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Fast Interpolating Spline for Diurnal Temperature Patterns

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Abstract: *The monthly maximum and minimum data describing monthly variation in temperature in the city of Owerribetween 2010 and 2019 were collected and applied for the evaluation of the diurnal temperature range (DTR). The results were useful for the derivation of models for the prediction of the amount of DTR for subsequent months through interpolation analysis of DTR change. The best-fitted model was the cubic spline interpolation model and was used to interpolate between known data points due to its stable and smooth characteristics for the intervals in the selected years. The month of June 2010 and 2019 was left missing and interpolated with percentage error results of 2.0444% and 12.2694% respectively. The study revealed that the interpolated results were consistent with the yearly data while the observed percentage error fell in a good and commendable range.*

Keywords: *Diurnal temperature, interpolation, cubic spline.*

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1.0 Introduction

Some studies have linked the occurrence of surface warming on the land to the relative increase in the daily minimum temperatures to the maximum temperatures over 50 years, even when minimum and maximum temperatures showed a significant increase (Karl *et al.*, 1993; Easterling *et al.*, 1997; New *et al.*, 2000). Since the trend in temperature can be due to either maximum or maximum changes, changes in daily maximum and minimum temperatures are useful indicators for climate variability than mean surface temperature. The DTR is the difference between the mean monthly maximum temperature (T_{max}) and minimum temperature (T_{min}). These changes indicate a strong connection to global warming (Karl *et al.*, 1991). Diurnal temperature range (DTR) has been reported to be decreasing over large continental areas of the globe since 1950, the decrease was attributed to a larger increase in minimum temperature relative to maximum temperature (Karl *et al*., 1991, 1993; Easterling *et al*., 1996, 1997; Vose*et al.*, 2005). According to Kalnay and Cai (2003), DTR reflects changes in minimum and maximum temperature and is susceptible to urban effects. Some studies showed that temperature rise due to a daily minimum increase at a faster rate or decrease at a slower rate than a daily maximum temperature can lead to a decrease in Diurnal Temperature Rise (DTR) for some parts of the world (Karl., 1993). DTR was first studied in the United State where the minimum temperature rate of increase was faster than the maximum rate (Gil-Alana, 2018). A Long-term study trend in DTR for different locations (Ding*et al.*, 2018; Wang *et al.* 2014), showed that DTR has decreased over many land areas

(Ding*et al.,* 2018; You *et al.*, 2016). In Nigeria, the average DTR has decreased significantly (−0.34°C per decade) over the Nigerian Sahel (north of 10°N), but with a slightly increasing trend (0.01°C per decade) over the Nigerian Guinea Coast (Victor *et al.,*2019).

The DTR can be affected by irrigation, desertification and variation in land use which causes an increase in cloud cover, greenhouse gases and tropospheric aerosols (Jackson, 2012). The DTR is also responsible for the current warming experienced in the world (Karl, 1995). A decrease in monthly DTR has a strong connection with global warming (Karl *et al.*, 1991). For an instant, in Nigeria, the increase in both annual and seasonal temperature over Ibadan showed that there is ongoing warming and it is linked to the anthropogenic global warming effect (Abatan*et al*.,2018) The reported decrease in the DTR for the Nigerian Sahel is associated with an increasing trend of annual and summer precipitation in the region, but the increasing DTR trend in the Nigerian Guinea Coast region is attributed to the decreasing trend of cloud cover over the region (Victor *et al.,*2019). Consequently, the forecasting of temperature has been tested and ascertained helpful (Wimmer, 2010).

Due to this change in weather pattern DTR data may not be uniform due to irregular weather patterns. Such data can be interpolated so that the amount of the DTR for subsequent months can be calculated and predicted. The acquisition and development of DTR data are significant to farmers for the monitoring of cultivated or developing plants. However, several models have been applied in acquiring and analyzing DTR information but have some setbacks. In this study, the cubic spline interpolation for the analysis is employed. The cubic spline model has been observed to have a smaller error margin with a smoother interpolation compared to the other methods (Jain *et al*., 2007; Young and Mohlenkamp 2017).

Spline interpolation is mostly used over polynomial interpolation because the interpolation error can be made small even when the spline involves a low degree polynomial. (Hazewinkel, 2001).

2.0 Materials and Methods

2.1 *The study area*

The study area, Owerri in Imo State, is located in the South-Eastern part of Nigeria and lies within the Latitude: 5.476310 DMS Lat: 5° 28' 34.7160" N and Longitude: 7.025853 DMS Long: 7° 1' 33.0708" E, minimum, maximum and elevations of the area are 38, 245 and 93 m respectively. Temperature is highest between February and April while the prevalent climatic condition in the area is associated with the rainy and dry seasons. The rainy season is from April to October, while the dry season begins from October through February. However, slight variation in the period has been reportedly linked to abnormal climatic changes.

The primary data used for the study were obtained from the Nigeria Meteorological Station, Abuja. Monthly average minimum and maximum temperatures covering 10 years were used for this study. The yearly data were computed from the average monthly data of 10 years for the period, 2010 to 2019. The DTR values were computed from the difference between the maximum and minimum temperature values.

2.2 The cubic splines

Interpolation general refers to a robust process of defining a function that takes on a specific value at a specified point. A typical example is the well-known n^{th} order spline model with flexible derivatives that can be applied in curve fitting (Wahba, 1990). The most popular among the spline functions is the cubic spline of the n = 3, order. A cubic spline function can be described by a piecewise cubic polynomial with continuous and minimal integrated squared second derivatives that guarantee its smoothness and high stability. Consequently, they are widely applied for smoothing data such as in interactive computer graphics (Smithand Graham,1974), and real-time digital signal processing (Feng *et al*., 1998). and satellite-based time series data (Chen, 2014; Wust, 2017; Mao *et al*.,2017; Singh *et al*., 2012; Yu *et al*., 2021). In order to provide a suitable continuous seasonal diurnal temperature $T(t)$pattern, where t denotes time, we need to construct an interpolating polynomial.

The constructed$T(t)$satisfies the following conditions (1) to (5);$T(t)$must be on the interval

$$[t_{k-1}\, t_k],\ k = 1,2,\dots,n. \tag{1}$$

$$T(t_k) = y_k,\ k = 0,1,2,\dots,n. \tag{2}$$

$$\lim_{t\to t_k^-} T'(t_k) = \lim_{t\to t_k^+} T'(t_k),\ \ k = 1,2,\dots,(n-1). \tag{3}$$

$$\lim_{t\to t_k^-} T''(t_k) = \lim_{t\to t_k^+} T''(t_k),\ \ k = 1,2,\dots,(n-1) \tag{4}$$

$$T''(t_0) = T''(t_n) = 0. \tag{5}$$

The diurnal temperatures,$T(t)$and the derivatives, $T'(t)$ and$T''(t)$ are taken to be continuous on the time interval $[\ t_{0,}\ \ t_n\ ]$. The spline of the diurnal temperature $T(t)$ is constructed starting with the introduction of the variables say $\mu_0, \mu_1, \dots, \mu_n$, such that $\mu_i = T''(t_k)$ for $k = 0,\dots,n$. Since $T(t)$ is a cubic spline on $[\ t_{j-1,}\ \ t_j\ ]$, this implies the second derivative $T''(t)$ is linear.

At the endpoint $t_{j-1}$and t_j, this gives

$$T''(t) = \mu_{j-1}\frac{t_{j-1} - t}{t_j - t_{j-1}} + \mu_j\frac{t - t_{j-1}}{t_j - t_{j-1}} \tag{6}$$

The spline $T(t)$ can then be obtained by taking the second antiderivative of equation (1) at the intervals $[\ t_{j-1}\, t_j\ ]$ and applying the interpolating condition $T(t_{j-1}) = y_{j-1}$ and $T(t_j) = y_j$ to obtain the following interpolating spline

$$T(t) = \frac{1}{6}\frac{(t_{j-1} - t)^3\mu_{j-1} + (t - t_{j-1})^3\mu_j}{(t_j - t_{j-1})} + \frac{(t_j - t)y_{j-1} + (t - t_{j-1})y_j}{(t_j - t_{j-1})} - \frac{1}{6}\left(t_j - t_{j-1}\right)\left((t_j - t)\mu_{j-1} + (t - t_{j-1})\mu_j\right) \tag{7}$$

Equations (6) and (7) guarantee the continuity of $T''(t)$ and $T(t)$ respectively so that $T(t)$ interpolates the given data. To guarantee the continuity of the first derivative $T'(t)$it also required that the anti-derivative of$T''(t)$ on $[\ t_{j-1,}\ \ t_j\ ]$ and $[\ t_{j,}\ \ t_{j+1}\ ]$ have the same value at the knott$_j$, $j = 1,2,\dots,n-1$. This yields the spline

$$\frac{1}{6}(t_j - t_{j-1})\mu_{j-1} + \frac{1}{3}(t_{j+1} - t_{j-1})\mu_{j-1} + \frac{1}{6}(t_{j+1} - t_{j-1})\mu_j = \frac{1}{6}\frac{y_{j+1} - y_j}{t_{j+1} - t_j} - \frac{y_j - y_{j-1}}{t_j - t_{j-1}} \tag{8}$$

where the constants $\mu_0 = \mu_n = 0,\ j = 1,2,\dots,n-1$, leads to the determination of the required spline. The interplant is evaluated numerically and such efficient numerical libraries that perform the calculations are embedded in some programs such as MATLAB as used in our computations.

3.0 Results and Discussion

In order to effectively validate the prediction using cubic spline interpolation of the diurnal temperature measured in Owerri city, the histogram and the proposed interpolation spline on the ten years of data are presented in figure (1), and the analysis of midmonth diurnal temperature obtained by spline interpolation (2010-2019) is presented in Table1.

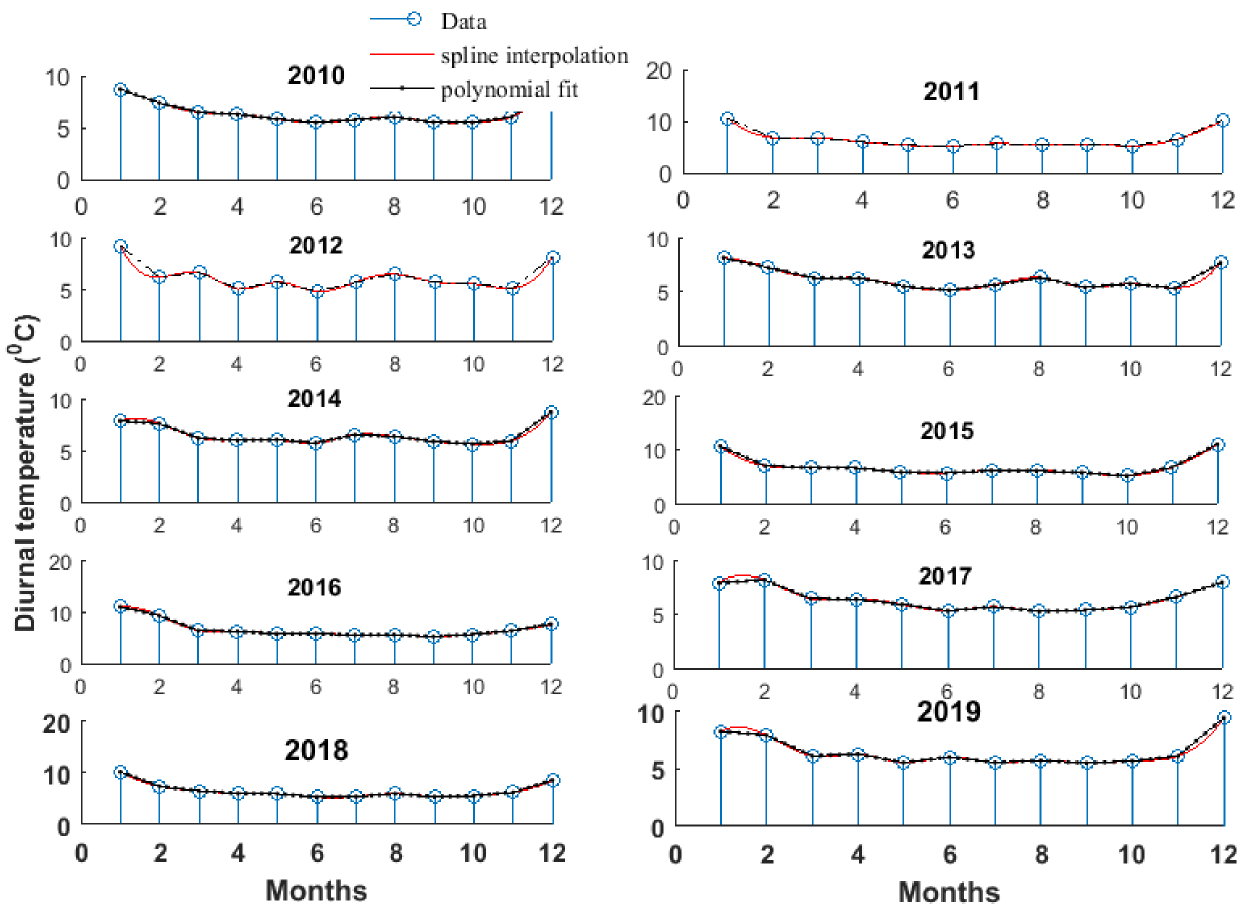

Fig. 1: Spline interpolation (red) compared with a 3-dregree polynomial fit (black) and the actual data (histogram) of the measured 10 years' diurnal temperature data

Table 1. Analysis of midmonth diurnal temperature obtained by spline interpolation (2010-2019)

Mid month	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
1.5	8.0569	7.8427	6.6401	7.7796	8.0863	8.2120	10.6413	8.6090	8.4708	8.6607
2.5	6.8520	6.6647	6.5026	6.6158	6.8469	6.6102	7.5583	7.2104	6.8746	6.8900
3.5	6.3589	6.4366	5.7849	6.2489	5.9418	6.6769	6.0976	6.2886	6.2512	6.1669
4.5	6.0929	5.6048	5.2714	5.9102	6.0684	6.2241	5.9887	6.2552	6.0423	5.9296
5.5	5.6083	5.1712	5.2716	5.1803	5.8075	5.6303	5.6829	5.4985	5.6717	5.7747
6.5	5.5841	5.4604	4.9928	5.2452	6.0878	5.8892	5.6240	5.4991	5.2609	5.8613
7.5	5.9751	5.5448	6.2729	6.0979	6.5569	6.1260	5.5375	5.5132	5.8080	5.6262
8.5	5.7970	5.5210	6.1002	5.8415	6.1005	5.9543	5.3497	5.2485	5.7477	5.6482
9.5	5.4640	5.3797	5.5675	5.5004	5.7205	5.3444	5.3206	5.4864	5.4088	5.5883
10.5	5.6263	5.5478	5.1806	5.5037	5.6098	5.5808	5.9370	6.0461	5.8422	5.7769
11.5	6.9949	7.9966	5.8279	5.8024	6.8603	8.5127	6.8348	7.2420	7.1139	7.2023

From Fig. 1, it can be seen that the boundary conditions are suitable for the diurnal data interpolation at Owerri from 2010-2019. Also, the values of the spline are positive and hence, a realistic one. Based on the interpolation, the missing diurnal data in June is analyzed and based on the error percentage for splines, it is seen that the spline is suitable for missing diurnal data imputation. Tables 1 and 2 summarize the prediction.

The proposed spline interpolation model was also applied in a real-time situation to analyze the splines where one or two months month of measured diurnal temperature data are made to be missing. Then, the missing month diurnal temperature data is compared to the actual data value by estimating the percentage error by

$$\frac{Actual\ value - spline\ interpolated\ value}{Actual\ value} \times 100\% \qquad (9)$$

Table 2. Error analysis of missing diurnal temperature data in (2010 and 2019) estimated by spline interpolation

Month	**Actual data**		**Spline data**		**% error**	
	2010	2019	2010	2019	0	0
1	8.7339	8.3032	8.7339	8.3032	0	0
2	7.4014	7.9582	7.4014	7.9582	0	0
3	6.4932	6.1503	6.4932	6.1503	0	0
4	6.2810	6.3003	6.2810	6.3003	0	0
5	5.8371	5.5729	5.8371	5.5729	0	0
6	5.5030	6.0460	5.6155	5.3042	2.0444	12.2694
7	5.7800	5.5823	5.7800	5.5823	0	0
8	6.0116	5.7458	6.0116	5.7458	0	0
9	5.5417	5.5223	5.5417	5.5223	0	0
10	5.5103	5.7106	5.5103	5.7106	0	0
11	6.0270	6.1243	6.0270	6.1243	0	0
12	8.8126	9.4603	8.8126	9.4603	0	0

Table 2 summarizes the prediction as well as the missing data imputation. The interpolation for the diurnal data in Owerri 2010 and 2019 for spline has no problem since the values are positive and correspond; it is within the acceptable range for a diurnal data type. From the analysis, June which is the missing month has shown the error percentage for both years to be 2.0444% and 12.2694% respectively which is a good value as the error percentage must be at least below 20%.

4.0 Conclusion

In this paper, the diurnal temperature range data of Owerri were collected and analyzed for ten years. The proposed spline interpolation is capable to interpolate and also, give the results. Furthermore, there were negligible discrepancies between the actual data and spline data. Also, from table 2, for DTR data in 2010 and 2019, the spline gives a positive value to the month left missing (June). The cubic spline interpolation method showed less percentage error. The percentage errors noted to fall in the range of good value. Finally, cubic spline interpolation method will produce good results when used to predict the diurnal temperature range, which is of great importance not majorly to farmers but also to aviation ministries.

5.0 References

Abatan, A.A., Osayomi, T., Akande, S.O., Abiodun, B.J. & Gutowski, W.J. (2018). Trends in mean and extreme temperatures over Ibadan, Southwest Nigeria.

Theoretical and Applied Climatology,pp- 131(3-4), pp. 1261-1272

Chen, S.; Hu, X.; Peng, S.; Zhou, Z.2014 Denoising of hyperspectral imagery by cubic smoothing spline in the wavelet domain. *High Technol. Lett.*, pp. 20, 54–62.

Ding, Z., Y. Wang, &Lu. R(2018). An Analysis of Changes in Temperature Extremes in the Three River Headwaters Region of the Tibetan Plateau during 1961–2016. *Atmospheric Research* 209: pp. 103–114. doi:10.1016/j.atmosres.2018.04.003.

Easterling, D. R., DR, Horton B, Jones PD, Peterson TC, Karl TR, Parker DE, Salinger MJ, Razuvayev V, Plummer N, Jamason P, (1997), Maximum and minimum temperature trends for the globe, *Science*,pp- 277, 364–367.

Feng, G.**(**1998) Data smoothing by cubic spline filters. *IEEE T. Signal Proces*.,46, pp.2790–2796.

Ferrer-Arnau, L.; Reig-Bolaño, R.; Marti-Puig, P.; Manjabacas, A.; Parisi-Baradad, V. (2013) Efficient cubic spline interpolation implemented with FIR filters. *Int. J. Comput. Inf. Syst. Ind*. Manag. Appl.,5, pp. 98-105.

Gil-Alana, L.A. (2018).Maximum and minimum temperatures in the United States: Time trends and persistence. *Atmospheric Science Letters*, 19: e810. DOI:10.1002/asl.810

Graham, N.Y.**(**1983) Smoothing with periodic cubic splines. *Bell Syst. Tech*. J.,62, pp.101-110.

Hou, H.&. Andrews, H. (1978**)**Cubic splines for image interpolation and digital filtering. *IEEE T. Acoust. Speech*., 26, pp.508–517.

Jackson, O. I., Isienyi, N. C.,Osudiala, C.S.,Odofin, B. J.,Adeyemi, A. A.,Odeleye, O. A.,&Amoo, V. O. (2012). Analysis of temperature trends of Ibadan, Nigeria over the period of 1965-2012. *Journal of Forest Reserve and Management., 9*, pp. 61-72.

Kalnay E. & Cai M. (2003). Impact of urbanization and land-use change on climate. *Nature*.423(6939): pp.528–531.

Karl, T.R., Kukla, G., Razuvayev, V.N., Changery, M.J., Quayle, R.G., Heim, R. R, Easterling, D.R.&Fu, C. B.(1991). Global warming: evidence for asymmetric diurnal temperature change. *Geophys Res Lett*.18(12): pp.2253–2256.

Karl, T. R., P. D. Jones, R.W. Knight, G. Kukla, N. Plummer, V. Razuvayev,K. P. Gallo, J. Lindseay, R. J. Charlson, & T. C. Peterson (1993) Asymmetric trends of daily maximum and minimum temperature, *Bull. Am. Meteorol. Soc*.,74, pp. 1007–1023.

Mao, F., Li, X., Du, H., Zhou, G., Han, N., Xu, X., Liu, Y., Chen, L & Cui, L. (2017) Comparison of two data assimilation methods for improving MODIS LAI time series for bamboo forests. *Remote Sens*., 9, Iss.5, 401. DOI:10.3390/rs9050401

New, M., Hulme., & Jones, P.D., (2000), Representing twentieth-century space-time climate variability. Part II: Development of 1901 – 1996 monthly grids of terrestrial surface climate, *J. Clim*.,13, pp.2217–2238.

Singh, R.K., Liu, S.,Tieszen, L.L.,Suyker, A.E.& Verma, S.B. (2012). Estimating seasonal evapotranspiration from temporal satellite images. *Irrig. Sci.*, 30, pp. 303–313.

Smith, R.E., Price, J. M. and Howser, L. M. (1974)*A Smoothing algorithm using cubic Splinefunctions; Technical note:NASA TN D-7397; National Aeronautics and Space Administration (NASA)*:Washington, DC, USA,.

Victor, D., Zhaohui, L., Yuxi, W & Hyacinth, N (2019):Observed trends in diurnal temperature range over Nigeria.*Atmospheric and Oceanic Science Letters*, DOI:10.1080/16742834.2019.1570688

Vose, R. S., Easterling, D. R.and Gleason, B. (2005) Maximum and minimum temperature trends for the globe: an update through 2004. *Geophys Res Lett*., 32:L23822. doi:10.1029/2005GL024379

Wahba, G.(1990). Spline models for observational data; cbms-nsf regional conference series in applied mathematics*; Society for Industrial and Applied Mathematics (SIAM)*: Philadelphia, PA, USA,; pp. 1–20.

Wang, F., Zhang C.,, Peng Y.,, & Zhou, H. (2014). "Diurnal Temperature Range Variation and Its Causes in a Semiarid Region from 1957 to 2006." *International Journal of Climatology* 34 (2): pp- 343–354. doi:10.1002/joc.2014.34.issue-2.

Wüst, S., Wendt, V., Linz, R.& Bittner, M.**(**2017), Smoothing data series by means of cubic splines: Quality of approximation and introduction of a repeating spline approach*. Atmos. Meas. Tech*. 10, pp. 3453–3462.

You, Q., Min, J., Yang, S., Mika, & Shichang, K. (2016).Observed trend of diurnal temperature range in the tibetan plateau in recent decades. *International Journal of Climatology, 6, pp.* 2633–2643. doi:10.1002/joc.4517.

Yu, G., Di, L.; Yang, Z., Chen, Z.& Zhang, B. (2012)*Crop condition assessment using high temporal resolutionsatellite images. In Proceedings of the 2012First International Conference on Agro-Geoinformatics*, *Shanghai,China*, 2–4 August.

Zhang, H., Pu, R. & Liu, X. (2016) A New Image Processing Procedure Integrating PCI-RPC and ArcGIS-Spline Tools to Improve the Orthorectification Accuracy of High-Resolution Satellite Imagery. *Remote Sens*., 8, 827. DOI:10.3390/rs8100827

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Authors. ' Contribution

Uchechi Ezere carried out data sorting, review, Implementation and explanations. Chijioke Oriaku carried out staistical analysis, review and implementation. Ozochi Akwuegbu carried out review, implementatin and conclusion.