Communication in Physical Sciences, 2021, 7(4): 363-369

Modification of Dual to Separate Product-type Exponential Estimator in Case of Post- Stratification

Abubakar Yahaya and Adam Rabiu

Received: 12 September 2021/Accepted 28 November 2021/Published online:25 December 2021

Abstract: *This article proposes a modification of dual to separate product-type exponential estimator in the case of post-stratification. The bias and Mean Square Error (MSE) of the suggested estimator have been studied up to the first degree of approximation. It has been shown that the suggested estimator is more efficient than Lone and Tailor estimators and in the same vein, efficiency conditions of which the proposed estimator would be better are derived. Finally, an empirical study has been carried out to demonstrate the performance of the suggested estimator*

Keywords: *Post-stratification, Separate estimator, bias, Mean Square Error*

Abubakar Yahaya
Department of Statistics
Ahmadu Bello University, Zaria
Kaduna State, Nigeria
Email: adamrabiutk@gmail.com

Adam Rabiu
Department of Statistics
Ahmadu Bello University, Zaria
Kaduna State, Nigeria
Email:

1.0 Introduction

In some literature, it has been shown by various researchers such as Reddy (1974), Srivenkaramana (1978) that the bias and MSE of the ratio and product estimators can be reduced with the applications of transformation on the auxiliary variable. Srivenkaramana (1980) used the transformation to propose a dual ratio estimate of the population mean in simple random sampling sampling.In the same vein, Lone and Tailor (2015) proposed a dual to separate product type exponential estimator in the post-stratification approach.

Ratio, Product type and linear regression estimator have been extensively used when the auxiliary information is available and it is well established that, there are good examples of estimators that make good use of auxiliary information at the estimation stage. When the study variable and the auxiliary variable are positively correlated and the line passes through the origin, the ratio type estimator is employed to improve the estimation of the population parameters under study provided by Lone and Tailor (2014). However, the product type estimators are used to improve the estimation of parameters when the interesting variable and auxiliary variable are negatively correlated as given by Lone and Tailor (2015). The regression type estimators are for estimation when the line of regression does not pass through the origin. It is observed that, all these methods capitulate biased estimators and of course bias decrease with increasing sample size Okafor (2002). Many researchers have made several efforts to develop more and more ratio-type or product-type estimators with less MSE than the existing estimator.

Recently Gamze (2015) proposed modified exponential type estimators for population mean in stratified sampling and Kadila (2016) proposed a new exponential type estimator for the population mean in simple random sampling. Moreover, Abubakar *et al* (2019) examined the estimator of Kadilar (2016) for combined ratio estimator in two-phase sampling. However, none of these studies examines these estimators under post-stratification sampling.

Many researchers including; Singh and Espeyo (2003), Vishwakarma and Singh (2011), Tailor *et al* (2011), Lone and Tailor (2015) among others contribute significantly to this area of research.

2.0 Materials and Methods

Consider a finite population $U = U_1, U_2, \ldots, U_N$ of size N. A sample of size n is drawn from population U using simple random sampling without replacement. After selecting the sample, it is observed that some units belongs to the h^{th} stratum. Let n_h be the size of the sample falling in h^{th} stratum such that $\sum_{h=1}^{L} n_h = n$. Here it is assumed that n is so large that possibility of n_h being zero is very small. Let x_{hi} be the observation on i^{th} unit that falls in h^{th} stratum for auxiliary variate x and y_{hi} be the observation on i^{th} unit that falls in h^{th} stratum for study variate y, then

$\bar{X}_h = \frac{1}{N}\sum_{i=1}^{Nh} x_{hi}$: h^{th}Stratum mean for auxiliary variate x

$\bar{Y}_h = \frac{1}{N}\sum_{i=1}^{Nh} y_{hi}$: h^{th}Stratum mean for study variate y

$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{Nh} x_{hi} = \sum_{h=1}^{L} W_h \bar{X}_h$: Population mean of auxiliary variate

$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{Nh} y_{hi} = \sum_{h=1}^{L} W_h \bar{Y}_h$: Population mean of interested variate y

In the case of post-stratification, the usual unbiased estimator of population mean $\bar{Y}$ is defined as:

$$\bar{y}_{ps} = \sum_{h=1}^{L} W_h \bar{y}_h$$

Where $W_h = \frac{N_h}{N}$ is called the weight of the h^{th} stratum

and $\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$ is called sample mean of n_h sample unit that falls in the h^{th} stratum

Using Stepha (1945), the variance of $\bar{y}_{ps}$ to the first degree of approximation is obtained as

$$Var(\bar{y}_{ps}) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{n^2}\sum_{h=1}^{L}(1 - W_h)S_{yh}^2 \quad (1)$$

Where $S_{yh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)^2$

Lone and Tailor (2015) defined the estimator with their respective MSE as:

$$\hat{\bar{Y}}_{pps}^{*} = \bar{y}_h \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h}{\bar{x}_h^{*}}\right) \quad (2)$$

The MSE of $\hat{\bar{Y}}_{pps}^{*}$

$$MSE\left(\hat{\bar{Y}}_{pps}^{*}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h S_{yh}^2 + \sum_{h=1}^{L} W_h R_h^2 a_h^2 S_{xh}^2 + 2\sum_{h=1}^{L} W_h a_h R_h S_{yxh}\right] \quad (3)$$

where $x_h^{*} = \frac{N_h \bar{X}_h - n_h \bar{x}_h}{N_h - n_h}$, $R_h = \frac{\bar{Y}_h}{\bar{X}_h}$ and $a_h = \frac{n_h}{N_h - n_h}$

$$\hat{\bar{Y}}_{ps}^{spe} = \sum W_h \bar{y}_h \left(\frac{\bar{x}_h - \bar{X}_h}{\bar{x}_h + \bar{X}_h}\right) \quad (4)$$

The MSE of equation (4) is given as

$$MSE\left(\hat{\bar{Y}}_{ps}^{spe}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h \left(S_y^2 + \frac{1}{4}R_y^2 S_{xy}^2 + R_h S_{yxh}\right) \quad (5)$$

where $S_{xy}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(x_{hi} - \bar{X}_h\right)^2$ and $S_{yx} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)\left(x_{hi} - \bar{X}_h\right)$

$$\hat{\bar{Y}}_{ps}^{pe} = \bar{y}_{ps}\exp\left(\frac{\bar{x}_{ps} - \bar{X}}{\bar{x}_{ps} + \bar{X}}\right) \tag{6}$$

The MSE of equation (6) up to the first degree of approximation is

$$MSE\left(\hat{\bar{Y}}_{ps}^{pe}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(S_{yh}^2 + \frac{1}{4}R_y^2 S_{xy}^2 + RS_{yxh}\right) \tag{7}$$

Where $R = \frac{\bar{Y}}{\bar{X}}$

$$\hat{\bar{Y}}_{ps}^{*pe} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left(\frac{\bar{X}_h - \bar{x}_h^*}{\bar{X}_h + \bar{x}_h^*}\right) \tag{8}$$

The MSE of equation (8) up to the first degree of approximation is

$$MSE\left(\hat{\bar{Y}}_{ps}^{*pe}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{4}\sum_{h=1}^{L} W_h a_h^2 R_y^2 S_{xy}^2 + \sum_{h=1}^{L} W_h a_h R_h S_{yxh}\right] \tag{9}$$

2.1 *Proposed estimator*

Motivated by Lone and Tailor (2015) and following the concept of Srivenkataramana (1980) and Bandyopadyhay (1980) transformations, we proposed modification of dual to separate product type exponential estimator under post-stratification sampling scheme as :

$$\hat{\bar{Y}}_{ps}^{**p} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{x}_h^*}{\bar{x}_h}\right)^{\lambda} \exp\left(\frac{\bar{x}_h - \bar{x}_h^*}{\bar{x}_h + \bar{x}_h^*}\right) \tag{10}$$

where $\bar{x}_h^* = \frac{N_h \bar{X}_h - n_h \bar{x}_h}{N_h - n_h}$ and λ is constant

To obtain the bias and MSE of the proposed estimator $\hat{\bar{Y}}_{ps}^{**p}$, we write

$y_h = \bar{Y}_h(1 + e_{oh})$, $\bar{x}_h = \bar{X}_h(1 + e_{1h})$, $\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h$, $\bar{X} = \sum_{h=1}^{L} W_h \bar{X}_h$, Such that

$E(e_{oh}) = E(e_{1h}) = 0$

$E\left(e_{oh}^2\right) = \left(\frac{1}{nwh} - \frac{1}{Nh}\right)C_{yh}^2$, $E\left(e_{1h}^2\right) = \left(\frac{1}{nwh} - \frac{1}{Nh}\right)C_x^2$ and $E\left(e_{oh}e_{1h}\right) = \left(\frac{1}{nwh} - \frac{1}{Nh}\right)\rho_{yxh}C_{yh}C_{xb}$

Where $C_y = \frac{S_{yh}}{\bar{Y}}$, $C_x = \frac{S_{xh}}{\bar{X}}$ and $\rho_{yxh} = \frac{S_{yxh}}{S_{yh}S_{xh}}$

$$\hat{\bar{Y}}_{ps}^{**p} = \sum_{h=1}^{L} W_h \bar{y}_h \left[\frac{N_h \bar{X}_h - n_h \bar{x}_h}{\bar{X}_h\left(N_h - n_h\right)}\right]^{\lambda} \exp\left\{\frac{\left[\bar{X}_h - \left(\frac{N_h \bar{X}_h - n_h \bar{x}_h}{N_h - n_h}\right)\right]}{\bar{X}_h + \left(\frac{N_h \bar{X}_h - n_h \bar{x}_h}{N_h - n_h}\right)}\right\}$$

$$\hat{\bar{Y}}_{ps}^{**p} = \sum_{h=1}^{L} W_h \bar{y}_h \left[\frac{N_h \bar{X}_h - n_h \bar{x}_h}{\bar{X}_h\left(N_h - n_h\right)}\right]^{\lambda} \exp\left[\frac{\left[\left(\frac{n_h \bar{x}_h - n_h \bar{X}_h}{N_h - n_h}\right)\right]}{\left(\frac{2N_h \bar{X}_h - n_h \bar{X}_h - n_h \bar{x}_h}{N_h - n_h}\right)}\right]$$

$$\hat{\bar{Y}}_{ps}^{**p} = \sum_{h=1}^{L} W_h \bar{y}_h \left[\frac{N_h\bar{X}_h - n_h\bar{x}_h}{\bar{X}_h(N_h-n_h)}\right]^{\lambda} \exp\left[\frac{\left(\frac{n_h\bar{x}_h}{N_h-n_h} - \frac{n_h\bar{X}_h}{N_h-n_h}\right)}{\left(\frac{2N_h\bar{X}_h}{N_h-n_h} - \frac{n_h\bar{X}_h}{N_h-n_h} - \frac{n_h\bar{x}_h}{N_h-n_h}\right)}\right]$$

$$\hat{\bar{Y}}_{ps}^{**p} = \sum_{h=1}^{L} W_h \bar{y}_h \left[\frac{N_h\bar{X}_h - n_h\bar{x}_h}{\bar{X}_h(N_h-n_h)}\right]^{\lambda} \exp\left[\frac{\gamma_h\bar{x}_{h} - \gamma_h X_{h}}{\left(\frac{2N_h\bar{X}_h}{N_h-n_h} - \gamma_h\bar{X}_{h_h} - \gamma_h\bar{x}_h\right)}\right]$$

Where $\gamma_h = \frac{n_h}{N_h-n_h}$ and recalling that $\bar{y}_h = \bar{Y}_h(1+e_{oh})$, $\bar{x}_h = \bar{X}_h(1+e_{oh})$

then

$$\hat{\bar{Y}}_{ps}^{**p} = \sum_{h=1}^{L} W_h \bar{y}_h \left[\frac{N_h\bar{X}_h - n_h\bar{X}_h(1+e_{oh})}{\bar{X}_h(N_h-n_h)}\right]^{\lambda} \exp\left[\frac{\gamma_h\bar{X}_h(1+e_{oh}) - \gamma_h X_{h}}{\left(\frac{2N_h\bar{X}_h}{N_h-n_h} - \gamma_h\bar{X}_{h_h} - \gamma_h\bar{X}_h(1+e_{oh})\right)}\right]$$

$$\hat{\bar{Y}}_{ps}^{**p} = \sum_{h=1}^{L} W_h \bar{y}_h (1+e_{oh})[1-\gamma_h e_{1h}]^{\lambda} \exp\left[\frac{\gamma_h\bar{X}_h e_{1h}}{2\bar{X}_h - \gamma_h\bar{X}_h e_{1h}}\right]$$

$$\hat{\bar{Y}}_{ps}^{**p} = \sum_{h=1}^{L} W_h \bar{y}_h (1+e_{oh})[1-\gamma_h e_{1h}]^{\lambda} \exp\left[\left(\frac{\gamma_h e_{1h}}{2}\right)\left(1-\frac{\gamma_h e_{1h}}{2}\right)^{-1}\right]$$

$$\hat{\bar{Y}}_{ps}^{**p} = \sum_{h=1}^{L} W_h \bar{y}_h (1+e_{oh})\left[1-\lambda\gamma_h e_{1h} + \frac{\lambda(\gamma_h-1)\gamma_h^2 e_{1h}^2}{2}\right]\left[1+\frac{\gamma_h e_{1h}}{2} + \frac{3\gamma_h^2 e_{1h}^2}{8}\right] \quad (12)$$

Expanding the right hand side of equation (12) and returning terms up to the second power of e's we have;

$$\hat{\bar{Y}}_{ps}^{**p} - \bar{Y} = \bar{Y}\left[\begin{array}{l}\frac{\gamma_h e_{1h}}{2} + \frac{3\gamma_h e_{ih}^2}{8} - \lambda\gamma_h e_{1h} - \frac{\lambda\gamma_h^2 e_{ih}^2}{2} + \frac{\lambda(\lambda-1)\gamma_h^2 e_{ih}^2}{2} + e_{0h} \\ + \frac{\gamma_h e_{0h}e_{1h}}{2} + \frac{3\gamma_h^2 e_{0h}e_{ih}^2}{8} - \gamma_h e_{0h}e_{1h} - \lambda\gamma_h e_{0h} + \frac{\lambda(\lambda-1)\gamma_h e_{0h}e_{1h}}{2}\end{array}\right] \quad (13)$$

To obtain bias, we take the expectation of both sides of equation (13)

$$E\left[\hat{\bar{Y}}_{ps}^{**p} - \bar{Y}\right] = \sum_{h=1}^{L} W_h \bar{Y}_h\left[\begin{array}{l}\frac{\gamma_h E(e_{1h})}{2} + \frac{3\gamma_h E(e_{ih}^2)}{8} - \lambda\gamma_h E(e_{1h}) - \frac{\lambda\gamma_h^2 E(e_{ih}^2)}{2} + \frac{\lambda(\lambda-1)\gamma_h^2 E(e_{ih}^2)}{2} + E(e_{0h}) \\ + \frac{\gamma_h E(e_{0h}e_{1h})}{2} + \frac{3\gamma_h^2 E(e_{0h}e_{ih}^2)}{8} - \gamma_h E(e_{0h}e_{1h}) - \lambda\gamma_h E(e_{0h}) + \frac{\lambda(\lambda-1)\gamma_h E(e_{0h}e_{1h})}{2}\end{array}\right]$$

$$E\left[\hat{\bar{Y}}_{ps}^{**p} - \bar{Y}\right] = \sum_{h=1}^{L} W_h \bar{Y}_h\left[\begin{array}{l}\frac{3\gamma_h E(e_{ih}^2)}{8} - \frac{\lambda\gamma_h^2 E(e_{ih}^2)}{2} + \frac{\lambda(\lambda-1)\gamma_h^2 E(e_{ih}^2)}{2} + \frac{\gamma_h E(e_{0h}e_{1h})}{2} \\ -\gamma_h E(e_{0h}e_{1h}) + \frac{\lambda(\lambda-1)\gamma_h E(e_{0h}e_{1h})}{2}\end{array}\right]$$

By squaring both side of equation (13) and then taking expectation and retaining terms up to the second power of e's, we get the MSE of the proposed estimator $\hat{\bar{Y}}_{ps}^{**p}$ up to the first degree of approximation; then we can say that

$$E\left[\hat{\bar{Y}}_{ps}^{**p} - \bar{Y}\right]^2 = \bar{Y}^2 E\left[\frac{\gamma_h e_{1h}}{2} - \lambda\gamma_h e_{1h} + e_{0h}\right]^2$$

$$E\left[\hat{\bar{Y}}_{ps}^{**p} - \bar{Y}\right]^2 = \bar{Y}^2\left[\frac{\gamma_h^2 E\left(e_{ih}^2\right)}{4} - \lambda^2\gamma_h^2 E\left(e_{ih}^2\right) + E\left(e_{0h}^2\right) - \lambda\gamma_h^2 E\left(e_{1h}^2\right) + \gamma_h E\left(e_{0h}e_{1h}\right) - 2\lambda\gamma_h E\left(e_{0h}e_{1h}\right)\right]$$

The mean square error $\hat{\bar{Y}}_{ps}^{**p}$ is

$$MSE\left(\hat{\bar{Y}}_{ps}^{**p}\right) = \bar{Y}^2\left[\begin{array}{l}\dfrac{\gamma_h^2 E\left(e_{ih}^2\right)}{4} - \lambda^2\gamma_h^2 E\left(e_{ih}^2\right) + E\left(e_{0h}^2\right) - \lambda\gamma_h^2 E\left(e_{1h}^2\right)\\ + \gamma_h E\left(e_{0h}e_{1h}\right) - 2\lambda\gamma_h E\left(e_{0h}e_{1h}\right)\end{array}\right] \quad (14)$$

We obtain the optimum value (λ) to minimize $MSE\left(\hat{\bar{Y}}_{ps}^{**p}\right)$ by differentiating $MSE\left(\hat{\bar{Y}}_{ps}^{**p}\right)$ with respect to λ and equating the derivative to zero

$$\frac{\partial MSE\left(\hat{\bar{Y}}_{ps}^{**p}\right)}{\partial\lambda} = \bar{Y}^2\left[2\lambda\gamma_h^2 E\left(e_{ih}^2\right) - \gamma_h^2 E\left(e_{1h}^2\right) - 2\gamma_h E\left(e_{0h}e_{1h}\right)\right] = 0$$

$$\hat{\lambda}_{opt} = \frac{2\bar{Y}_h^2 E\left(e_{0h}e_{1h}\right) + \gamma_h \bar{Y}_h^2 E\left(e_{1h}^2\right)}{2\gamma_h \bar{Y}_h^2 E\left(e_{1h}^2\right)} \quad (15)$$

Substitute equation (15) in equation (14), we get the minimum value of $MSE\left(\hat{\bar{Y}}_{ps}^{**p}\right)$ by saying that;

$$MSE_{\min}\left(\hat{\bar{Y}}_{ps}^{**p}\right) = \bar{Y}^2\left\{E\left(e_0^2\right) - \frac{\left[E\left(e_{0h}e_{1h}\right)\right]^2}{E\left(e_1^2\right)}\right\}$$

$$MSE_{\min}\left(\hat{\bar{Y}}_{ps}^{**p}\right) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\sum_{h=1}^{L} W_h^2 \bar{Y}_h^2\left[C_{yh}^2 - \frac{\rho_{yxh}^2 C_{yh}^2 C_{xh}^2}{C_{Xh}^2}\right]$$

$$MSE_{\min}\left(\hat{\bar{Y}}_{ps}^{**p}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(C_{yh}^2 - \rho_{yxh}^2 C_{yh}^2\right)$$

$$MSE_{\min}\left(\hat{\bar{Y}}_{ps}^{**p}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h C_{yh}^2 \bar{Y}_h^2\left(1 - \rho_{yxh}^2\right)$$

2.2 Efficiency comparison of the proposed estimator $\hat{\bar{Y}}_{ps}^{**p}$

A theoretical comparison of the proposed estimator has been made with other competing estimators of the population mean. Therefore, a comparison of (1), (3), (5), (7) and (9) shows that the proposed estimator $\hat{\bar{Y}}_{ps}^{**p}$ would be more efficient than

$\bar{y}_{ps}$ if $\beta > -\sum_{h-1}^{L} W_h S_{yh}^2$

$\hat{\bar{Y}}_{ps}^{s}$ if $\beta > \sum_{h-1}^{L} W_h R_h \left(R_h S_{xh}^2 - S_{yxh} \right)$

$\hat{\bar{Y}}_{ps}^{*}$ if $\beta > \sum_{h-1}^{L} W_h a_h R_h \left(a_h R_h S_{xh}^2 + 2S_{yxh} \right)$

$\hat{\bar{Y}}_{ps}^{spe}$ if $\beta > \sum_{h-1}^{L} W_h R_h \left(\frac{1}{4} R_h S_{xh}^2 + S_{yxh} \right)$

$\hat{\bar{Y}}_{ps}^{pe}$ if $\beta > \sum_{h-1}^{L} W_h R_h \left(\frac{1}{4} a_h R_h S_{xh}^2 + S_{yxh} \right)$

where $\beta = \sum_{h-1}^{L} \frac{S_{yxh}^2}{S_{xh}^2}$

3.0 Empirical study

Explicit understanding of the advantages of the proposed estimator was validated by the treatment of some natural population data as shown in Table 1 while the MSE and percentage relative efficiency (PRE) of the proposed estimator and some conventionally related estimators over sample mean is given in Table 2

Table 1: Population data

Constant	Stratum 1	Stratum 2
n_h	4	4
N_h	10	10
$\overline{X_h}$	1629.99	2035.96
$\overline{Y_h}$	142.8	91.0
S_{xi}^2	1044.71	1066.63
S_{yi}^2	37.31	43.16
p_{yxh}	-0.38	-0.35
S_{yxh}	-239.25	-240.45

Table 2: MSE and PRE

Constant	Stratum 1	Stratum 2
$\bar{y}_{ps}$	2.4141	100.00
$\dot{\hat{\bar{Y}}}_{ps}^{*}$	2.5366	95.17
$\dot{\hat{\bar{Y}}}_{ps}^{*spe}$	2.5366	108.58
$\hat{\bar{Y}}^{*pe}$	2.1178	113.99
$\dot{\hat{\bar{Y}}}_{ps}^{**p}$	1.9805	121.8935

4.0 Conclusion

Section 2 provided the conditions under which the proposed estimator is more efficient than conventional estimators. An empirical study reveals that the proposed estimator $\dot{\hat{\bar{Y}}}_{ps}^{**p}$ has high percentage relative efficiency in comparison to Lone and Tailor's (2015) estimators for population I where the interested variate Y and the auxiliary X are negatively correlated. From the theoretical discussion and the numerical result from Table 2, we conclude that the proposed estimator is better than the mentioned estimators. Therefore, the proposed estimator is recommended to use in practice for the estimation of the population mean provided conditions given in section 2 are satisfied.

3.0 References

Agrawal, M. & Panda, K. (1993). An efficient estimator in post-stratification. Metron, 51, 3, 4, pp. 179–187.

Bandyopadhyay, S. (1980). Improved ratio and product estimators. *Sankhya*,42,1,2.pp. 45–49.

Chouhan, S. (2012). *Improved estimation of parameters using auxiliary information in sample surveys.* Ph.D.Thesis, Vikram University, Ujjain, MP India.

Gamze, Ö. (2015). Modified exponential type estimator for population mean using auxiliary variables in stratified random sampling. *Alphanumeric Journal*,3, 2, pp.49–56.

Holt, D.,& Smith, T. F. (1979). Post-stratification. *Journal of the Royal Statistical Society: SeriesA(General)*,142,1, pp. 33–46.

Lone,H.A & Tailor, R. (2014). Dual to separate ratio type exponential estimator in post stratification. *Journal of Statistics Applications&Probability*,3,3, pp. 425-432

Lone,H.A.,&Tailor, R. (2015). Dual to separate product type exponential estimator in post-stratification. *Journal of Statistics Applications and Probability*,4,3, pp. 429.

Lone, H. A.& Tailor, R., &Singh, H.P. (2016). Generalized ratio-cum-product type exponential estimator in stratified random sampling. *Communications in Statistics- Theory and Methods*,45,11, ,pp.3302–3333

Srivenkataramana,T. (1980). A dual to ratio estimator in sample surveys. *Biometrika*,67,1, pp.199–204.

Vishwakarma, G.K.,& Singh,H.P. (2011). Separate ratio-product estimator for estimating the population mean using auxiliary information. *Journal of Statistical Theory and Applications*,10,4, pp.653-664.

Conflict of Interest

The authors declared no conflict of interest