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An Economic Production Quantity model with shortages, a variable lead time and a variable holding cost

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Abstract: *An economic production quantity model with shortages, a variable lead time and a variable holding cost represents a common real-life observation. It provides quantitative insight into a serious practical problem where costs are amplified due to production delay. Previous models incorporating variable lead time assume that the holding cost is constant for the entire inventory cycle. A mathematical model has been developed for determining the optimal production quantity and the optimal cycle time. The holding cost is considered as an increasing function of the ordering cycle length. Differential calculus is used for finding the optimal solution. A numerical example is used to validate the proposed model.*

Keywords: *Inventory model, variable lead time, variable holding cost, shortage cost, Optimization*

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1.0 ` Introduction

The primary operation strategies and goals of most manufacturing firms are to seek for a high satisfaction to customer's demands and to become a low-cost producer. To achieve these goals, the company must be able to effectively utilize resources and minimize costs. In the manufacturing sector, when items are produced internally instead of being obtained from an external supplier, the economic production quantity (EPQ) model can be employed to determine the optimal production lot size that minimizes overall production/inventory costs. In the classical economic production quantity (EPQ) model, lead time and holding cost are constant. The usefulness of an inventory model in managerial decision making requires some of its usual parameters to be decision variables. However, variable lead time and variable holding cost are factors that are not strange to purchasing managers Also, in most inventory problems, the shortage cost is one of the components in the objective function, but in practice, the shortage cost includes intangible components such as loss of goodwill and potential delay to the other parts of the system.

For the past decade, some scholars have been challenging the assumptions of constant lead time and constant holding cost. For example, Nasri *et al.* (1990) investigated the impact of setup cost reduction on economic order quantity (EOQ) model under the variable lead time environment. Paknejad *et al*. (1992) however, extended the work of Nasri *et al*. (1990) by including the option of investing in lead time variability reduction. Study conducted by Sarker and Coates (1997) presented an economic manufacturing quantity (EMQ) model with variable lead time and considered a finite number of opportunities for setup cost reduction investment. According to Ouyang and Chuang (2001), under most market behavior as shortages occur, the length of lead time and the amount of shortage will increase while the proportion of customers who wait and the backorder rate will decreased. Beltran and Krass (2002) analyzed a dynamic lot sizing problem with positive or negative demands and allowed disposal of excess inventory. They assumed deterministic time-varying demands and concave holding costs where an efficient dynamic programming algorithm is developed for the finite time horizon of the problem. Goh (1994) apparently provides the only existing inventory model in which the demand is stock dependent and the holding cost is time dependent. Actually, Goh (1994) considers two types of holding cost variation: (a) a nonlinear function of storage time and (b) a nonlinear function of storage level. Alfares (2007) developed an inventory model with inventory-level dependent demand rate and variable holding cost. He considered holding cost as a step function of storage time in two cases: retroactive holding cost increase and incremental holding cost increase. Ghasemi and

Nadjafi (2013) developed inventory models with varying holding cost. They developed two models. The first model was optimized without shortages while the second model considered shortages. Yang (2014) studied an inventory model with both stock-dependent demand rate and stock-dependent holding cost rate. He formulated two models. The first model considered shortage cost while the second was optimized without shortages.

Though both the lead time and holding cost have been recognized as cruxes of elevating productivity, there has been little literature simultaneously examining the effects of these two factors on the inventory-control systems. And hence, we would like to investigate such an issue and extend the recent study presented by Chang (2004) in this paper. The layout of this paper is organised as follows. In Section 2, we give relevant notations and assumptions for the proposed model. In section 3, we present a mathematical formulation of the model. Numerical example is given in section 4. Finally, we summarize our findings in section 5 and provide some suggestions for future research.

1.1 Notations and Assumptions

The following are notations applied in the development of the model:

D – Demand Rate

P – Production Rate, P > D

K – Set-up Cost per set-up

T – Inventory cycle length

Q – Economic Production Quantity, where **Q** = DT

h – Holding cost per unit per unit of time, a random variable with a uniform distribution,

$$f(h) = \frac{1}{T}, \quad 0 \le h \le T$$
$$= 0, \quad \text{otherwise.}$$

b – Backorder (shortage) cost per unit per unit of time

t – Reorder time, (time from the start of the cycle)

L – Lead time, a random variable with a uniform distribution,

$$f(L) = \frac{1}{\beta - \alpha}, \quad \alpha \le L \le \beta$$
$$= 0, \quad \text{otherwise.}$$

In addition, the following assumptions are made in developing our mathematical model:

i. The demand rate for the product is known and is a constant
ii. The delivery of the product is wholesale
iii. A single item is considered
iv. The production rate is known and is a constant that is greater than the demand rate
v. The replenishment lead time is stochastic and finite range
vi. During lead time, shortages may be allowed and all shortages are backlogged
vii. Holding cost is an increasing function of period length

2.0 Model formulation

In this model, we assume that the total amount of the ordered materials will be delivered to the store where shortages are allowed. The behaviours of the model is presented in figure 1.

The Expected Annual Total Cost is calculated as follows:

EAC = Set-up Cost + Holding Cost + Backorder Cost

$$= \frac{K}{T} + \int_{\alpha}^{\beta}\left[\left\{\frac{1}{T}\int_{0}^{T}\frac{hD(1-D/P)}{2T}\left[T - \frac{(L-T)}{(1-D/P)}\right]^2 dt\right\} + \frac{bD(L-T)^2}{2T(1-D/P)}\right] f(L)dL \qquad (1)$$

We simplify (1) to have:

$$\frac{1}{T}\int_{0}^{T}\frac{hD(1-D/P)}{2T}\left[T - \frac{(L-t)}{(1-D/P)}\right]^2 dt = \frac{hD\rho}{2T^2}\left[T^3 - \frac{2T^2L}{\rho} + \frac{T^3}{\rho} + \frac{L^2T}{\rho^2} - \frac{LT^2}{\rho^2} + \frac{T^3}{3\rho^2}\right] \qquad (2)$$

$$= \frac{K}{T} + \int_{\alpha}^{\beta}\left[\frac{hD\rho}{2T^2}\left[T^3 - \frac{2T^2L}{\rho} + \frac{T^3}{\rho} + \frac{L^2T}{\rho^2} - \frac{LT^2}{\rho^2} + \frac{T^3}{3\rho^2}\right] + \frac{bD}{2T\rho}\left[L^2 - 2Lt + t^2\right]\right] f(L)dL \qquad (3)$$

$$= \frac{K}{T} + \frac{hD\rho}{2T^2}\left[T^3 - \frac{2T^2\mu_L}{\rho} + \frac{T^3}{\rho} + \frac{(\sigma_L^2+\mu_L^2)T}{\rho^2} - \frac{\mu_L T^2}{\rho^2} + \frac{T^3}{3\rho^2}\right] + \frac{bD}{2T\rho}\left[(\sigma_L^2 + \mu_L^2) + 2\mu_L t + t^2\right] \qquad (4)$$

$$= \frac{K}{T} + \frac{hD}{2}\left[\rho T - 2\mu_L + T + \frac{(\sigma_L^2+\mu_L^2)}{\rho T} - \frac{\mu_L}{\rho} + \frac{T}{3\rho}\right] + \frac{bD}{2T\rho}\left[\sigma_L^2 + (\mu_L - t)^2\right] \qquad (5)$$

$$= \frac{K}{T} + \frac{hD}{2}\left[\frac{T(3\rho^2+3\rho+1)}{3\rho} - \frac{\mu_L(2\rho+1)}{\rho} + \frac{(\sigma_L^2+\mu_L^2)}{\rho T}\right] + \frac{bD}{2T\rho}\left[\sigma_L^2 + (\mu_L - t)^2\right] \qquad (6)$$

$$EAC = \frac{K}{T} + \frac{hD}{2}\left[\frac{T(3\rho^2+3\rho+1)}{3\rho} - \frac{\mu_L(2\rho+1)}{\rho} + \frac{(\sigma_L^2+\mu_L^2)}{\rho T}\right] + \frac{bD}{2T\rho}\left[\sigma_L^2 + (\mu_L - t)^2\right] \qquad (7)$$

$$\mu_L = \int_{\alpha}^{\beta} L f(L)dL, \quad \sigma_L^2 = \int_{\alpha}^{\beta} L^2 f(L)dL - \mu_L^2, \quad \rho = 1 - D/P$$

μ_L and σ_L^2 are the mean and variance of the lead time respectively.

2.1 The Optimal Inventory Policy for the model

Using the classical optimization technique, we derive the optimal value for T and Q such that EAC is minimum.

Differentiating equation (7) with respect to T and solving for T, we have:

$$\frac{\partial EAC}{\partial T} = \frac{-K}{T^2} + \frac{hD}{2}\left[\frac{3\rho^2+3\rho+1}{3\rho} - \frac{(\sigma_L^2+\mu_L^2)}{\rho T^2}\right] - \frac{bD}{2T^2\rho}\left[\sigma_L^2 + (\mu_L - t)^2\right] = 0 \quad (8)$$

$$\frac{K}{T^2} + \frac{hD(\sigma_L^2+\mu_L^2)}{2\rho T^2} + \frac{bD(\sigma_L^2+(\mu_L-t)^2)}{2\rho T^2} = \frac{hD}{2}\left[\frac{3\rho^2+3\rho+1}{3\rho}\right] \quad (9)$$

$$\frac{2\rho K + hD(\sigma_L^2+\mu_L^2) + bD(\sigma_L^2+(\mu_L-t)^2)}{2\rho T^2} = \frac{hD(3\rho^2+3\rho+1)}{6\rho} \quad (10)$$

$$\frac{2T^2\rho}{6\rho} = \frac{2\rho K + D[h(\sigma_L^2+\mu_L^2) + b(\sigma_L^2+(\mu_L-t)^2]}{hD(3\rho^2+3\rho+1)} \quad (11)$$

$$T^2 = \frac{6\rho K + 3D[h(\sigma_L^2+\mu_L^2) + b(\sigma_L^2+(\mu_L-t)^2]}{hD(3\rho^2+3\rho+1)} \quad (12)$$

$$T^* = \sqrt{\frac{6\rho K + 3D[h(\sigma_L^2+\mu_L^2) + b(\sigma_L^2+(\mu_L-t)^2]}{hD(3\rho^2+3\rho+1)}} \quad (13)$$

The Optimal Economic Production Quantity follows by the relation;

$$Q^* = DT^* \quad (14)$$

$$Q^* = D\sqrt{\frac{6\rho K + 3D[h(\sigma_L^2+\mu_L^2) + b(\sigma_L^2+(\mu_L-t)^2]}{hD(3\rho^2+3\rho+1)}} \quad (15)$$

3.0 Numerical example

To illustrate the proposed models and the results derived, let us consider an inventory system with the following known parameters as used in Ouyang and Chang (2000):

D = 5200 units/year, K = \$500/setup, h = \$10/unit/year, b = \$20/unit/year,

μ_L = 0.028846 years (1.5 weeks), P = 10400, σ_L^2= 0.0000308 (year^2).

Consider the cases that t = 0 to 10 years when ρ = 0.5 and μ_L^2 = 0.000863.

In **Table 1**, the optimal solutions for selected values of t ranging from 0 to 10 are examined. The results shows that the economic production quantity increases as t increases: the lot size is inflated when t approaches the final point due to the shortage cost. The expected annual total cost increases as the economic production quantity increases as well as T* (order cycle length)

Table 1: optimal solutions for different values of t when backorders is allowed

t	T*	Q*	EAC*
0	0.1063	553	8976.08
1	1.3232	6881	146084.98
2	2.6801	13936	298959.06
3	4.0382	20999	451975.03
4	5.3966	28063	605025.76
5	6.7552	35127	758090.30
6	8.1138	42192	911161.70
7	9.4724	49257	1064237.00
8	10.8311	56322	1217314.75
9	12.1898	63387	1370394.12
10	13.5485	70452	1523474.63

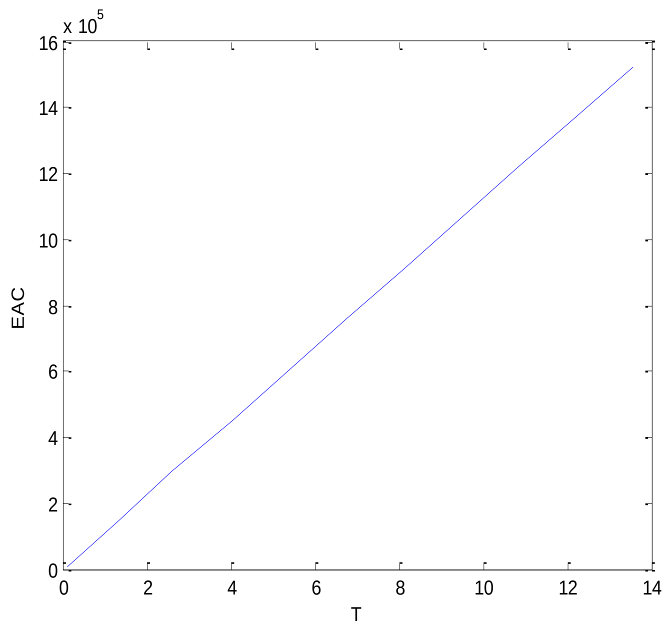

Fig. 1: Expected Annual Total Cost versus cycle length.

4.0 Conclusion and Future Research

In this paper, a model describing an inventory system with shortages, a variable lead time and a variable holding cost has been presented. The holding cost is considered as an increasing function of ordering cycle length. A single item was considered. The newly developed model was examined using a numerical example. The preliminary result from the numerical example showed that the expected average cost for the model, the cycle time and the production quantity increase when the reorder time is increased.

The model presented in this study provides a basis for several possible extensions. For future research, this model can be extended to include a partial backorder case and a non-instantaneous receipt of orders. Another possible extension may be to consider the holding cost as a decreasing step function of time.

6.0 References

Alfares, H.K. (2007). Inventory model with stock-level dependent demand rate and variable holding cost. *International Journal of Production Economics* 108, pp. 259- 265.

Beltran, J.L. & Krass, D. (2002). Dynamic lot sizing with returning items and disposals. *IIE Transactions,* 34, 5, pp. 437–448

Chang, H. C. (2004): A note on the EPQ model with shortages and variable lead time. *Information and Management Sciences*,15(1), 61-67.

Ghasemi, N. & Nadjafi, A.B. (2013). EOQ models with varying holding cost. *Journal of industrial mathematics*. Hindawi publishing company. doi:10.1155 /2013 /749 -21

Nasri, F., Affisco, J. F. & Paknejad, M. J. (1990). Setup cost reduction in an inventory model with finite range stochastic lead-times. *International. Journal of production resources*, 28, pp. 199-212.

Ouyang, L. Y. & Chang, H. C. (2000). EMQ model with variable lead time and imperfect production process. *Information and Management Sciences*, 11, 1, pp. 1-10.

Ouyang, L. Y. & Chuang, B. R. (2001). Mixture inventory model involving variable lead time and controllable backorder rate, *Computers and Industrial Engineering*, 40, 4, pp.339–348.

Paknejad, M. J., Nasri, F. & Affisco, J. F. (1992) Lead-time variability reduction in stochastic inventory models. *European Journal of Operations Research*, 62, pp. 311-322.

Sarker, B. R. & Coates, E. R. (1997). Manufacturing setup cost reduction under variable lead times and finite opportunities for investment. *International Journal of Production Economics,* 49, pp. 237-247.

Yang, C (2014). An Inventory model with both stock-dependent demand and stock-dependent holding cost. *International Journal of Production Economics,* 155, pp. 214 – 221.

Conflict of Interest

The authors declared no conflict of interest