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Effects of Injection and Heat Sink/Source on Convective Kuvshinki Fluid through a Porous Medium

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Abstract: *This study investigated the effects of injection on convective boundary layer Kuvshinshiki fluid through a porous medium in the presence of a heat source/sink. The equations governing the flow were modelled as non-dimensionless partial differential equations (PDEs). The transformed dimensionless ordinary differential equations (ODEs) were solved analytically using the perturbation method. Expressions for Skin-friction, Nusselt and Sherwood numbers were obtained. The numerical values of the Kuvshinshiki fluid velocity, temperature and concentration at the plate were shown graphically for varied values of the physical parameters. It was observed that the velocity of the fluid increased as Dufour number, Kuvshinski parameter, thermal and mass Grashof numbers were increased and a reverse trend was noticed as heat source/sink increased.*

Keywords: *Kuvshinski fluid, free convection flow, heat source; Dufour effect, porous medium*

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1.0 Introduction

Convective heat transfer plays an important role in the dispensation of non-Newtonian fluids through channels such as tubes, pipes, corrugation as well as parallel walls. Oldroyd-B, Casson, walters' B and Kuvshinski are all fundamental models of non-Newtonian which have been studied by many researchers (Dada and Disu, 2015; Disu and Dada 2017a; Salawu and Disu, 2020).

Amongst the non-Newtonian models that have been developed, Kuvshinski model is most used to analysis theoretical validation of experimental investigation. Given this, the theoretical significance of Kuvshinski fluids over a porous plate was studied by Agrawal *et al* (2012) and Gurudatt and Varshney (2013). The researchers ignored induced magnetic fields in comparison to the applied magnetic field.

Kumar *et al.* (2012) presented the effects of Kuvshinski fluid on double-diffusive convective-radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and the Soret effect. Also, Kumar and Raman (2010) investigated the effects of a chemical reaction and mass transfer on radiation and MHD free convection flow of Kuvshinki fluid through a porous medium. Devasena and Leela (2014) also investigated the combined influence of chemical reaction thermo-diffusion and thermal radiation on the convective heat and mass transfer flow of a kuvshinski fluid past a vertical plate. All these studies are necessary because of the applications of radiation heat transfer in nuclear power, rockets, satellites, propulsion devices for aircraft, gas turbines, missiles, etc.

Reddy *et al.* (2015) investigated unsteady MHD free flow of convection of a Kuvshinski fluid in the presence of homogeneous chemical reaction, radiation absorption and heat source/sink. The authors reported that the heat source/sink reduced the fluid flow in the boundary layer. However, Vidyasagar *et al*. (2014) considered unsteady MHD free convection boundary layer flow of radiation-absorbing kuvshinski fluid through a porous medium. The study of fluid flow in a porous medium is very important because of its applications in water purification, underground hydrology, irrigation process in farming, chemical, agricultural, ceramic and metallurgical engineering (Disu and Dada (2017b, 2018)).

Suction/injection plays a critical function in boundary layer control, particularly in engineering, aerodynamics, space research and medicine. These applications gained the attention of researchers to investigate the effects of suction/injection in different types of fluids. As a result, Sandeep and Sulachana (2015) investigated the influence of non-uniform heat sink/source, mass transfer and chemical reaction on an unsteady mixed convection boundary layer flow. Muondwe *et al.* (2018) performed an unsteady, laminar hydrodynamic flow of an incompressible, viscous and electrically conducting Newtonian fluid over a porous contacting sheet. Effects of suction (injection) on magnetohydrodynamic (MHD) steady flow of a viscous and electrically conducting fluid were analyzed by Hamza (2019).

Motivated by the aforementioned studies and applications of a porous medium, we present the effects of injection and heat sink/source on convective Kuvshinki fluid through a porous medium. To the aim of this study, the fluid flow is governed by a modeled coupled nonlinear system of partial differential equations (PDEs) which are transformed into non-dimensional form by introducing necessary non-dimensional variables. The ODEs in non-dimensional form were solved analytically by perturbation method to obtain approximate solutions for the velocity, temperature and concentration. Further, the effects of physical parameters associated with the fluid flow were examined and discussed.

2.0 Mathematical Formulation

We considered the two-dimensional boundary layer flow of a visco-elastic Kuvshinski type fluid in a uniform porous medium is studied. A transverse magnetic field is applied in the direction of the flow field. The effects of injection and heat absorption/generation are taken into account. According to the coordinate system the x^*-axis is taken along the porous plate in the upward direction and y^*- axis is normal to it (See Figure 1). The radiative heat flow in x^*-direction is considered negligible in comparison with that in the y^*- direction. The following assumptions were further made: (1) the porous plate is of infinite length in x^*-direction (2) since the flow of the fluid is assumed to be in the direction of x^*- direction, the physical quantities are functions of y' and t' only (3) the injection normal to the plate is constant (4) visco-elastic and Darcy's resistance terms are taken into consideration with constant permeability of the medium (5) Boussinesq's approximations holds.

The equations describing the flow can be written in the Cartesian frame of reference as follows:

$$\frac{\partial v^*}{\partial y^*}=0 \tag{1}$$

$$\left(1+\lambda^*\frac{\partial}{\partial t'}\right)\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}-g\left[\beta\left(T^*-T_\infty^*\right)-\beta^*\left(C^*-C_\infty^*\right)\right]+\left(1+\lambda^*\frac{\partial}{\partial t^*}\right)\left(\frac{\sigma B_0^2}{\rho}+\frac{\nu}{k^*}\right)u^*=0 \tag{2}$$

$$\rho\left[\frac{\partial T^*}{\partial t^*}+v^*\frac{\partial T^*}{\partial y^*}-\frac{D_m k_T}{c_s c_p}\frac{\partial^2 C^*}{\partial y^{*2}}\right]-\frac{1}{Cp}\left[k_0\frac{\partial^2 T^*}{\partial y^{*2}}-\frac{\partial q_r}{\partial y'}-Q\left(T^*-T_\infty^*\right)\right]=0 \tag{3}$$

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} - D_m \frac{\partial^2 C^*}{\partial y^{*2}} + K_m \left(C^* - C_\infty^*\right) = 0 \tag{4}$$

And, the boundary conditions associated with the models are given as:

$$\begin{aligned} &u^* = 0, T^* = T_w^*, C^* = C_w^* \quad at\ y = 0 \\ &u^* \to 0, T^* \to T_\infty^*, C^* \to C_\infty^* \quad as\ \ y \to \infty \end{aligned} \tag{5}$$

where u and v are the velocity components in x^* and y^* directions respectively, T is the temperature, t is the time, g is the acceleration due to gravity, β is the thermal expansion coefficient, β^* is the concentration expansion coefficient, υ is the kinematic viscosity, D_m is the chemical molecular diffusivity, k_T is the thermal diffusion ratio, C_S is the concentration susceptibility, C_p is heat capacity at constant pressure, B_0 is a constant magnetic field intensity, σ is the electrical conductivity of the fluid, k_0 is the thermal conductivity, q_r is the radiative heat flux, k' is the Darcy permeability, ρ is the density, λ is the viscoelastic Kuvshinski parameter, K_m is the chemical reaction parameter, T_w is the wall temperature, T_∞ is the free stream temperature, C_w is the species concentration at the plate surface, C_∞ is the free stream concentration, Q is the heat source/sink coefficient.

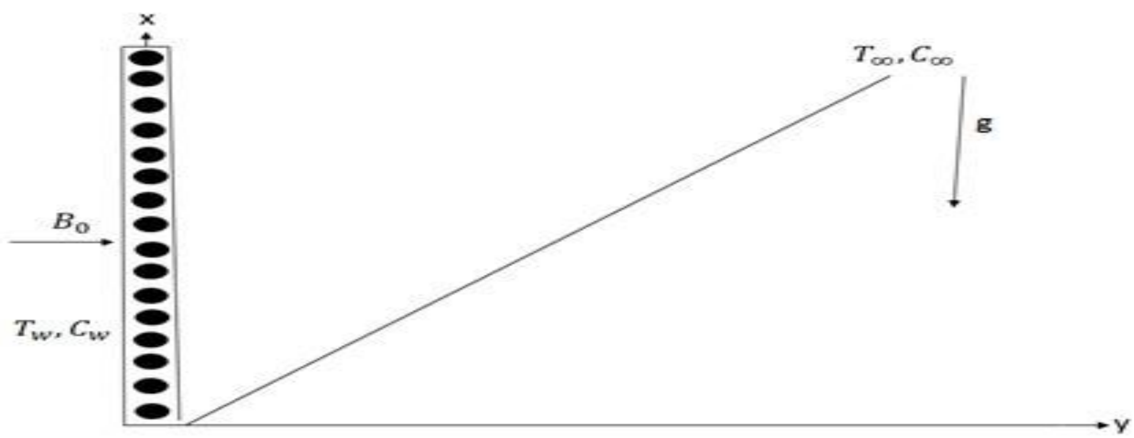

Fig. 1: Schematic diagram.

It is clear from equation 5 above that the injection velocity at the plate surface is a real positive constant it takes the following form:

$$v^* = v_0 \tag{6}$$

For the case of an optically thin gray gas, the local radiant absorption is expressed by

$$\frac{\partial q_r}{\partial y'} = -4 a_R \sigma \left(T_\infty^{*4} - T^{*4}\right) \tag{7}$$

we assume that the temperature difference within the flow is sufficiently small such that T^{*4} may be expressed as a linear function of the temperature. This is accomplished by expanding T^{*4} in Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T^{*4} \cong 4T_\infty'^3 T^* - 3T_\infty^{*4} \tag{8}$$

By using equations (6) and (8), equation (2) reduces to

$$\rho\left[\frac{\partial T^*}{\partial t^*}+v^*\frac{\partial T^*}{\partial y^*}-\frac{D_m k_T}{c_s c_p}\frac{\partial^2 C^*}{\partial y^{*2}}\right]-\frac{1}{Cp}\left[k_0\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{16a_R\sigma v T^*_{\infty}}{U_0^2}-Q\left(T^*-T^*_{\infty}\right)\right]=0 \tag{9}$$

To obtain the non-dimensional form of equations (2) to (4), the following dimensionless quantities are introduced:

$$y=\frac{u_0}{v}y^*,t=\frac{u_0^2}{v}t^*,u=\frac{u^*}{u_0},T=\frac{T^*-T^*_{\infty}}{T^*_w-T^*_{\infty}},C=\frac{C^*-C^*_{\infty}}{C^*_w-C^*_{\infty}} \tag{10}$$

Using the dimensionless quantities in equation (10), the dimensionless form of equations (2) to (4) are:

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t}+\gamma\frac{\partial u}{\partial y}-\frac{\partial^2 u}{\partial y^2}+\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\frac{1}{K}+M\right)u-GrT-GcC=0 \tag{11}$$

$$\frac{\partial T}{\partial t}+\gamma\frac{\partial T}{\partial y}-\frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2}-ST-\frac{\partial^2 C}{\partial y^2}=0 \tag{12}$$

$$\frac{\partial C}{\partial t}+\gamma\frac{\partial C}{\partial y}-\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}+RcC=0 \tag{13}$$

Boundary conditions (5) in dimensionless forms is:

$$\begin{aligned}&u=0,\theta=1,C=1\quad at\ y=0\\&u\to 0,\theta\to 0,C\to 0\quad as\ \ y\to\infty\end{aligned} \tag{14}$$

where $\Pr$ is the Prandtl number, Sc is the Schmidt number, Gr is the thermal Grashof number, Gc is the mass Grashof number, M is the Magnetic parameter, γ is the injection, λ is the viscoelastic Kuvshinski parameter, K is the permeability parameter, S is the heat source/sink, R is the radiation absorption parameter, Du is the Dufour number and Rc is the chemical reaction parameter and their expressions are provided in the appendix.

3.0 Solution of the Problem

We assume solution of equations (11) to (13) as

$$u\left(y,t\right)=u_0\left(y\right)e^{-wt} \tag{15}$$

$$T\left(y,t\right)=T_0\left(y\right)e^{-wt} \tag{16}$$

$$C\left(y,t\right)=C_0\left(y\right)e^{-wt} \tag{17}$$

Substituting Equations (15) to (17) into Equations (11) to (14), gives

$$\frac{d^2u_0}{dy^2}-\gamma\frac{du_0}{dy}-\left[\left(1-w\lambda\right)\left(M+\frac{1}{K}-w\right)\right]u_0+GrT_0+GcC_0=0 \tag{18}$$

$$\frac{d^2\theta_0}{dy^2}-\gamma\Pr\frac{d\theta_0}{dy}+\left(w+S-R\right)\theta_0+Du\Pr\frac{\partial^2 C}{\partial y^2}=0 \tag{19}$$

$$\frac{d^2C_0}{dy^2}-\gamma Sc\frac{dC_1}{dy}+Sc\left(w-Rc\right)C_0=0 \tag{20}$$

The corresponding boundary condition can be written as

$$\begin{aligned}&u_0=0,T_0=C_0=1\quad at\ y=0\\&u_0\to 0,T_0=\to 0,C_0\to 0\quad as\ \ y\to\infty\end{aligned} \tag{21}$$

Solving Equations (18) to (20) subject to the initial and boundary conditions in (21), and substituting the obtained approximate solutions into Equations (15) tp (17), gives

3.1 Velocity distribution

$$u(y,t)=\left(E_7e^{-m_6y}+E_8e^{-m_4y}+E_9e^{-m_2y}+E_{10}e^{-m_2y}\right)e^{-wt} \quad (22)$$

3.2 Temperature distribution

$$T(y,t)=\left(E_4e^{-m_4y}+E_5e^{-m_2y}\right)e^{-wt} \quad (23)$$

3.3 Concentration distribution

$$C(y,t)=\left(e^{-m_2y}\right)e^{-wt} \quad (24)$$

The Skin friction, Nusselt and Sherwood numbers respectively at the surface are as follows:

$$\tau=\left[-u'(0)\right]=\left(m_6E_7+m_4E_8+m_2E_9+m_2E_{10}\right)e^{-wt} \quad (25)$$

$$Nu=\left[-T'(0)\right]=\left(m_4E_4+m_2E_5\right)e^{-wt} \quad (26)$$

$$Sh=\left[-C'(0)\right]=m_2e^{-wt} \quad (27)$$

4.0 Results and Discussion

A MATLAB program is written to generate line graphs for the expressions obtained in the previous section for the velocity, temperature and concentration. The Influence of various physical parameters associated with the flow as well the skin friction coefficients, Nusselt and Sherwood numbers were plotted and computed numerically through Figures and Tables.

Physical sketch of the problem is illustrated in the Figure 1. Also, to focus our attention on numerical values of the result obtained in this study thermal Grashof and mass Grashof numbers greater than zero is considered which symbolizes cooling of the plate.

Velocity profiles are shown in Figs. 2 – 4. Fig. 2(a) presents velocity profiles for varied values of Dufour number. It is observed that, the velocity profiles rises with higher values of Du

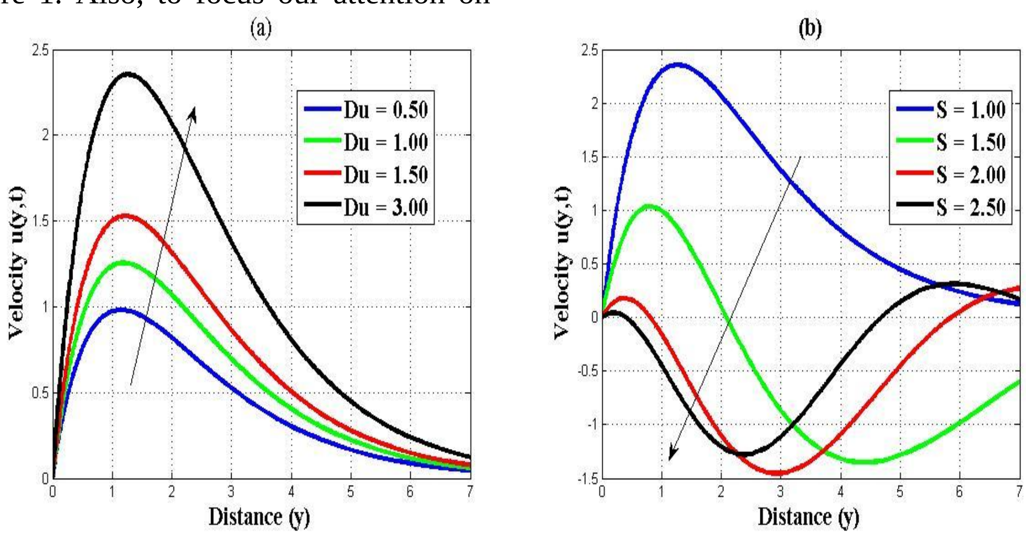

Fig. 2: Influences of (a) Du **and (b)** S **on velocity profiles.**

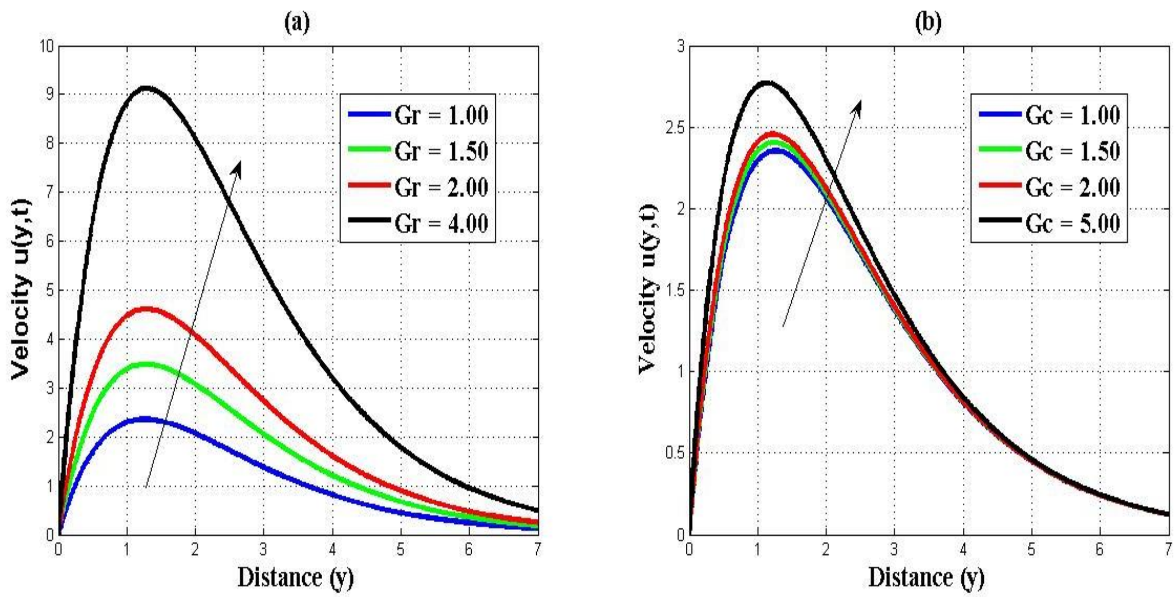

Fig. 3: Influences of (a) $Gr > 0$ and (b) $Gc > 0$ on velocity profiles.

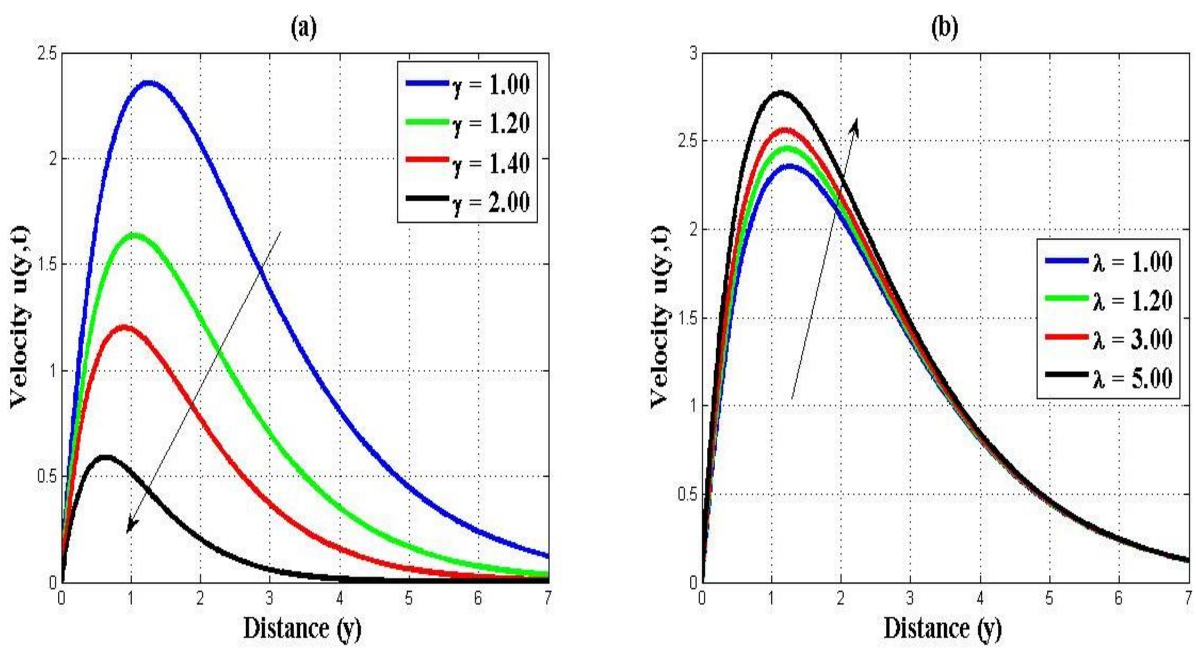

Fig. 4: Influences of (a) γ and (b) λ on velocity profiles.

Also, increasing values of heat source/sink parameter has a reverse trend on the velocity as depicted in Fig. 2(b). In Figs. 3(a) and 3(b), the influence of $Gr > 0$ and $Gc > 0$ on velocity profiles are seen. It is clear that the momentum boundary layer thickness is more as both mass and thermal Grashof numbers are raised respectively. This is because of buoyancy which acts on the fluid particles due to gravitational force. The influence of γ on velocity profiles is described in Fig. 4(a). It is found that the velocity improves as injection strength grows. Fig. 4(b) explains the influence of λ on the velocity. From this Figure, a retarding impact due to higher values of λ is revealed.

Temperature profiles are presented through Figures 5(a) and (b). From Fig. 5(a), it is shown that as the Prandtl number is increased the thermal boundary layer thickness reduces. This occurs because when Pr is increased, the thermal boundary layer thickness lowers rapidly. Hence, the fluid viscosity rises. Fig. 5(b) connotes that higher values of S leads to increase in the temperature profiles.

Concentration profiles are provided in Figures 6(a) and (b). Figure 6(a) displays concentration profiles for varied values of $Rc > 0$. The concentration reduces growth in the Chemical reaction parameter. In Figure 6(b), the influence of Sc on the concentration is demonstrated. Since the Schmidt number is a dimensional number defined as the ratio of momentum and mass diffusivity which is used to characterize fluid flows with simultaneous momentum and mass diffusion convection processes.

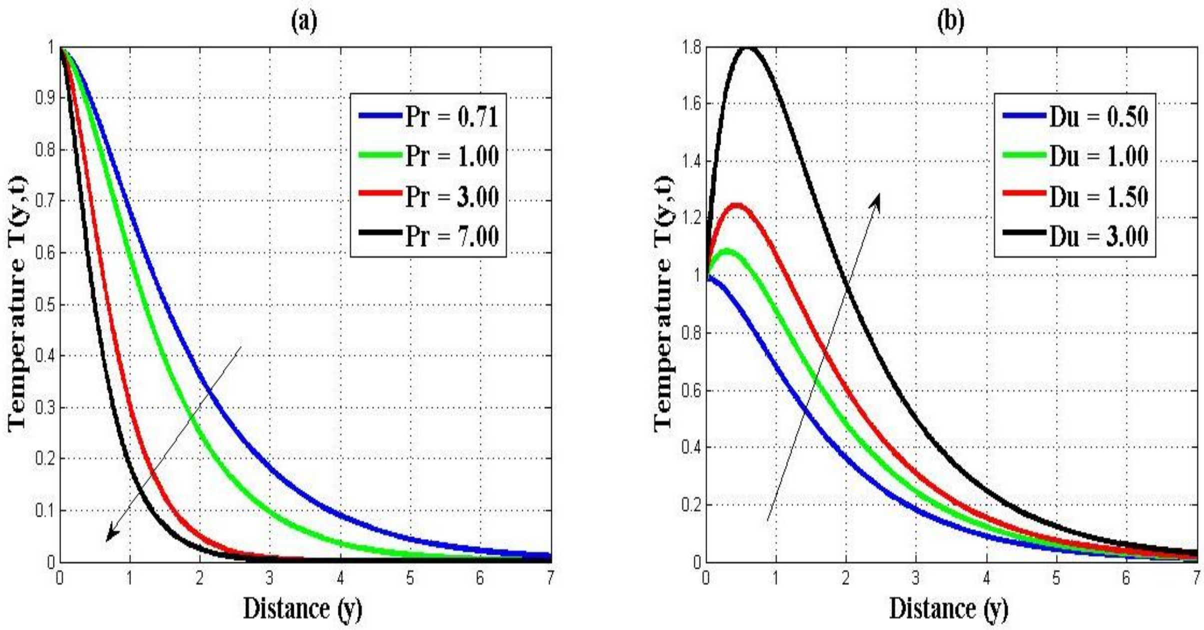

Fig. 5: Influence of (a) $\Pr$ **and (b)** Du **on temperature profiles.**

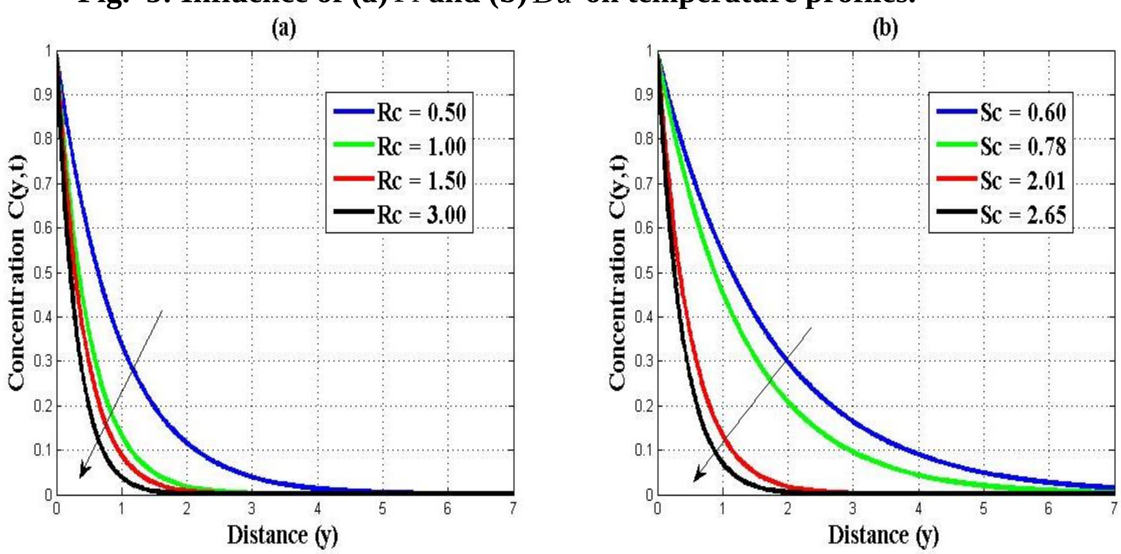

Fig. 6: Influence of (a) Rc **and (b)** Sc **on concentration profiles.**

Skin friction (τ_0), Nusselt number (Nu) and Sherwood number (Sh) presented in Tables 1 – 3. It was revealed that an increase in convection currents decreases the skin friction whereas there is no effect on the rate of heat and mass transfer. An increase in the Magnetic field increases the skin friction and a reverse trend is seen for a higher time on the Nusselt number. Prandtl number tends to enhance the rate of heat transfer of the fluid. Sherwood number decreases due to higher values of Schmidt number.

Table 1: Skin friction

Pr	Gr	γ	nt	M	Sr	t	τ_0
0.71	51	0.5	$\frac{\pi}{6}$	0.5	0.5	0.1	3.00986
7.00	1	0.5	$\frac{\pi}{6}$	0.5	0.5	0.1	3.27158
0.71	**3**	0.5	$\frac{\pi}{6}$	0.	0.5	0.1	2.836886
0.71	1	**1.0**	$\frac{\pi}{6}$	0.5	0.5	0.1	2.91963
0.71	1	0.5	$\frac{\pi}{4}$	0.5	0.5	0.1	3.03000
0.71	1	0.5	$\frac{\pi}{6}$	**1.0**	0.5	0.1	3.06462
0.71	1	0.5	$\frac{\pi}{6}$	0.5	**1.0**	0.1	2.650109
0.71	1	0.5	$\frac{\pi}{6}$	0.5	0.5	**0.5**	2.672267

Table 2: Nusselt number

Pr	t	Nu
0.71	0.1	2.566
7	0.1	3.958
0.71	**0.5**	2.488

Table 3: Sherwood number

Sc	Sr	Sh
0.60	0.5	2.814
2.01	0.5	2.729
0.60	**1.0**	2.725

5.0 Conclusion

The role of injection and heat source/sink on Kuvshinski fluid through porous medium has been studied. From this study, we conclude that:

1. The velocity is boosted as Dufour number, Kuvshinski parameter, thermal and mass Grashof numbers are increased and a reverse trend is noticed as heat source/sink becomes more.
2. The temperature boundary layer thickness rises for increased values of Dufour number and falls as Prandtl number is increased.
3. The mass boundary layer thickness reduces because of rise in chemical reaction parameter and Schmidt number.

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Authors' contributions
Akeem B. Disu: Conceptualisation, data curation, writing – original draft, writing – review and editing. **Emmanuel Omokhuale:** data curation, writing – original draft, writing – review and editing

Appendix

$$K=\frac{k'u_0^2}{\nu^2},\Pr=\frac{\nu C_P}{\rho},Sc=\frac{\nu}{D},M=\frac{\sigma B_0^2\nu}{\rho u_0^2},R=\frac{16a^*\sigma\nu T'^3_\infty}{\rho C_p u_0^2},Gr=\frac{g\beta\nu\left(T'_w-T'_\infty\right)}{u_0^3},Gc=\frac{g\beta^*\nu\left(C'_w-C'_\infty\right)}{u_0^3},$$

$$\eta_1=w-Rc,\ \eta_2=\left(w+S-R\right)\Pr,\ \eta_3=\left(1-w\lambda\right)\left(M+\frac{1}{K}-w\right),\ b_1=\frac{\sqrt{\left(\gamma Sc\right)^2-4Scw_1}}{4},$$

$$a_2=\frac{\sqrt{\left(\gamma\Pr\right)^2-4\eta_2}}{4},\ b_3=\frac{\sqrt{\left(\gamma\right)^2+4\eta_3}}{4},m_2=\frac{\gamma Sc}{2}+b_1,m_4=\frac{\gamma\Pr}{2}+a_2,m_6=\frac{\gamma}{2}+b_3,$$

$$E_5=-\frac{Duf_2^2\Pr}{\left(f_2^2-\gamma\Pr f_2+\eta_2\right)},E_4=1-E_5,E_7=-E_8-E_9-E_{10},E_8=-\frac{GrE_4}{\left(m_4^2-\gamma m_4-\eta_3\right)},$$

$$E_9=-\frac{GrE_5}{\left(m_2^2-\gamma m_2-\eta_3\right)},E_{10}=-\frac{Gc}{\left(m_2^2-\gamma m_2-\eta_3\right)}$$