Communication in Physical Sciences, 2024, 12(2): 162-174

Specification Procedure for Symmetric Smooth Transition Autoregressive (STAR) Models

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Received: 19 November 2024/Accepted: 26 January 2025/Published: 05 February 2025
https://dx.doi.org/10.4314/cps.v12i2.1

Abstract: *In this paper, we assess the first-rate specification accuracy of the Escribano-Jorda procedure (EJP) over Terasvirta procedure (TP) in the selection of true symmetric STAR model for the financial time series. Daily nonstationary BETAGLASS stock index (BSI) totaling 2472 observations were obtained from Nigerian Exchange Limited for empirical illustrations. Terasvirta sequential tests and Escribano-Jorda tests were carried out; first-order logistic function classified as asymmetric transition function and exponential function classified as symmetric transition function were specified by TP and EJP, respectively. Both symmetric and asymmetric STAR models were justifiably fitted to percentage BETAGLASS stock returns (PBSR) and the best model determined at the evaluation stage. The empirical assessment of the fits of both symmetric STAR models and asymmetric STAR models revealed that symmetric STAR models outperformed asymmetric STAR models under consideration. Hence, EJP has greater specification power over TP particularly when the true model for the financial time series is any symmetric STAR model. Owing to the presence of autoregressive conditional heteroscedastic (ARCH) effects, STAR-generalized ARCH (STAR-GARCH) models and autoregressive-GARCH (AR-GARCH) models were specified and fitted to PBSR. On balance, SPLSTAR-GARCH (1, 1) model with generalized hyperbolic skew-student's t innovations outperformed the competing models. Also, the overall prediction performance of SPLSTAR-GARCH (1 1) model is better than its linear counterpart based on the Akaike information criterion and forecast root mean square error.*

Keywords: *ARCH effects, Symmetric, Transition function, Asymmetric, Sequential tests, Nonstationary*

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I.0 Introduction

The robustness and optimality of any specified smooth transition autoregressive (STAR) model depend on the selection approach used to specify the transition function of the STAR model. Many analysts have applied either Terasvirta procedure (TP) of Terasvirta (1994) or Escribano-Jorda procedure (EJP) of Escribano and Jorda (2001) or both selection procedures to select either ESTAR (symmetric) model or LSTAR (asymmetric) model (Terasvirta, 1994; Terasvirta and Anderson, 1992; Sarantis, 1999; Anderson, 1997; Boero and Marrocu, 2002; Dijk et al., 2002; Liew *et al.*, 2003; Siliverstovs, 2005; Terasvirta *et al.*, 2005; Baharumshah and Liew, 2006; Shangodoyin *et al.*, 2009; Nor *et al.*, 2015; .Effiong *et al.*, 2023 and Effiong *et al.*, 2024). Only Yaya and Shittu, 2016 used TP and EJP to unfold improved symmetric properties of the absolute error logistic STAR (AELSTAR) model and quadratic logistic STAR (QLSTAR) model over the existing symmetric STAR (ESTAR) model. What happens when EJP and TP specify different transition functions for a particular financial time series? Escribano and Jorda (2001) revealed that EJP has better properties than TP and very relevant when the true model is ESTAR model. The aim of this paper is to empirically determine the specification accuracy of EJP over TP in the selection of symmetric STAR model for the financial time series (FTS).

Also, time series analysts in recent years have made considerable efforts to model stock returns with STAR and STAR-GARCH models. Hsu and Chiang (2011) who assessed the effect of monetary policy on stock returns using STAR model asserted positive and nonlinear relationship between the monetary policy and excess returns on stock prices. Yaya and Shitu (2014) who fitted STAR-GARCH model to financial data under non-normality assumption of innovations of the conditional mean model, revealed that the selection of LSTAR models was dependent of the structure of the innovations and improved as sample size increased. Khemiri (2011) modelled the dynamics of four international stock indexes using smooth transition GARCH (STGARCH) model and the result showed that STGARCH model performs better than the symmetric GARCH model. Midilic (2020) applied STAR–GARCH model with Iteratively Weighted Least Squares (IWLS) algorithm to model US dollar/Australian dollar and the FTSE Small Cap index returns which the result of one-day ahead out-of-sample forecast reveals improvement in STAR–GARCH model with IWLS algorithm over the benchmark (linear) model. Oyewale (2021) modelled monthly Nigerian gross domestic product from 1997 to 2019 with ESTAR-GARCH and LSTAR-GARCH models; LSTAR-GARCH and ESTAR-GARCH models perform better than the standard GARCH model. Hsiao *et al.* (2021) provide a mathematical and statistical approach for the estimation of heteroscedasticity in Taiwan' stock price index. The fitted heteroscedastic models revealed the influence of COVID-19 on the fluctuations of the return rates of Taiwan stock price index. The paper seeks to determine the applicability of Power logistic STAR (PLSTAR-GARCH) model in modelling the volatility clustery of Nigerian stock index.

This paper is organized as follows: Section 2 is material and methods. Section 3 discusses data analysis and results, Section 4 deals with discussion of findings and Section 5 concludes.

2.0 Material and Methods

2.1 Data

Daily BETAGLASS stock index comprising 2472 observations spanning from 2nd January, 2014 to 29th December, 2023 obtained from Nigerian Exchange Limited are used for empirical analyses. The percentage change of BETAGLASS stock index $(z_t) = 100\nabla \ln y_t$, where y_t is the BETAGLASS stock index at time t.

2.2 Methods

We adopt Escribano-Jorda procedure of Escribano and Jorda (2001) and the procedures put forward by Terasvirta (1994) for specification of STAR models. The methods of estimation of STAR and STAR-GARCH models are nonlinear least squares method and two-stage estimation procedure, respectively. The two-stage procedure is the estimation of conditional variance (GARCH) model using residuals obtained from nonlinear estimation of conditional mean (STAR) model with appropriate distribution (Gaussian or non-Gaussian) of the residuals. Model evaluation measures are used for the diagnostic checks of the fitted models.

2.3 Model

2.3.1 Smooth transition autoregressive (STAR) models

A two regime STAR model for a univariate time series z_t, which is observed at $t = 1 - p, 1 - (1 - p), \dots, -1, 0, 1, \dots, T - 1,\ T$ is given by

$$z_t = (\pi_{1,0} + \pi_{1,1}z_{t-1} + \cdots + \pi_{1,p}z_{t-p})(1 - L(z_{t-d}; \delta, \lambda)) + (\pi_{2,0} + \pi_{2,1}z_{t-1} + \cdots + \pi_{2,p}z_{t-p})L(z_{t-d}; \delta, \lambda)) + \nu_t. \quad (1)$$

(1) can be rewritten as

$$y_t = \mathbf{\Pi}_1'\mathbf{w_t}(1 - L(z_{t-d}; \delta, \lambda)) + \mathbf{\Pi_2'}\mathbf{w_t}L(z_{t-d}; \delta, \lambda) + \boldsymbol{\nu}_t, \quad (2)$$

where $\mathbf{\Pi}_i = (\pi_{i,0}, \pi_{i,1}, \cdots, \pi_{i,p})'$ for i = 1,2, $\mathbf{w}_t = (1, z_{t-1}, \dots, z_{t-p})'$. The $\nu_t{}'s$ are assumed to be a martingale difference sequence with respect to the history of the time series up to time $t - 1$, which is donated as $\Omega_{t-1} = \{z_{t-1}, z_{t-2}, \dots, z_{1-(p-1),}\ z_{1-p}\}$, $E[\nu_t/\Omega_{t-1}] = 0$, and $E[\nu_t^2/\Omega_{t-1}] = \sigma^2$ and $L(z_{t-d}; \delta, \lambda)$ is a transition function (Dijk *et at.*, 2002).

2.3.2 Transition functions

Transition function denoted by $L(z_{t-d}; \delta, \lambda)$ is a distribution function which is at least twice differentiable. Here we assume that the transition variable z_{t-d} is a lagged endogenous variable for certain integer $d > 0$. The most commonly used transition functions that give rise to different types of regime switching behaviour are the following:

The first-order logistic function (LSTR1) proposed by Terasvirta (1994) given by

$$L(z_{t-d}; \delta, \lambda) = (1 + \exp[-\delta(z_{t-d} - \lambda)])^{-1},\ \delta > 0. \quad (3)$$

and the STAR model (2) with (3) is called the logistic STAR (LSTAR) model.

The second-order logistic (LSTR2) function is given by

$$L(z_{t-d}; \delta, \lambda) = (1 + \exp[-\delta(z_{t-d} - \lambda_1)(z_{t-d} - \lambda_2)])^{-1}, \lambda_1 \le \lambda_{2,}\ \delta > 0, \quad (4)$$

where $\lambda = (\lambda_1, \lambda_2)'$, as proposed by Jansen and Terasvirta (1996).

The exponential function proposed by Terasvirta (1994) given by

$$L(z_{t-d}; \delta, \lambda) = 1 - \exp[-\delta(z_{t-d} - \lambda)^2],\ \delta > 0. \quad (6)$$

Model (2) with (6) is called exponential STAR (ESTAR) model.

The Power Logistic (PL) Function proposed by Effiong *et al*. (2023) is given by

$$L(z_{t-d}; \delta, \lambda) = \{1 + 0.5\exp[-\delta(z_{t-d}^i - \lambda)]\}^{-2} - \frac{1}{1.5^2}, \delta > 0,\ i = 1,2, \quad (7)$$

Model (2) with (7) is called Power logistic STAR (PLSTAR) model.

(7) is called asymmetric power logistic (APL) function when $i = 1$ and the corresponding STAR model is APLSTAR, while (7) is symmetric power logistic (SPL) function when $i = 2$ and the corresponding STAR model is SPLSTAR model.

2.4 Specification of STAR model

2.4.1 Terasvirta Procedure (TP)

The Lag length (p) of the linear model based on percentage BETAGLASS stock returns (PBSR) is determined by Schwarz Bayesian criterion (BIC), followed by tests of linearity against the alternative of STAR nonlinearity using PBSR. If linearity is rejected, delay parameter, d, is determined from the range of values $1 \le d \le P$ considered appropriate. Also, the rejection of null hypothesis of linearity, $H_{01}: K_1 = K_2 = K_3 = 0$, is based on the following auxiliary regression equation proposed by Luukkonen *et al.* (1988) given by

$$z_t = K_0'w_t + K_1'w_t z_{t-d} + K_2'w_t z_{t-d}^2 + K_3'w_t z_{t-d}^3 + \varepsilon_t \quad (8)$$

Equation 8 is used to surmount the problem of unidentified parameters under the null hypothesis. Then, sequence of F tests proposed by Terasvirta (1994) based on the following hypotheses is used to identify suitable transition function for STAR model:

$H_{02}: K_3 = 0$
$H_{03}: K_2 = 0 | K_3 = 0 H_{04}: K_1 = 0 | K_2 = K_3 = 0$
LSTAR model is appropriate If H_{02} is rejected. It is ESTAR, If H_{03} is rejected, while H_{04} is rejected, If the true model is a LSTAR model. According to Terasvirta (1994), it is better to compare the strengths of the rejections. If LSTAR is the model, H_{02} and H_{04} are rejected more strongly than H_{03}. If the model is ESTAR, the opposite is the case. Thus, if p-value of the test corresponding to H_{03} is the smallest, an ESTAR model should be selected, while in all other cases, LSTAR model should be chosen.

2.4.2 Escribano-Jorda procedure

Escribano and Jorda procedure (EJP) according to Escribano and Jorda (2001) is based on the following two hypotheses within the auxiliary regression equation:

$$z_t = K'_0 w_t + K'_1 w_t z_{t-d} + K'_2 w_t z_{t-d}^2 + K'_3 w_t z_{t-d}^3 + K'_4 w_t z_{t-d}^4 + \varepsilon_t \quad (9)$$

$H_{0L}: K_2 = K_4 = 0$ with an F-test (F_L)
$H_{0E}: K_1 = K_3 = 0$ with an F-test (F_E)
If the minimum p-value corresponds to F_E, select LSTAR model. Otherwise select ESTAR model. EJP has better properties than TP and preferred to TP especially when the true model is ESTAR.

2.5 ARCH/GARCH Models

Consider the autoregressive moving average (ARMA) model:

$$z_t = \pi_{1,1} z_{t-1} + \cdots + \pi_{1,p} z_{t-p} + \varrho_{2,1} \nu_{t-1} + \cdots + \varrho_{2,q} \nu_{t-q} + \nu_t \quad (10)$$

Letting $\sigma_t^2 = \text{Var}[\nu_t | \Omega_{t-1}]$ denote the conditional variance of ν_t given the past Ω_{t-1}, then the basic ARCH(l) model can be formulated as

$$\nu_t = \sigma_t z_t, \quad (11)$$

$$\sigma_t^2 = \lambda_0 + \lambda_1 \nu_{t-1}^2 + \lambda_2 \nu_{t-2}^2 + \cdots + \lambda_l \nu_{t-l}^2, \quad (12)$$

where $\{z_t\}$ is a sequence of independent and identically distributed (iid) random variables with mean zero and variance 1, $\lambda_0 > 0$, and $\lambda_i \geq 0$ for $i = 1,2,\ldots,l-1$, and $\lambda_l > 0$. The additional constraints are imposed to ensure the conditional variance σ_t^2 is positive. The constraint $\sum_{i=1}^{l} \lambda_i < 1$ ensures that ν_t are covariance stationary with finite unconditional variance (σ_ν^2).

According to Bollerslev (1986), the GARCH(l, m) model is given by

$$\sigma_t^2 = \lambda_0 + \sum_{i=1}^{l} \lambda_i \nu_{t-i}^2 + \sum_{j=1}^{m} \beta_j \sigma_{t-j}^2, \quad (13)$$

where $\lambda_0 > 0$, and $\lambda_i \geq 0$ for $i = 1,2,\ldots,l-1$, and $\lambda_l > 0$, $\beta_j \geq 0$ for $j = 1,2,\ldots,m-1$, and $\beta_m > 0$.

PLSTAR-GARCH model is the combination of (2), (7), (11) and (13).

2.5.1 Test for ARCH/GARCH Effects

Lagrange multiplier test of ARCH effects proposed by Engle 1982 is given by

$$\vartheta = kR^2, \quad (14)$$

to test the null hypothesis

$$H_0: \varsigma_i, i = 1,2,\ldots,l \text{ (No ARCH effects)}, \quad (15)$$

where k is the sample size and R^2 is the coefficient of determination computed from the following auxiliary regression equation:

$$\hat{\nu}_t^2 = \varsigma_0 + \varsigma_1 \hat{\nu}_{t-1}^2 + \cdots + \varsigma_u \hat{\nu}_{t-l}^2 + \eta_t \quad (16)$$

If the ARCH effect is found to be significant, the ARCH order is determined using the PACF of $\hat{\nu}_t^2$.

2.5.2 Evaluation measures

Relative forecast performance is used as a model selection criterion or as an alternative or complement to an in-sample comparison of different models (Dijk *et al*., 2002). The ratio of the forecast root mean square error (FRMSE) of the nonlinear model to that of the corresponding linear model chosen by AIC or BIC provide the relative performance of the two models. The standard error of the residuals will be used to determine the efficiency of the models.

If $z_t, t = 1,2, \dots, h$ are the actual values of the observations used in the estimation of the model, $g_t, t = 1,2, \dots, h$ are the forecasted values, then, $e_t = z_t - g_t$, $t = 1,2, \dots, h$ are the forecast errors. The FRMSE of the in-sample forecast is the square root of the Mean square error (MSE) given by

$$\text{RMSE}= \sqrt{\frac{1}{h}\sum_{i=1}^{h} e_t^2}, \qquad (17)$$

where h is the total number of forecast errors.

3.0 Data Analysis and Results

3.1 Preliminary Analysis

The time series plot of BETAGLASS stock index (BSI) appears to be increasing over time, but the rate of growth (rise and fall) of BSI is rather slow, except in 2020 when sharp fall was witnessed due to Covid-19 pandemics. Hence, BSI series is nonstationary (Fig. 1)).

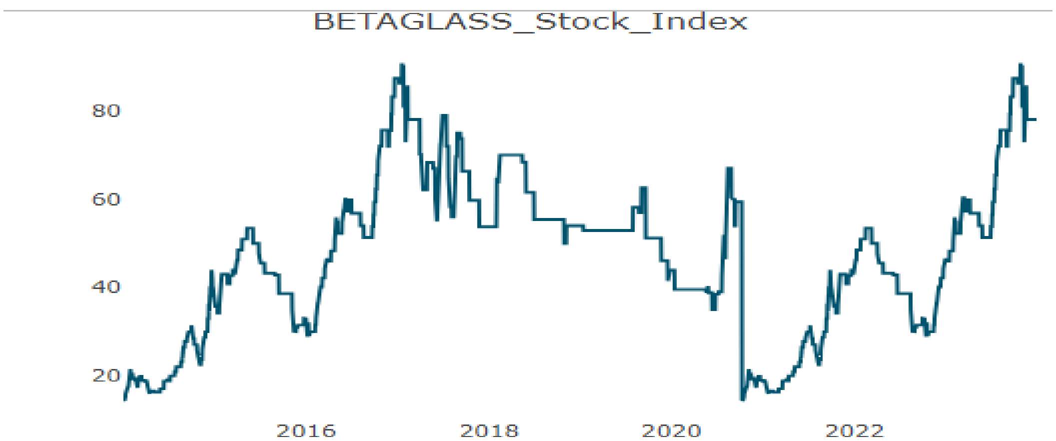

Fig. 1: Time series plot of BETAGLASS stock index

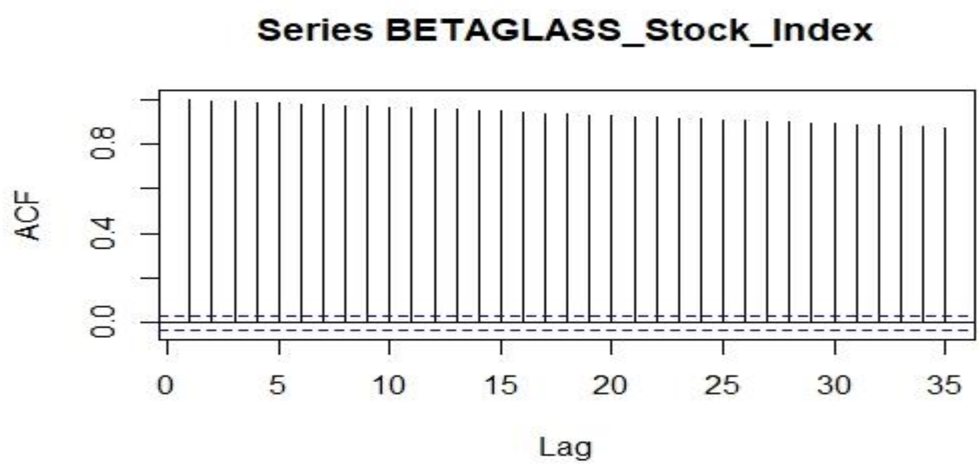

Fig. 2: Autocorrelation function (ACF) of BETAGLASS stock index

The pattern of ACF of BSI confirms the nonstationarity of BSI (Fig. 2). The time series plot of percentage BETAGLASS stock returns (PBSR) reveals the presence of volatility cluster; the mean and variance of PBSR appear to be stable (Fig. 3).

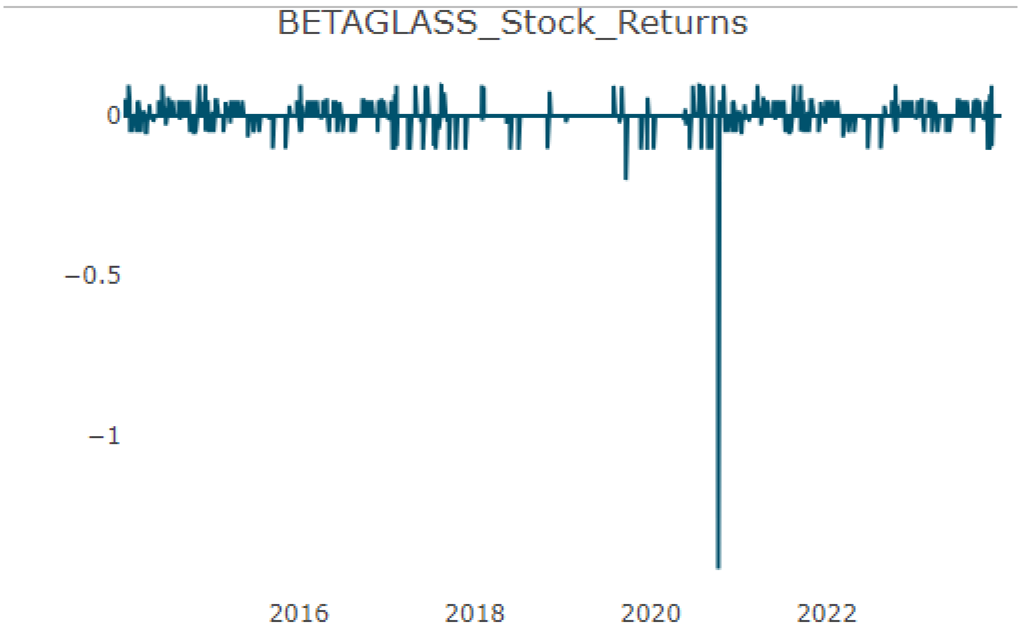

Fig. 3: Time series plot of percentage BETAGLASS stock returns

Based on Table 1, the p-values of Dickey-Fuller statistics for both BSI and PBSR indicate that the BSI contains unit root, but that PBSR is stationary at 5% level of significance.

Table 1: Results of augmented Dickey-Fuller tests

Variable	**Original series**			**Transformed series**				
				Logarithmic series		**Lag order**	**Difference logarithmic series**	
BETAGLASS stock index	Dickey Fuller Statistic	ρ-Value	Lag Order	Dickey Fuller Statistic	ρ-Value	15	Dickey Fuller Statistic	ρ-Value
	-2.486	0.373	15	-2.595	0.326		-14.783	0.01

3.2 Identification of linear model and test for ARCH effects

In accordance with Table 2, AR (2) and AR (5) models are specified by BIC and AIC, respectively. We chose AR (2) model, even though AR (2) is too parsimonious. Based on Table 3, AR (2) and AR (5) models are free from serial correlation since the p-value= $0.9926 > 0.05$ and p −value= $0.9692 > 0.05$, respectively and AR (2) has the smaller standard error of the residuals. Lagrange multiplier (LM) test reveals the presence of ARCH effects in the residuals obtained from the fitted AR (2) model. Hence, the need for AR-GARCH model.

The null hypothesis of linearity is rejected (p-value= $0.000 < 0.05$) in favour of STAR type nonlinearity as shown in Table 4 and the delay length with the smaller value of residual sum of squares (RSS) is 2. Hence, the delay parameter is 2.

Table 2: Determination of Lag length of an autoregressive model

Lags	Loglik	p(LR)	AIC	BIC	HQC
1	6329.51509		-5.132616	-5.130259	-5.131760
2	6333.85489	0.00322	-5.135324	-5.130612*	-5.133612*
3	6334.81635	0.16554	-5.135293	-5.128225	-5.132725
4	6335.92395	0.13666	-5.135380	-5.125956	-5.131956
5	6337.60431	0.06677	-5.135932*	-5.124151	-5.131652

The asterisks above indicate the best values of the respective information criteria, AIC = Akaike criterion, BIC = Schwarz Bayesian criterion and HQC = Hannan-Quinn criterion

Table 3: Parameter estimates of linear models fitted to PBSR

Model	Parameter	Estimate	Ljung-Box Statistic	σ_{v_t}	LM statistic
AR(5)	ϕ_1	0.0890 (0.000)	8.5863e-05 (0.9926)	3.98875	6606952 (< 2.2e-16)
	ϕ_2	0.0541 (0.007)			
	ϕ_3	0.0232 (0.252)			
	ϕ_4	0.0267 (0.187)			
	ϕ_5	0.0369 (0.067)			
AR(2)	ϕ_1	0.027236 (0.000)	0.0014947 (0.9692)	3.27	6606821 (< 2.2e-16)
	ϕ_2	0.025474 (0.003)			

Where the values in the parentheses are p-values of estimated parameters and σ_{v_t} is the standard error of the residuals.

Table 4: Linearity Test

Hypothesis	F- Statistic	Regime	Residual sum of squares (RSS) of the Delay Length (d)	
			1	**2**
H_{01}	8.657488 (0.000)	2	2345.374745	2342.533730

The value in the parenthesis is the p-value of F- Statistic.

3.3 Specification of transition functions

In accordance with Table 5, Terasvirta Sequential Tests specify first-order logistic function (asymmetric function) since H_{03} is not rejected ($p-$value$=0.0618>0.05$) and H_{04} is rejected ($p-$value$=0.000<0.05$). Escribano-Jorda Tests specify exponential function (symmetric function) since the p-value of $H_{0L}<$ the p-value of H_{0E}. We decided to model PBSR with both symmetric and asymmetric STAR models and determine the best model, hence, the robustness of the selection procedure at the evaluation stage.

From Table 6, the residuals of APLSTAR, LSTAR, ESTAR and SPLSTAR models fitted to PBSR are uncorrelated based on Ljung-Box statistic of $\hat{v}_t$, which signifies no lack of fit of the models,. Still, theyare heteroscedastic because the p-values of LM statistics are less than 0.05. The most efficient STAR model based on the standard error of the residuals is ESTAR model, followed by SPLSTAR model and AR (2) model. Hence, symmetric models fitted well to PBSR. Thus, EJP is more robust than TP when the true model for the FTS is symmetric STAR model.

Owing to the presence of ARCH effects, SPLSTAR-GARCH, ESTAR-GARCH and AR-GARCH models were estimated using the residuals obtained from modelling PBSR by SPSTAR, ESTAR and AR models, respectively. SPLSTAR-GARCH (1, 1) and ESTAR-GARCH (1, 1) models with generalized hyperbolic skew-student's t (ghst) innovations and AR(2)-GARCH (1,1) models with snorm innovations were justifiably specified for PBSR

Table 5: Terasvirta Sequential and Escribano-Jorda Tests

Terasvirta Sequential Tests			**Escribano-Jorda Tests**		
Null hypothesis	**F- Statistic**	**Transition function**	**Null hypothesis**	**F- Statistic**	**Transition function**
H_{02}	-	First-order logistic function	H_{0L}	4.890444 (0.0022)	Exponential Function
H_{03}	2.787053 (0.0618)		H_{0E}	2.409339 (0.0652)	
H_{04}	11.57591 (0.0000)				

The values in the parentheses are p-values of F- Statistics.

Table 6: STAR models fitted to PBSR

Parameters	**ASPLSTAR**	**LSTAR**	**ESTAR**	**SPLSTAR**
$\boldsymbol{\Phi_{10}}$	-0.02936 (0.84366)	-0.0033843 (0.981307)	-	-
$\boldsymbol{\Phi_{11}}$	0.35934 (0.07691)	0.0134278 (0.677902)	-0.14034 (0.02611)	0.35934 (0.03691)
$\boldsymbol{\Phi_{12}}$	0.15106 (0.05334)	0.0042383 (0.825747)	0.99910 (0.00302)	-0.39608 (0.00725)
$\boldsymbol{\Phi_{20}}$	-0.35301 (0.03382)	1.1807129 (0.039432)	0.02481 (0.24630)	0.15106 (0.00334)
$\boldsymbol{\Phi_{21}}$	-0.35301 (0.02382)	0.2371352 (0.005849)	-2.37065 (0.00800)	0.04805 (0.04322)
$\boldsymbol{\Phi_{22}}$	0.40787 (0.09835)	-0.1158016 (0.255107)	0.19479 (0.00267)	-0.35301 (0.08382)
λ	-1.98239 (0.00353)	0.6398412 (0.406423)	1.42861 (4.53e-05)	-1.98239 0.19279
δ	1.42861 (0.00755)	7.7413925 (0.682817)	1.5582 (0.00244)	5.40787 (0.09835)
$\boldsymbol{\sigma_{\hat{v}_t}}$	3.394609	3.89452	3.18023	3.22654

Ljung statistic of $\hat{v}_t$	0.02573 (0.9951)	0.0068492 (0.934)	0.010244 (0.9194)	3.8266e-05 (0.9951)
LM statistics	3033396 (< 2.2e-16)	3033333 (< 2.2e-16)	6600375 (< 2.2e-16)	3033430 (< 2.2e-16)

Where $\hat{v}_t$ is the estimates of the residuals, $\sigma_{\hat{v}_t}$ is the standard error of the residuals and the values in the parentheses are p-values of estimated parameters.

Table 7: Parameter estimates of ESTAR-GARCH (1, 1) model

Type of Distribution	Parameter	Estimate	LM Test (ARCH(5))	Ljung-Box Statistics for Standardized Residuals	
				$\tilde{v}_t$	$(\tilde{v}_t)^2$
Ghst	λ_0	0.000164 (0.00000)			
	λ_1	0.030577 (0.0000)	0.003769 (0.9510)	0.4336 (0.9674)	0.1002 (0.7515)
	β_1	0.838298 (0.0000)			
AIC			2.3405		

The values in the parentheses are p-values of estimated parameters and ghst is generalized hyperbolic skew-student's t-distributions.

Table 8: Parameter estimates of SPLSTAR-GARCH (1, 1) model

Type of Distribution	Parameter	Estimate	LM Test (ARCH(5))	Ljung-Box Statistics for Standardized Residuals	
				$\tilde{v}_t$	$(\tilde{v}_t)^2$
Ghst	λ_0	0.000036 (0.00000)		0.09779 (0.9982)	0.008594 (0.9261)
	λ_1	0.008854 (0.0000)	0.020483 (0.9986)		
	β_1	0.900567 (0.0000)			
AIC			1.9739		

The values in the parentheses are p-values of estimated parameters and ghst is generalized hyperbolic skew-student's t-distribution.

Based on Table 7, The parameters of ESTAR-GARCH (1, 1) model with ghst innovations are significant at 5% level of significance. Portmantaeu tests of standardized $\tilde{v}_t$ (p −value$= 0.9674 > 0.05$) and standardized $(\tilde{v}_t)^2$ (p −value$= 0.7515) > 0.05$) reveal that the model is free from serial correlation. The LM test for remaining ARCH effects reveals absence of ARCH effects.

Based on Table 8, the parameters of SPLSTAR-GARCH (1, 1) model with ghst innovations specified for modeling PBSR are significantly different from zero and no lack of fit of the model based on Ljung-Box statistics of standardized residuals and standardized squared residuals is revealed. No remaining ARCH effects presence based on LM test.

Table 9: Parameter estimates of AR(2)-GARCH (1, 1) model

Type of Distribution	Parameter	Estimate	LM Test (ARCH(5))	Ljung-Box Statistics for Standardized Residuals	
				$\tilde{\nu}_t$	$(\tilde{\nu}_t)^2$
	ϕ_1	0.072446 (0.000609)			
	ϕ_2	0.061061 (0.003261)			
Snorm					
	λ_0	0.473896 (0.00000)	0.002636 (0.9999)	0.02832 (0.8664)	0.0008406 (0.9769)
	λ_1	0.042815 (0.00000)			
	β_1	0.814073 (0.00000)			
AIC		4.0527			

Where the values in the parentheses are p-values of estimated parameters and snorm is skewed normal distribution of innovations.

AR(2)-GARCH (1, 1) model is specified and fitted to PBSR. From Table 9, the parameters of AR(2)-GARCH (1, 1) model are statistically significance and the model fitted well to PBSR based on the results of Portmanteau tests of standardized residuals and standardized squared residuals. Moreover, no remaining ARCH effects based on LM test.

Table 10: Evaluation of Models

Estimated Model	FRMSE	AIC
SPLSTAR-GARCH (1, 1)	0.07630439	1.9739
ESTAR-GARCH (1, 1)	0.2240533	2.3405
AR(2)-GARCH (1, 1)	0.2150709	4.0527
AR(2)	2.988752	18348.01

Where FRMSE is the forecast root mean square error

In terms of AIC value and FRMSE, SPLSTAR-GARCH (1, 1) model is the best model for describing PBSR. Also, based on FRMSE, the overall prediction performance of SPLSTAR-GARCH (1 1) model is better than its linear counterpart.

4.0 Discussion of Findings

Daily BSI totaling 2472 observations were obtained from Nigerian Exchange Limited for empirical illustrations. The time series plot of BSI indicates overall positive trend pattern with slow growth rate, except in 2020 when BSI witnessed sharp fall due to Covid-19 pandemics, while the time series plot of PBSR reveals the presence of volatility cluster and the mean and variance of PBSR which appear to be stable.

The rejection of null hypothesis of linearity leads to the determination of delay length. Terasvirta Sequential Tests specify first-order logistic function (asymmetric function), while Escribano-Jorda Tests specify exponential function (symmetric function). Both symmetric and asymmetric STAR models were fitted to PBSR to determine the relevant of EJP in the detection of symmetric transition function whenever symmetric STAR model is the true model for the FTS, while delaying the determination of the best model to the evaluation stage. Based on standard error of the residuals, Symmetric STAR models outperformed its asymmetric counterparts. Hence, the superiority of EJP over TP in the detection of symmetric model, when indeed, it is the true model for the FTS. This is similar to Escribano and Jorda (2001) who applied both specification procedures (TP and EJP) and ESTAR model that EJP selected was chosen on the basis of best fit. However, some analysts applied both specification procedure and either LSTAR or ESTAR model was selected (Dijk *et al*, 2002; Effiong et al, 2023 and Effiong et al, 2024)

Owing to the presence of ARCH effects, SPLSTAR-GARCH (1, 1) and ESTAR-GARCH (1,1) models with generalized hyperbolic skew-student's t (ghst) innovations and AR(2)- GARCH (1,1) models with snorm innovations were specified and fitted to PBSR. SPLSTAR-GARCH (1, 1) model fitted well to PBSR in terms of AIC value and FRMSE. Also, the overall prediction performance of SPLSTAR-GARCH (1 1) model is better than its linear counterpart which according to Tong and Lim (1980), it is one of the requirements for applying nonlinear time series model.

5.0 Conclusion

BETAGLASS stock index experience gradual growth rate over time, except in 2020 when BSI witnessed sharp fall due to Covid-19 pandemics. Escribano-Jorda procedure is more accurate than Terasvirta procedure particularly when the true model for the FTS is any of the symmetric STAR models. SPLSTAR-GARCH (1, 1) and ESTAR-GARCH (1,1) models with ghst innovations fitted well to PBSR compare to AR(2)- GARCH (1,1) models with snorm innovations and AR (2) model. SPLSTAR-GARCH (1, 1) and ESTAR-GARCH (1, 1) model have better overall prediction performance than its linear counterpart, SPLSTAR-GARCH (1 1) model is the most efficient model for modeling PBSR based on AIC and FRMSE.

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Acknowledgement

The authors are grateful to the Tertiary Education Trust Fund of Nigeria (TETfund) for supporting this work. The authors also acknowledge the support of the Akwa Ibom State Polytechnic for providing the facilities needed for the study.

Compliance with Ethical Standards Declaration

Ethical Approval

Not Applicable

Competing interests

The authors declare that they have no known competing financial interests

Funding

The research that produced this work was fully sponsored by the TETfund through an institutional Based Research grant

Authors' Contributions

Both authors contributed equally to the design, analysis and writing of the paper.