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Lehmann Type II-Lomax Distribution: Properties and Application to Real Data Set

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Abstract: *In this paper, we proposed a new compound probability distribution known as the Lehmann Type II-Lomax (LTL) distribution generated from the Lehmann Type II Family of distribution and derived some of its mathematical properties such as entropy, moments, moment-generating functions, and order statistics. Parameters of the new distribution were estimated using a maximum likelihood estimator. One dataset was used to illustrate the usefulness of the model. The newly developed model outperformed its competitors.*

Keywords: *Lehmann Type II, Lomax, compound distribution, moment, entropy*

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1.0 Introduction

The development of more robust distributions by expanding the classical distributions by introducing additional shape parameters into the baseline model has been of major interest. Over the last two decades, several generalized distribution families have been pro-positioned and studied to model data in many fields of use, such as economics, engineering, biological sciences, and environmental science. Some of the recently developed compound distributions include: Lehmann Type II Weighted Weibull Distribution by Badmus *et al*. (2014), Bur XII-Exponentiated Weibull distribution by Hamed (2018), Exponentiated Gamma Exponential Distribution by Jabeen and Para (2018), Lomax Exponential Distribution with application to real life data by Ijaz *et al*. (2019), Type II Topp Leone Generalized Inverse Rayleigh Distribution by Yahia (2019), Odd Generalized Exponential Weibull Exponential Distribution by Muhammad and Al-Kadim (2019), Weibull Inverse Lomax Distribution by Hassan and Mohamed (2019), Lehmann Type II Inverse Weibull Distribution by Tomazella *et al*., (2020), Modified by Iqbal *et al*., (2021), a new generalization of Lehmann type-II distribution by Balogun et al., (2021) and Lehmann Type II Frechet Poisson Distribution by Ogunde *et al*., (2021) and Lehmann Type II Generalized Half-

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Logistic Distribution by Awodutire *et al.* (2020).

Badmus *et al*. (2014) proposed a new compound distribution called Lehmann Type II-Weighted Weibull Distribution to obtain a distribution that is better than both weighted Weibull and Weibull distribution in terms of an estimate of their characteristics and their parameter. Some basic properties of the newly proposed distribution including moments and moment generating function, survival rate function, hazard rate function, asymptotic behaviour and the estimation of parameters were also presented.

Awodutire *et al*. (2020) proposed a new compound distribution called the Lehmann type II generalized half-logistic distribution. Estimates of the parameters of the new distribution under complete and censored observations would be obtained using the maximum likelihood estimation method. Simulation studies were carried out to assess the consistency of the maximum likelihood estimates. Application of the new distribution to data showed that it performed better than the type I generalized half logistic distribution.

Elgarhy *et al*. (2017) proposed a new four-parameter continuous model, called the exponentiated Weibull exponential distribution based on exponentiated Weibull-G family which contains some new distributions as well as some former distributions and also derived some of its mathematical properties such as the quantile function, expansion of distribution and density functions, moments, moment generating function, Renyi and q – entropies and order statistics. The estimation of the model parameters is discussed using the maximum likelihood method. The practical importance of the new distribution is demonstrated through real data set where the proposed model was compared with several lifetime distributions and it turns out that the EWE distribution provided a better fitting in comparison to several other former lifetime distributions. Applications showed that the EWE models can be used instead of other known distributions.

Lehmann (1953) came up with a new family of distribution which is known as the Exponentiated-G Family (EXP-G) and add one parameter to an existing distribution. According to Lehmann (1953), If $G(x)$ is a cumulative distribution function (cdf), then

$$F(x;\ \alpha,\xi) = G(x;\ \xi)^{\alpha},\ \ \alpha > 0 \qquad (1)$$

Is also a cumulative distribution function and it is called Exponentiated-G (Exp-G) distribution with power parameter γ. The distribution G is called the baseline distribution in this and in similar contexts (other G classes). This class is also called Lehmann type I distribution. The cdf of the Lehmann type II distribution is given by

$$F(x;\ \gamma,\xi) = 1 - [1 - G(x;\ \xi)]^{\gamma}, \quad \alpha > 0 \qquad (2)$$

The probability density function (pdf) of the type II Lehmann G family of a probability distribution is given by

$$f(x;\ \gamma;\ \xi) = \gamma g(x;\ \xi)[1 - G(x;\ \xi)]^{\gamma-1} \qquad (3)$$

1.1 Lomax distribution

Let a positive random variable X has the Lomax distribution with parameters α and λ, then the cumulative distribution function (cdf) takes the form:

$$F(x,\alpha,\lambda) = 1 - \left(1 + \frac{x}{\lambda}\right)^{-\alpha}, x > 0, \alpha > 0, \lambda > 0 \qquad (4)$$

and the corresponding pdf is given by:

$$f(x,\alpha,\lambda) = \frac{\alpha}{\lambda}\left(1 + \frac{x}{\lambda}\right)^{-(\alpha+1)} \qquad (5)$$

where $\alpha > 0$ and $\lambda > 0$ are the shape and scale parameters respectively

Recently, the Lomax distribution attracted the attention of many researchers. It was studied and applied to model different datasets. It was applied in breaking stress of carbon fibers by Ijaz et al., (2019), in losses due to wind catastrophes recorded in 1977 by Ijaz et al., (2020) and in remission times of bladder cancer patients by Chaudhary and Kumar (2020).

Bukoye *et al*., (2021) introduced and studied a new distribution called Type II half logistic

Exponentiated Lomax (TIIHLEE) distribution established from Type II half logistic G family of distribution and investigated some of its mathematical and statistical properties such as the explicit form of the ordinary moments, moment generating function, conditional moments, mean deviations, residual life and Renyi entropy. The maximum likelihood method was used to estimate the model parameters and they also conducted a simulation study to assess the finite sample behavior of the maximum likelihood estimators. Finally, they illustrated the importance and applicability of the model by analyzing bladder cancer patients.

Falgore and Doguwa (2021) proposed New Weibull Inverse Lomax distribution and carried out a simulation study with different sample sizes ranging from 30, 50, 100, 200, 300, to 500 were considered. Also, 1,000 replications were considered for the experiment in their study. NWIL is a fat tail distribution. Higher moments are not easily derived except with some approximations. However, the estimates have higher precisions with low variances. Finally, the usefulness of the NWIL distribution was illustrated by fitting it to the breaking strength of 100 Yarn data.

2.0 The New Lehmann Type II Lomax Distribution

By inserting the probability density function and the cumulative density function of the Lomax distribution into the pdf of the Lehmann Type II distribution, we obtained a new probability distribution known as the Lehmann Type II-Lomax Distribution as given in the equation.

$$f(x;\,\gamma,\alpha,\lambda)=\frac{\gamma\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma-1}\qquad \gamma,\lambda,\alpha,x>0 \tag{6}$$

The cdf is also given by

$$F(x;\,\gamma,\alpha,\lambda)=1-\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma} \tag{7}$$

The survival function $S(x)$, hazard function $h(x)$, reverse hazard function $r(x)$ and the cumulative hazard function $H(x)$ of the Lehmann Type II-Lomax Distribution are given below:

$$S(x)=1-F(x)=\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma} \tag{8}$$

$$h(x)=\frac{f(x)}{1-F(x)}=\frac{\frac{\gamma\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma-1}}{\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma}} \tag{9}$$

$$r(x)=\frac{f(x)}{F(x)}=\frac{dF(x)/dx}{F(x)}=\frac{d}{dx}(\ln F(x))=\frac{\frac{\gamma\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma-1}}{1-\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma}} \tag{10}$$

$$H(x)=\int_0^x\frac{f(t)}{1-F(t)}dt=\int_0^x\frac{\frac{\gamma\alpha}{\lambda}\left(1+\frac{t}{\lambda}\right)^{-(\alpha+1)}\left\{1-\left[1-\left(1+\frac{t}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma-1}}{\left\{1-\left[1-\left(1+\frac{t}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma}}dt$$

$$=-\ln\left\{1-\left[1-\left(1+\frac{t}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma} \tag{11}$$

2.1 The Quantile function

This function is derived by inverting the cumulative density function (cdf) of any given continuous probability distribution. It is used for obtaining some moments like skewness and kurtosis as well as the median and for the generation of random variables from the distribution in question (Hyndman & Fan 1996). The quantile function of the LTII-L distribution can be obtained as follows:

$$Q(u) = F^{-1}\left\{\lambda\left[1-\left(1-(1-u)^{\frac{1}{\gamma}}\right)\right]^{-\frac{1}{\alpha}}\right\} \qquad (12)$$

2.2 Mixture Representation

The pdf of the proposed Lehmann Type II Lomax Distribution is given by:

$$f(x;\gamma,\alpha,\lambda) = \frac{\gamma\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma-1}$$

$$\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)} = \sum_{i=0}^{\infty}(-1)^i\binom{\alpha+i}{i}\left(\frac{x}{\lambda}\right)^i$$

$$\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma-1} = \sum_{j=0}^{\infty}(-1)^j\binom{\gamma-1}{j}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^j$$

$$\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^{\boldsymbol{j}} = \sum_{k=0}^{\infty}(-1)^k\binom{j}{k}\left(1+\frac{x}{\lambda}\right)^{-\alpha j}$$

$$\left(1+\frac{x}{\lambda}\right)^{-\alpha j} = \sum_{l=0}^{\infty}(-1)^l\binom{\alpha j}{l}\left(\frac{x}{\lambda}\right)^l$$

$$f(x;\gamma,\alpha,\lambda) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}(-1)^{i+j+k+l}\binom{\alpha+i}{i}\binom{\gamma-1}{j}\binom{j}{k}\binom{\alpha j}{l}x^{i+l}\lambda^{-(i+l)}$$

$$Let\ \wp = (-1)^{i+j+k+l}\binom{\alpha+i}{i}\binom{\gamma-1}{j}\binom{j}{k}\binom{\alpha j}{l}\lambda^{-(i+l)}$$

Therefore, the pdf simplifies to:

$$f(x;\gamma,\alpha,\lambda) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\wp\, x^{i+l}$$

The cdf of the Lehmann Type II Lomax Distribution can also be expressed as:

$$F(x;\gamma,\alpha,\lambda) = 1-\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma}$$

$$\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma} = \sum_{m=0}^{\infty}(-1)^m\binom{\gamma}{m}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^m$$

$$\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^m = \sum_{n=0}^{\infty}(-1)^n\binom{m}{n}\left(1+\frac{x}{\lambda}\right)^{-\alpha n}$$

$$\left(1+\frac{x}{\lambda}\right)^{-\alpha n} = \sum_{p=0}^{\infty}(-1)^p\binom{-\alpha n+p-1}{p}\lambda^{-p}x^p$$

$$F(x;\gamma,\alpha,\lambda) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}(-1)^{m+n+p}\binom{\gamma}{m}\binom{m}{n}\binom{-\alpha n+p-1}{p}\lambda^{-p}x^{p}$$

Let $\mathfrak{H} = (-1)^{m+n+p}\binom{\gamma}{m}\binom{m}{n}\binom{-\alpha n+p-1}{p}\lambda^{-p}$

Therefore, the cdf of the Lehmann Type II Lomax Distribution can be expressed as:

$$F(x;\gamma,\alpha,\lambda) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\mathfrak{H}\,x^{p}$$

3.0 Mathematical Properties

The mathematical properties of the Lehmann Type II Lomax distribution such as the rth moment, moment generating function, entropy and order statistics were derived and presented as follows:

3.1 Moment

In this subsection, we discuss the r^{th} moment for Lehmann Type II Lomax distribution. The r^{th} moment of the random variable X with probability density function $f(x)$ is given by

$$\mu_r' = \int_{-\infty}^{\infty} x^r f(x;\gamma,\alpha,\lambda)dx = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\wp\int_{-\infty}^{\infty}x^{r+i+l}\,dx$$

$$Let\ \psi = \int_{0}^{\infty}x^{r+i+l}$$

Therefore, the rth moment of the Lehmann Type II distribution is given by:

$$\mu_r' = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\wp\,\psi \tag{14}$$

3.2 Moment generating function

The moment-generating function of a random variable X is the expected value of e^{tx}. We say that the moment-generating function exists, if there exists a positive constant a such that $M_x(t)$ is finite for $\forall\ t \in [-a,a]$.

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty}e^{tx}f(x;,a,b)dx$$

$$= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\wp\int_{0}^{\infty}e^{tx}x^{i+l}\,dx$$

$$Let\ \Upsilon = \int_{0}^{\infty}e^{tx}x^{i+l}\,dx$$

Therefore, the Moment Generating Function of the Lehmann Type II Lomax distribution is given by:

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\wp\,\Upsilon \tag{15}$$

3.3 Entropy

Entropy is used as a measure of information or uncertainty, which is present in a random observation of its actual population. There will be greater uncertainty in the data if the value of entropy is large. The Shannon entropy for the true continuous random variable X is defined as:

$$I_\theta(x) = \frac{1}{1-\theta}\log\int_{-\infty}^{\infty} f(x;\gamma,\alpha,\lambda)^\theta dx$$

$$f(x;\gamma,\alpha,\lambda)^\theta = \left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\wp\, x^{i+l}\right)^\theta$$

$$f(x;\gamma,\alpha,\lambda)^\theta = \left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\wp\right)^\theta \left(x^{i+l}\right)^\theta$$

$$Let\ \beta = \left(x^{i+l}\right)^\theta$$

Then,

$$I_\theta(x) = \frac{1}{1-\theta}\left[\left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\wp\right)^\theta \log\int_{-\infty}^{\infty}\beta^\theta dx\right] \tag{16}$$

3.4 Order statistics

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a continuous population having a pdf $f(x)$ and distribution function (cdf) F(x), Let $X_{1:n} \le X_{2:n} \le X_{n:n}$ be the corresponding order statistics (OS). David (1970) defined the pdf of $X_{r:n}$ that is the r^{th} OS by:

$$f_{r:n}(x;\gamma,\alpha,\lambda) = \frac{\left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\wp\, x^{i+l}\right)}{B(r,n-r+1)}\left(\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\mathfrak{H}\, x^p\right)^{r-1}$$

$$\left(1-\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\mathfrak{H}\, x^p\right)^{n-r} \tag{17}$$

4.0 Parameter Estimation

4.1 Maximum Likelihood Estimator

Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with pdf $f(x;\ \theta)$, the likelihood function $L(\theta)$ is given by:

$$L(\theta, \underline{x}) = f(x_1, x_2, \ldots, x_n;\ \theta) \qquad \theta \in \Omega,$$

where Ω is the domain for the values of θ. Here, $f(x_1, x_2, \ldots, x_n;\ \theta)$ is the value of the joint probability density function of the random variables $X_1, X_2, \ldots, X_n$ at $X_1 = x_1,\ X_2 = x_2, \ldots, X_n = x_n$. For the Lehmann Type II Lomax distribution, the maximum likelihood is given by

$$L\big(f(x;\gamma,\alpha,\lambda)\big) = \prod_{i=0}^{n}\left(\frac{\gamma\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}^{\gamma-1}\right) \tag{13}$$

$$log\ \ell\big(f(x;\gamma,\alpha,\lambda)\big) = n\log\gamma + n\log\alpha + n\log\lambda - (\alpha + 1)\sum_{i=0}^{n}\log\left(1+\frac{x}{\lambda}\right) + (\gamma-1)\sum_{i=1}^{n}\log\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}$$

$$\frac{\partial\ell}{\partial\gamma} = \frac{n}{\gamma} + \sum_{i=1}^{n}\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}^{-1} \tag{13}$$

$$\frac{\partial\ell}{\partial\alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n}\log\left(1+\frac{x}{\lambda}\right) + (\gamma-1)\sum_{i=1}^{n}\frac{\left(1+\frac{x}{\lambda}\right)^{-\alpha}\log\left(1+\frac{x}{\lambda}\right)}{\left\{1-\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}} \tag{14}$$

$$\frac{\partial\ell}{\partial\lambda} = \frac{n}{\lambda} + (\alpha+1)\sum_{i=1}^{n}x\left(1+\frac{x}{\lambda}\right)^{-1}\lambda^{-2} \tag{16}$$

Equation (13), (14) and (15) gives the maximum likelihood estimators of the parameters γ, $\lambda\ and\ \alpha$ respectively.

5.0 Monte Carlo Simulation and Application

5.1 Simulation study

The construction of a model depends heavily on assumptions associated with uncertainty, Monte Carlo simulation help to explain the impact of risk and uncertainty in prediction and forecasting models. In this work, we adopt the Monte Carlo simulation to ascertain our proposed distribution of the risk in modeling lifetime data. To assess the performance of the new Lehmann Type II Lomax distribution, the simulation study was conducted using the Monte Carlo Simulation method to compute the mean, bias and mean square error of the estimated parameters from the maximum likelihood estimates. The Simulated data is generated using the quantile function defined in equation (11) for different sample sizes: n= 20, 50, 100, 250, 500 and 1000 times. For each sample size $\gamma = 0.7, \alpha = 0.5,$ and $\lambda = 3$. Table 1 presents the results of the estimates, bias and mean square error from the new distribution.

Table 1: Estimate, Bias and MSE of the new Lehmann Type II Lomax Distribution

N	Properties	$\gamma = 0.7$	$\alpha = 0.5$	$\lambda = 3$
	Est.	0.8545	0.9027	9.4288
20	Bias	0.1545	0.4027	6.4288
	MSE	0.7405	0.8061	8.9166
	Est.	0.7748	0.8297	8.0473
50	Bias	0.0948	0.3297	5.0473
	MSE	0.5534	0.6345	5.8962
	Est.	0.7862	0.8368	7.5939
100	Bias	0.0862	0.3368	4.5939
	MSE	0.5446	0.6443	5.0296
	Est.	0.8203	0.8063	7.2586
250	Bias	0.1203	0.3063	4.2586
	MSE	0.5633	0.6048	4.5386
	Est.	0.8736	0.7688	7.1153
500	Bias	0.1736	0.2688	4.1153

	MSE	0.5827	0.5826	4.3161
	Est.	0.9826	0.7064	7.0775
1000	Bias	0.2826	0.2064	4.0775
	MSE	0.6333	0.5758	4.2449

The results of the Monte Carlo Simulations are shown in Table 1 above. These findings demonstrate that the bias tends to zero as the sample size increases, demonstrating the suitability and low risk of the suggested distribution for modeling lifetime datasets.

5.2 Application

The dataset considered in this work is 100 observations taking on the breaking stress of carbon fibres obtained from Isa *et al*., (2020) for illustrative purposes and comparison with the baseline distribution (Lomax), Lehmann Type II Weighted Weibull (LTWW), and Lehmann Type II Half Logistic (LHL) Distributions. For each data set, we estimate the unknown parameters of each distribution by the maximum-likelihood method. We obtain the values of the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Consistent Akaike Information Criterion (CAIC) and Hannan-Quinn Information Criteria (HQIC).
The dataset is given below:
3.7, 2.74, 2.73, 2.5, 3.6, 3.11, 3.27, 2.87, 1.47, 3.11, 4.42, 2.41, 3.19, 3.22, 1.69, 3.28, 3.09, 1.87, 3.15, 4.9, 3.75, 2.43, 2.95, 2.97, 3.39, 2.96, 2.53, 2.67, 2.93, 3.22, 3.39, 2.81, 4.2, 3.33, 2.55, 3.31, 3.31, 2.85,2.56, 3.56, 3.15, 2.35, 2.55, 2.59, 2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68, 2.97, 1.36, 0.98, 2.76, 4.91, 3.68, 1.84, 1.59, 3.19,1.57, 0.81, 5.56, 1.73, 1.59, 2, 1.22, 1.12, 1.71, 2.17, 1.17, 5.08, 2.48, 1.18, 3.51, 2.17, 1.69,1.25, 4.38, 1.84, 0.39, 3.68, 2.48, 0.85, 1.61, 2.79, 4.7, 2.03, 1.8, 1.57, 1.08, 2.03, 1.61, 2.12, 1.89, 2.88, 2.82, 2.05, 3.65

Table 2: Summary statistics of the datasets

Data	Minimum	Q_1	Media	Mean	Q_3	Maximum
Dataset	0.390	1.840	2.700	2.621	3.220	5.560

Table 3: MLE, AIC, CAIC, BIC, and HQIC of the data sets

Distribution	MLE	AIC	$CAIC$	BIC	$HQIC$
LTL	53.80245	113.6049	113.8549	116.8152	116.7690
LHL	176.5809	361.5829	361.5829	362.3722	365.3793
Lomax	196.3709	396.7417	396.8654	401.9521	398.8504
LTWW	53.80245	115.6049	116.0260	116.8125	119.8223

Table 2 presents the results of the analysis of the datasets. The results of the analysis of the Lehmann Type II Lomax Distribution were compared with some of its competitors such as the baseline distribution (Lomax), Lehmann Type II Weighted Weibull, and Lehmann Type II Half Logistic Distributions. The proposed Lehmann Type II Lomax distribution is confirmed to be the better model because it has the least AIC, CAIC, BIC and HQIC.

6.0 Conclusion

In this article, a new compound probability distribution called Lehmann Type II Lomax Distribution was developed using the generator proposed by Recife (2017). Some of its properties such as the moment, entropy, survival function, hazard function, reverse hazard function and cumulative hazard were also derived. The parameters of the distribution were also estimated using the method of maximum likelihood. This newly developed model is more flexible than some of its competitors as illustrated in Table 2.

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