood Flow through a Lipid Saturated Porous Channel with Metabolic Heat and Magnetic Field

K.W. Bunonyo* **and I.C. Eli**

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Abstract: *This research investigates an oscillatory blood flow through a lipid saturated porous channel with metabolic heat and magnetic field. The study involves formulating mathematical model for blood momentum equation, the energy equation and the lipid concentration equation, and the coupled PDE were reduced to set of nonlinear ODE using the perturbation method. The set of ODEs are solved and the blood velocity, temperature and lipid concentration profiles were obtained, with some governing parameters. Numerical computation was carried out using Mathematica software to by varying the governing parameters within some specific range in order to the study the effects of the parameters on the flow profiles. The results revealed that the flow profiles were influenced with the varying pertinent parameters such as Prandtl number, radiation parameter, metabolic heat parameter, Hartmann number, Grashof number, solutal Grashof number, Schmidt number and the oscillatory frequency parameters respectively.*

Key Words: *Blood, Lipid, Magnetic Field, Cardiovascular system, Heat transfer, ODE and PDE.*

Kubugha Wilcox Bunonyo
Department of Mathematics and Statistics
Federal University Otuoke
Yenogoa, Nigeria
Email: wilcoxbk@fuotuoke.edu.ng
Orcid id: 0000-0002-1261-9041

Innocent C. Eli
Department of Mathematics and Statistics
Federal University Otuoke
Yenogoa, Nigeria
Email: innocenteli35@gmail.com

1.0 Introduction

Cardiovascular system is made of the heart, the blood vessel and blood. The heart is the pumping system of the human body. It pumps and circulates blood to the entire body within a short period of time and it is made up of four chambers, including the left and right ventricles and left and right atria. The left ventricle pumps oxygenated blood while the right ventricle pumps deoxygenated blood to the lungs Schmidt *et al.,* (2017).

Blood vessels are the channels through which blood flows to and from the organs and tissues of the body. There are basically different types of blood vessels, namely, the aorta, the arteries, arteriole and the micro vessels. Blood is the suspension of formed elements in an aqueous solution called plasma. The formed elements are the cells in the blood, which are made up of 45% of the total blood volume while the plasma is made up of about 55% of the blood volume, and they the red blood cells (RBC) are also known as erythrocyte, the white blood cells (WBC) are known as leukocytes and the platelets are called thrombocyte, Bunonyo and Amos (2020). The erythrocytes are the carriers of hemoglobin at the lungs and they flow through the capillary-veins, via the venules to the left atrium and finally to the left ventricles before the onward transport to rest of the body. The red blood cells (RBC) are made up of about 96%, the white blood cells (WBC) are 3% and the platelets are made up of 1% of the formed elements. The plasma fluid is the yellowish topmost fluid after it's been centrifuged, and it is made up of 90% water, 7% blood protein like fibrinogen, 0.9% inorganic salt and 2.1% others includes minerals, glucose and hormones. The leukocytes are basically anti-invaders, they fight against any unwanted substances well recognized in the body, while the platelets or thrombocytes are the injuries healers, and they help in healing and preventing bleeding if injury is sustained (Bunonyo *et al*., 2018).

Blood lipids (or blood fats) are lipids in the blood which are either free or bound to other molecules. They are mostly transported in a protein capsule, and the density of the lipids and type of protein determines the fate of the particle and its influence on metabolism. The concentration of blood lipids depends on intake and excretion from the intestine, and uptake and secretion from cells. Blood lipids are mainly fatty acids and cholesterol. Hyperlipidemia is the presence of elevated or abnormal levels of lipids and/or lipoproteins in the blood, and is a major risk factor for cardiovascular disease (Durrington, 2003).

Radiation effect in blood flow is very important subject in cardiovascular research, because it is applicable in biomedical engineering and other medical treatment procedures, particularly the thermal therapeutic procedure. For instance, infrared radiation is one of the frequently used methods for implementing heat treatment to various parts of human body. The method is preferred in heat treatment because it is possible to directly heat the capillary of the affected region in the body. Misra *et al*. (2010) theoretically derived the estimates of blood flow in arteries during the therapeutic procedure of electromagnetic hyperthermia used for cancer treatment. Misra and Shit (2007) also studied the role of slip velocity in blood flow through stenosed arteries.

In muscle treatment, spams, myalgia (muscle pain), chronic wide-spread pain (fibromyalgia, in medical terms) and permanent shortening of muscle (called, contracture), heat therapy is found to be very effective. It is also used in the treatment of bursitis, that is, inflammation of the fluid-filled sac (bursa) that lies between tendon and bone, or between tendon and skin. There are several other studies on heat therapy such as those reported by Kobu (1999), Inoue and Kabaya (1989) and Nishimoto *et al*. (2006) who independently examined the effects of infrared radiation/ultrasonic radiation on blood flow. The effect of radiative heat transfer on blood flow in a stenosed artery was also studied theoretically by Prakash and Makinde (2011). By using a numerical model, He *et al.* (2006) reported the effect of temperature on blood flow in human breast tumor under laser irradiation. Effects of pulsatile blood flow in arteries during thermal therapy were studied by Craciunescu and Clegg (2011) and by Horng *et al*. (2007). Shrivastava and Roemer (2005) studied heat transfer rate from one blood vessel to another. Blood flow through radiative heat transfer has also been reported by Ogulu and Bestman (1994) using theoretical modelling. On Extended work reported by Shah *et al*. (2005) was achieved through the design of an instrument for measuring the heat convection coefficient on the endothelial surface of arteries.

Study on the effect of radiation on human blood is of significant interest to scientists and clinicians who employed therapeutic procedure of hyperthermia that has a well-recognized effect in oncology. Its effect is achieved by overheating the cancerous tissues by means of electromagnetic radiation (Szasz, 2007).

In view of the vast research need in modelling blood flow in human system, this study is aimed at investigating an oscillatory blood flow through a lipid saturated porous channel with metabolic heat in the presence of external perpendicular magnetic field. The aim shall be achieved by formulating formulate a system of governing equation and incorporate into the body force of the Navier-Stoke momentum equation governing general flow and also formulate energy and lipid concentration equations with the radiation and metabolic source terms. The governing dimensional equations shall be made dimensionless using the non-dimensional variables, with the biophysical parameters such as $Pr, Ha, Sc, Rd_1, Rd_2 Da, Gr, Gc$ and ω obtaine d, and the dimensionless equation are reduced to ODE using the perturbation method and the pulsatile blood velocity, temperature and lipid concentration profiles were obtained. In order to study the effect of the aforementioned physical parameters on blood velocity, blood temperature and lipid concentration in the blood we carry out numerical computation using Mathematica. The simulated results were presented graphically showcasing each parameter effect.

2.0 Mathematical Formulation

In initiating the primary formulation, human blood was considered as a viscous mixture of formed elements and cholesterol, electrically conducting and Newtonian fluid. Consequently, the–flow is assumed to take place in a channel with a distance R_0 length l_0. Also, channel is filled with concentration of cholesterol with some levels of permeability that allows the flow in the axial direction. We also assumed that the flow takes place in the presence of external magnetic field $\vec{B}$ and it is driven by heat source Q_0 and absorption arising from metabolism Q_1 and the flow is unidirectional owing to the valves in the channel with a velocity $\vec{w}^* = \left(0, 0, w^*\right)$.

Through the above assumptions, the following governing equation was derived:

Continuity Equation

$$\frac{\partial w^*}{\partial x^*} + \frac{\partial u^*}{\partial y^*} = 0 \qquad (1)$$

Momentum Equation

$$\rho_b \frac{\partial w^*}{\partial t^*} = -\frac{\partial P^*}{\partial x^*} + \mu_b \frac{\partial^2 w^*}{\partial y^{*2}} - \sigma_e B_0^2 w^* - \frac{\mu_b \varphi}{k^*} w^* + \rho_b g \beta_T \left(T^* - T_\infty\right) + \rho_b g \beta_C \left(C^* - C_\infty\right) \qquad (2)$$

Energy Equation

$$\rho_b c_p \frac{\partial T^*}{\partial t^*} = k_T \frac{\partial^2 T^*}{\partial y^{*2}} + Q_0 \left(T^* - T_\infty\right) + Q_1 \left(C^* - C_\infty\right) \qquad (3)$$

Lipid Concentration Equation

$$\frac{\partial C^*}{\partial t^*} = D_m \frac{\partial^2 C^*}{\partial y^{*2}} - k_0 \left(C^* - C_\infty\right) \qquad (4)$$

Therefore, the corresponding boundary conditions are presented as equation (5)

$$\left.\begin{aligned} w^* = 0, T^* = T_\infty, C^* = C_\infty \quad \text{at } y^* = 0 \\ w^* = 0, T^* = T_w, C^* = C_w \quad \text{at } y^* = R \end{aligned}\right\} \qquad (5)$$

where k_T is the thermal conductivity of blood, D_m is the molecular diffusivity, T_∞ is the surrounding temperature of blood, T^* is blood temperature, T_w^* is blood temperature at wall, ρ_b is the density of blood, σ_e is electrical conducting, B_0 is the magnetic induction, c_{bp} is the specific heat capacity of blood, β_T is the volumetric expansion, β_C is the volumetric expansion due to concentration, μ_b is the dynamic viscosity of blood, φ is the porosity, k^* is the permeability of the porous medium.

Consideration of the dimensional equation expressed in equation (6), led to the simplification of equations (2) to (5) into equations (7) to (9)

$$\left.\begin{aligned} x = \frac{x^*}{l_0}, y = \frac{y^*}{R_0}, w = \frac{w^* R_0}{\upsilon}, t = \frac{\upsilon t^*}{R_0^2}, Rd_1 = \frac{Q_1 R_0^2}{k_T} \frac{\left(C^* - C_\infty\right)}{\left(T_w - T_\infty\right)}, \\ \phi = \frac{C^* - C_\infty}{C_w - C_\infty}, \theta = \frac{T^* - T_\infty}{T_w - T_\infty}, P = \frac{P^* R_0^3}{l_0 \upsilon \mu_b}, Rd_2 = \frac{Q_0 R_0^2}{k_T} \end{aligned}\right\} \qquad (6)$$

$$\frac{\partial w}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 w}{\partial y^2} + Gr\theta + Gc\phi - M^2 w - \frac{w}{Da} \qquad (7)$$

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + Rd_2 \theta + Rd_1 Pr \phi \qquad (8)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - Rd_3 \phi \qquad (9)$$

The derived models were subjected to the boundary conditions expressed in equation (10),

$$\left.\begin{aligned} w = 0,\ \phi = 0, \theta = 0 \quad \text{at } y = 0 \\ w = 0,\ \phi = 1, \theta = 1 \quad \text{at } y = 1 \end{aligned}\right\} \qquad (10)$$

where:

$$\left.\begin{aligned} Ha = B_0 R_0 \sqrt{\frac{\sigma_e}{\mu_b}}, Da = \frac{k^*}{R_0^2}, Gc = \frac{g \beta_C \left(C_w - C_\infty\right) R_0^3}{\upsilon^2}, \\ Sc = \frac{\upsilon}{D_m}, Gr = \frac{g \beta_T \left(T_w - T_\infty\right) R_0^3}{\upsilon^2}, Pr = \frac{\mu_b c_p}{k_T} \end{aligned}\right\} \qquad (11)$$

3.0 Results and Discussion

3.1 Method of Solution

Since the flow is purely oscillatory due to the pumping action of the ventricle, it is appropriate to seek an oscillatory solution in order to reduce the dimensionless coupled partial differential equation (7)-(10) to ordinary differential equations in the following form:

$$\left.\begin{aligned} w(y,t) &= w_0(y)e^{i\omega t} \\ \theta(y,t) &= \theta_0(y)e^{i\omega t} \\ \phi(y,t) &= \phi_0(y)e^{i\omega t} \\ -\frac{\partial P}{\partial x} &= P_0 e^{i\omega t} \end{aligned}\right\} \quad (12)$$

Simplifying the dimensionless equations (7)-(10) using equation (12), we have:

$$\frac{\partial^2 w_0}{\partial y^2} - \left(i\omega + M^2 + \frac{1}{k}\right)w_0 = P_0 - Gr\theta_0 - Gc\phi_0 \quad (13)$$

$$\frac{\partial^2 \theta_0}{\partial y^2} + \left(Rd_2 - i\omega\right)Pr\theta_0 = -Rd_1\phi_0 \quad (14)$$

$$\frac{\partial^2 \phi_0}{\partial y^2} - \left(Rd_3 + i\omega\right)Sc\phi_0 = 0 \quad (15)$$

Subject to the following boundary conditions:

$$\left.\begin{aligned} &w_0 = 0,\ \phi_0 = 0, \theta_0 = 0 \qquad \text{at } y = 0 \\ &w_0 = 0,\ \phi_0 = e^{-i\omega t}, \theta_0 = e^{-i\omega t} \quad \text{at } y = 1 \end{aligned}\right\} \quad (16)$$

Solving the lipid concentration Equation (3.15), we have:

$$\phi_0(\chi) = A_1 sinh\left(\sqrt{\beta_3}\chi\right) + B_1 cosh\left(\sqrt{\beta_3}\chi\right) \quad (17)$$

where $\beta_3 = \left(Rd_3 + i\omega\right)Sc$. Solving for the constant coefficients in equation (17) using the appropriate boundary condition in equation (16), equation (18) was obtained.

$$\phi_0(y) = \frac{e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)} sinh\left(\sqrt{\beta_3}y\right) \quad (18)$$

Substitution of equation (18) into equation (14), yielded equation (19):

$$\frac{\partial^2 \theta_0}{\partial y^2} + \left(Rd_2 - i\omega\right)Pr\theta_0 = -\frac{Rd_1 e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)} sinh\left(\sqrt{\beta_3}y\right) \quad (19)$$

Let $\beta_4 = -\frac{Rd_1 e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)}$, so that equation (19) is reduced to:

$$\frac{\partial^2 \theta_0}{\partial y^2} + \beta_2\theta_0 = \left(\beta_4 sinh\left(\sqrt{\beta_3}\chi\right)\right) \quad (20)$$

where $\beta_2 = \left(Rd_2 - i\omega\right)Pr$

Solution of the homogenous part of equation (20) is:

$$\theta_{0h}(\chi) = A_2 sin\left(\sqrt{\beta_2}y\right) + B_2 cos\left(\sqrt{\beta_2}y\right) \quad (21)$$

and the solution of the particular part of equation (20) is:

$$\theta_{0p} = \frac{\beta_4}{\left(\beta_3 + \beta_2\right)} sinh\left(\sqrt{\beta_3}y\right) \quad (22)$$

Hence, the general exact solution of equation (20) is:

$$\theta_0(y) = A_2 sin\left(\sqrt{\beta_2}y\right) + B_2 cos\left(\sqrt{\beta_2}y\right) + \frac{\beta_4}{\left(\beta_3 + \beta_2\right)} sinh\left(\sqrt{\beta_3}y\right) \quad (23)$$

where $B_2 = 0, A_2 = \frac{e^{-i\omega t}}{sin\left(\sqrt{\beta_2}\right)} - \frac{\beta_4}{\left(\beta_3 + \beta_2\right)} \frac{sinh\left(\sqrt{\beta_3}\right)}{sin\left(\sqrt{\beta_2}\right)}$ Substitute the lipid concentration equation (18) and temperature profile equation (23) in the dimensionless momentum equation (13), we have:

$$\frac{\partial^2 w_0}{\partial y^2} - \left(i\omega + M^2 + \frac{1}{k}\right)w_0 = P_0 - \left(A_2 Gr sin\left(\sqrt{\beta_2}y\right) + \frac{\beta_4 Gr}{\left(\beta_3 + \beta_2\right)} sinh\left(\sqrt{\beta_3}y\right)\right) - \left(\frac{Gc e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)} sinh\left(\sqrt{\beta_3}y\right)\right) \quad (24)$$

Simplifying equation (24) further, we have:

$$\frac{\partial^2 w_0}{\partial y^2} - \left(i\omega + M^2 + \frac{1}{k}\right) w_0 = P_0 - A_2 Gr sin\left(\sqrt{\beta_2} y\right) - \left(\frac{Gce^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)} + \frac{\beta_4 Gr}{\left(\beta_3 + \beta_2\right)}\right) sinh\left(\sqrt{\beta_3} y\right) \quad (25)$$

Let $\beta_1 = h\left(i\omega + M^2 + \frac{1}{Da}\right)$, $\beta_5 = \left(\frac{Gce^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)} + \frac{\beta_4 Gr}{\left(\beta_3 + \beta_2\right)}\right)$ so that equation (25) is reduced to:

$$\frac{\partial^2 w_0}{\partial y^2} - \beta_1 w_0 = P_0 - A_2 Gr sin\left(\sqrt{\beta_2} y\right) - \beta_5 sinh\left(\sqrt{\beta_3} y\right) \quad (26)$$

The homogenous solution of equation (26) is:

$$w_{0h}\left(y\right) = A_6 sinh\left(\sqrt{\beta_1} y\right) + B_6 cosh\left(\sqrt{\beta_1} y\right) \quad (27)$$

The particular solution of equation (26) is in the form:

$$w_{0p} = A_{31} + A_4 sin\left(\sqrt{\beta_2} y\right) + A_5 sinh\left(\sqrt{\beta_3} y\right) \quad (28)$$

where $A_{31} = -\frac{P_0}{\beta_1}, A_4 = \frac{A_2 Gr}{\left(\beta_2 + \beta_1\right)}, A_5 = \frac{\beta_5}{\left(\beta_1 - \beta_3\right)}, B_4 = 0, B_5 = 0$

the general solution of equation (26) is:

$$w_0\left(y\right) = A_6 sinh\left(\sqrt{\beta_1} y\right) + B_6 cosh\left(\sqrt{\beta_1} y\right) + A_{31} + A_4 sin\left(\sqrt{\beta_2} y\right) + A_5 sinh\left(\sqrt{\beta_3} y\right) \quad (29)$$

Solving the constant coefficients in equation (29), we use the boundary condition (16) so that the solution to equation (26) is:

$$w_0\left(y\right) = A_6 sinh\left(\sqrt{\beta_1} y\right) - A_{31} cosh\left(\sqrt{\beta_1} y\right) + A_{31} + A_4 sin\left(\sqrt{\beta_2} y\right) + A_5 sinh\left(\sqrt{\beta_3} y\right) \quad (30)$$

where:

$$A_6 = A_{31}\frac{cosh\left(\sqrt{\beta_1}\right)}{sinh\left(\sqrt{\beta_1}\right)} - \frac{A_{31}}{sinh\left(\sqrt{\beta_1}\right)} + A_4\frac{sin\left(\sqrt{\beta_2}\right)}{sinh\left(\sqrt{\beta_1}\right)} - A_5\frac{sinh\left(\sqrt{\beta_3}\right)}{sinh\left(\sqrt{\beta_1}\right)}$$

Therefore, the blood velocity profile is obtained after substituting equation (30) into equation (12), which is:

$$w\left(y,t\right) = \left(A_6 sinh\left(\sqrt{\beta_1} y\right) - A_{31} cosh\left(\sqrt{\beta_1} y\right) + A_{31} + A_4 sin\left(\sqrt{\beta_2} y\right) + A_5 sinh\left(\sqrt{\beta_3} y\right)\right) e^{i\omega t} \quad (31)$$

The blood temperature profile is obtained after substituting equation (23) into equation (12), which is:

$$\theta\left(y,t\right) = \left(A_2 sin\left(\sqrt{\beta_2} y\right) + B_2 cos\left(\sqrt{\beta_2} y\right) + \frac{\beta_4}{\left(\beta_3 + \beta_2\right)} sinh\left(\sqrt{\beta_3} y\right)\right) e^{i\omega t} \quad (32)$$

and the lipid concentration profile is obtained after substituting equation (32) into equation (12), which is:

$$\phi\left(y,t\right) = \left(\frac{e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)} sinh\left(\sqrt{\beta_3} y\right)\right) e^{i\omega t} \quad (33)$$

The volumetric flow rate can be calculated analytically using equation (31) as follows:

$$Q = \int_0^1 \left(A_6 sinh\left(\sqrt{\beta_1} y\right) - A_{31} cosh\left(\sqrt{\beta_1} y\right) + A_{31} + A_4 sin\left(\sqrt{\beta_2} y\right) + A_5 sinh\left(\sqrt{\beta_3} y\right)\right) e^{i\omega t} dy \quad (34)$$

$$Q=\begin{pmatrix} \frac{A_6}{\sqrt{\beta_1}}cosh\left(\sqrt{\beta_1}y\right)-\frac{A_{31}}{\sqrt{\beta_1}}sinh\left(\sqrt{\beta_1}y\right)+A_{31}-\frac{A_4}{\sqrt{\beta_2}}cos\left(\sqrt{\beta_2}y\right) \\ +\frac{A_5}{\sqrt{\beta_3}}cosh\left(\sqrt{\beta_3}y\right)-\frac{A_6}{\sqrt{\beta_1}}+\frac{A_4}{\sqrt{\beta_2}}-\frac{A_5}{\sqrt{\beta_3}} \end{pmatrix}e^{i\omega t} \quad (35)$$

The rate of heat transfer at the wall of the vessel is calculated as:

$$Nu=-\left.\frac{\partial\theta}{\partial y}\right|_{y=1}=-\left(A_2\sqrt{\beta_2}cos\left(\sqrt{\beta_2}y\right)-B_2\sqrt{\beta_2}sin\left(\sqrt{\beta_2}y\right)+\frac{\beta_4\sqrt{\beta_3}}{\left(\beta_3+\beta_2\right)}cosh\left(\sqrt{\beta_3}y\right)\right)e^{i\omega t} \quad (36)$$

The rate of lipid mass transfer at the wall of the vessel is calculated as:

$$Sh=-\left.\frac{\partial\phi}{\partial y}\right|_{y=1}=-\left(\frac{\sqrt{\beta_3}e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)}cosh\left(\sqrt{\beta_3}y\right)\right)e^{i\omega t} \quad (37)$$

3.2 Presentation of Graphical Results

We have obtained the analytical solution for blood velocity, blood temperature and lipid concentration in Equations (31)-(37) respectively. However, in this section, we carry out numerical computation of the flow profiles by varying the governing parameters in order to study the parameters effects on each of the flow profiles. The parameters under consideration are: the magnetic field parameter, Schmidt number Sc, radiation parameter Rd_1, the Grashof number (Gr), solutal Grashof number Gc, the radiation absorption Rd_2, the Prandtl number (Pr), the volumetric flow rate Q, the rate of heat transfer Nu are simulated using Mathematica with variation of the parameters within the range: $0\le Rd_1\le 3,\ 0.2\le\omega\le 1,$ $0\le Rd_2\le 3, 0\le Sc\le 10,$ $Pr=\frac{\mu_b c_p}{k_T}=21, \mu_b=3.2\times10^3 kg/ms, c_p=14.65J/kg,$ $k_T=2.2\times10^{-3}$. The results are presented as following:

,

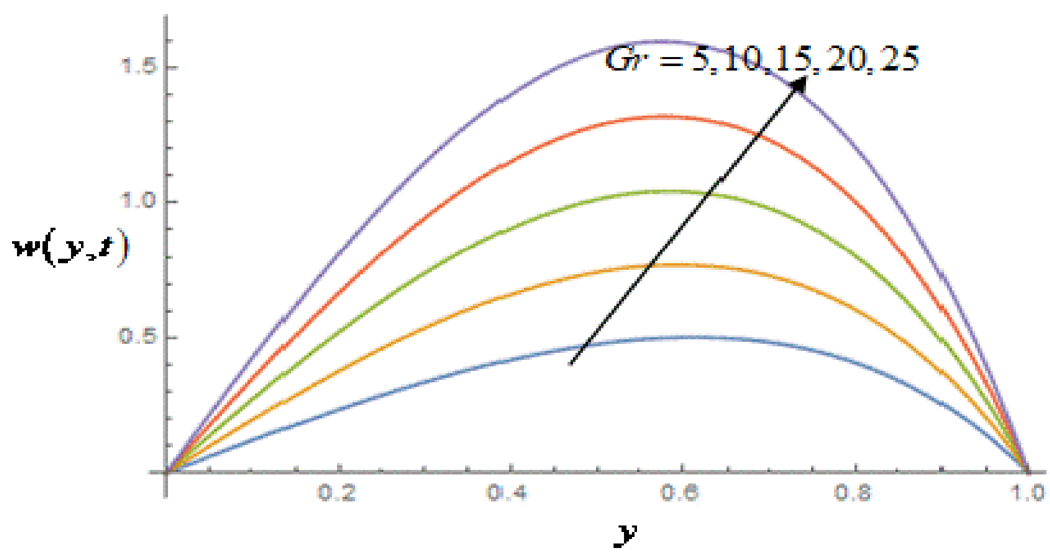

Fig. 1 Effect of Gr values on blood velocity profile

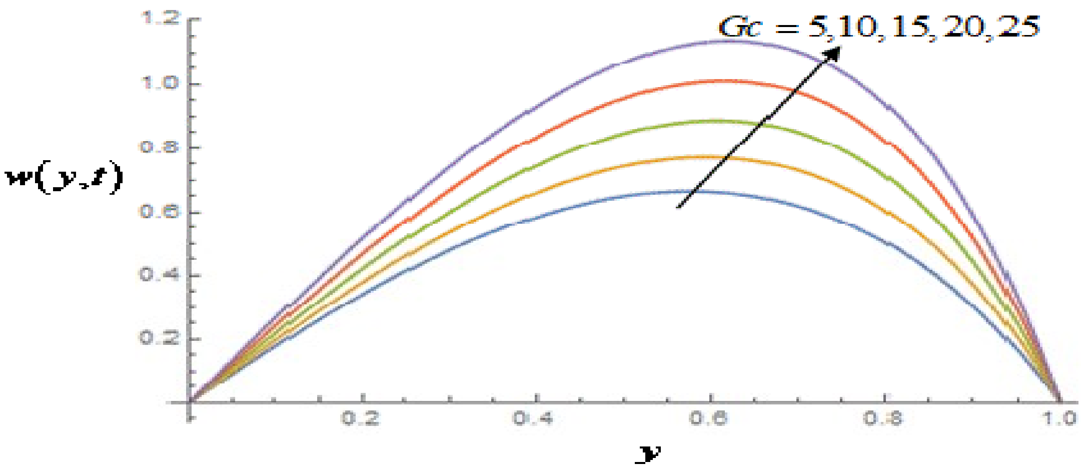

Fig. 2 Effect of Gc values on blood velocity profile

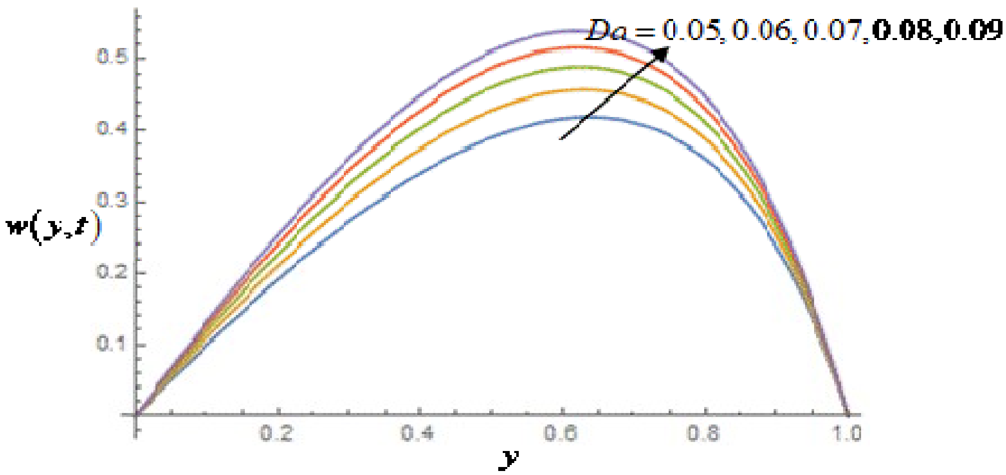

Fig. 3 Effect of Da values on blood velocity profile

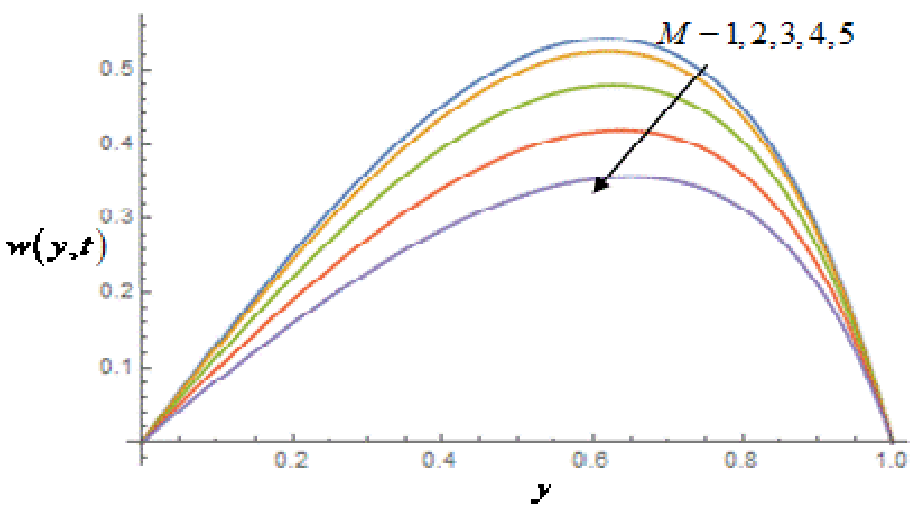

Fig. 4 Effect of M values on blood velocity profile

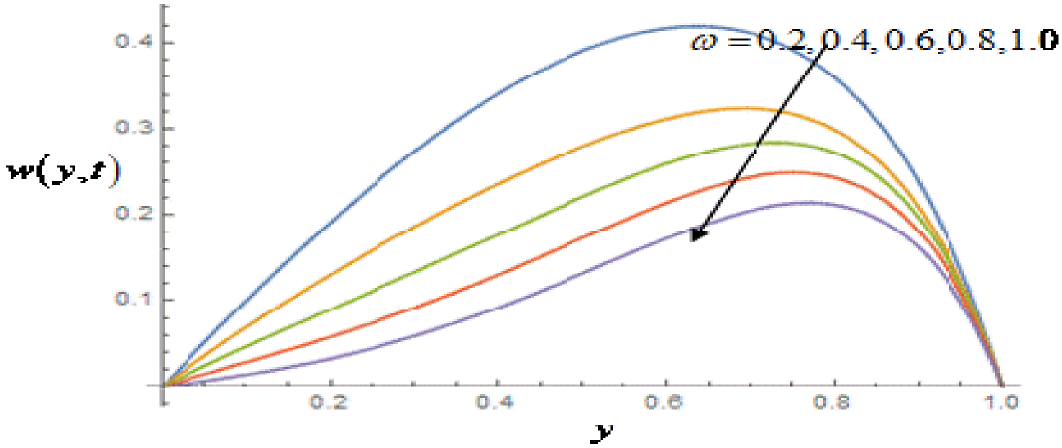

Fig. 5 Effect of ω **values on blood velocity profile**

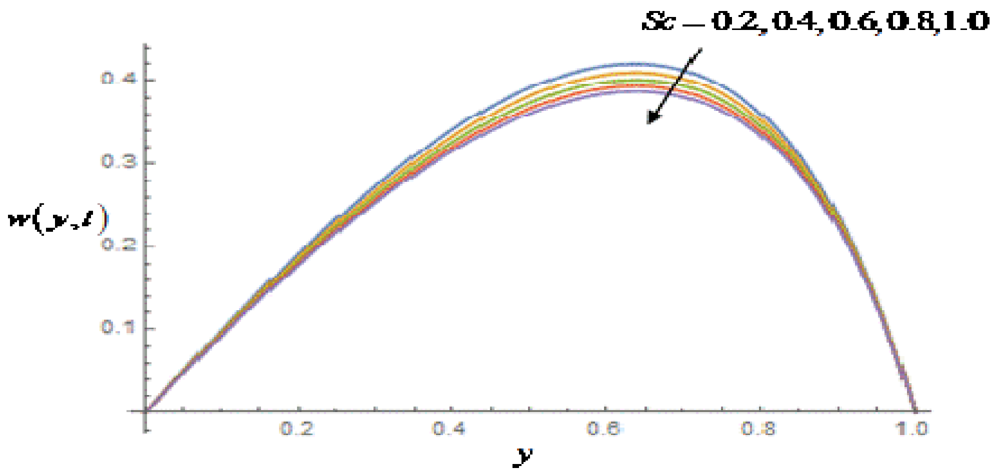

Fig. 6 Effect of Sc **values on blood velocity profile**

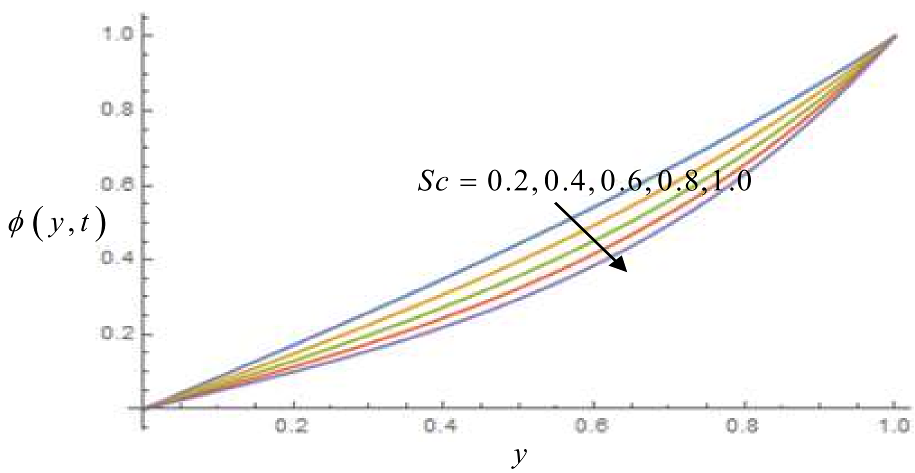

Fig. 7 Effect of Sc **values on blood lipid concentration profile**

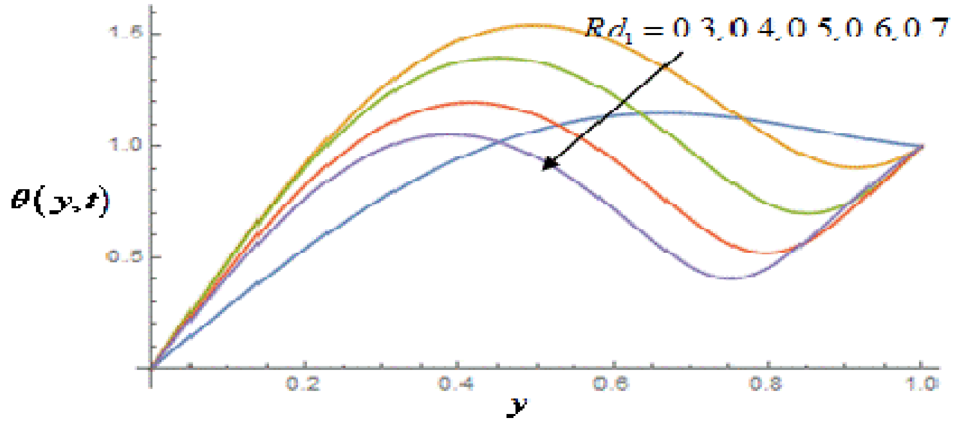

Fig.8 Effect of Rd_1 values on blood temperature profile

Fig. 1 shows the diagrammatic illustration depicting the porous vessel saturated with lipid. Figures 1 to 8 illustrate the blood velocity profiles under the influence of $Gr, Rd_1, Rd_2, M, Gc, Sc$ and ω respectively.

Grashof number is the ration of buoyancy force to viscous. Fig. 1 also depicts that buoyancy force causes the blood to accelerate and this is in agreement with known theory of physics. Fig. 2 illustrates the variation of blood velocity at different metabolic heat source Rd_2 and from the plot; it is evident that the velocity of blood increases as the metabolic heat source increase. Fig 3 depicts that buoyancy force due to lipid concentration causes the blood to accelerate and this is in agreement with known theory of physics. Fig.5 shows that the velocity of the blood increases for different values of Darcy number Da, owing to the fact that the permeability of the medium allows blood to flow. This is of the view that increase intravascular flow caused by decrease in vessel radius. Hartmann number is associated with ratio of magnetic body force to viscous force. Fig. 4 clearly indicates an increase in the values of Hartmann number causes retardation of the fluid, depicting the fact that perpendicularly applied magnetic field decreases the flow and in turn the boundary layer thickness of the velocity gets diminished. The oscillatory frequency is a function of the pulse frequency, that is $\omega = 2\pi f$. Increase in oscillatory frequency as indicated in Figure 5, is of the view that there is increase in pulse rate. This figure shows decrease in velocity of blood for different values of the oscillatory frequency. Fig. 6 shows that the velocity of blood decreases for the increase in the values of Schmidt number Sc. Fig.7 shows a decrease in lipid concentration level, for different values of the Schmidt number Sc.

The thermal radiation increases the temperature of the fluid, hence could cause rise in temperature of the fluid. Fig.8 illustrated the fact that temperature profile decreases for different values of thermal radiation parameter.

4.0 Conclusion

In this paper, we have investigated the oscillatory MHD blood flow through a porous channel saturated with lipid concentration in the presence of magnetic field with heat sources. The effect of the governing parameters was also investigated, and the following are the significant findings:

i. An increase in Grashof number Gr and solutal Grashof number Gc caused the dimensionless velocity of blood to increase.

ii. Radiation parameter Rd_1 and metabolic radiation parameter Rd_2 increase caused the dimensionless velocity of blood to increase.

iii. The Darcy number Da increases resulted to an increase in the dimensionless velocity of blood.

iv. The dimensionless velocity of blood decreases for the increasing values of the Hartmann number M, Schmidt number Sc and the oscillatory frequency parameter ω.

v. The lipid concentration of the fluid decreases for the increase in the value of Schmidt number Sc.

vi. The temperature of the fluid decrease for the increasing values of the radiation parameter Rd_1 but the temperature increases for the increase in the metabolic source parameter Rd_2.
vii. The temperature of the fluid decreases for the increasing values of the oscillatory frequency parameter ω.
viii. The dimensionless temperature of the fluid increases for the increase in the values of Prandtl number Pr.
ix. The volumetric flow rate increases against Schmidt number and Darcy number for the different values increase of the metabolic heat source parameter Rd_2.
x. The rate of heat transfer increases against Schmidt number Sc for different values of radiation parameter Rd_1.

5.0 References

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Conflict of Interest

The authors declare no conflict of interest