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Computational Modelling of Dynamical System and the Type of Stability

Eli Innocent Cleopas and Abanum Godspower Chukwunedum*
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Abstract: *The study of computational modelling of a dynamical system and the type of stability was investigated using ODE45 numerical techniques. Due to the decrease and increase of the growth rates of yeast species 1 and 2 otherwise called environmental perturbation on the prediction of the extent of the proportion decrease and increase in biodiversity. A biodiversity gain was observed when the growth rates increased together from 101% - 150%. When growth rates are decreased together by 50%, it was also found that, there is a biodiversity loss of yeast species. Finally, the region of instability was found since the pairs of eigenvalues are positive.*

Keywords*: Mathematical model; ODE45; Delay, Dynamical System, and stability*

Eli Innocent Cleopas
Department of Mathematics/Statistics, Federal University Otuoke, Yenagoa, Nigeria.
Email: eliic@fuotuoke.edu.ng
Orcid id: 0009-0008-7022-5375

Abanum Godspower Chukwunedum
Department of Mathematics, Dennis Osadebay University, Asaba, Nigeria.
Email: abanum.godspower@dou.edu.ng
Orcid id: 0000-0001-8173-226X

1.0 Introduction

A dynamic system is a system in which motion takes place under the influence of some forces. Dynamical systems are key foundation of every evolving real-world situation and the system can be used to describe the asymptotic behavior of a natural or man-made system.

In recent years, the dynamical system has had many applications to science and engineering from many researchers (Bertoin, 2016; Chellaboina *et al*., 2003; Edward and Ford, 2003; Eli and Abanum, 2020; *Godspower* et *al*., 2020; *Hale*, 1969; Yan, Y. & Ekakaa, 2011) in Mathematical Modeling and Ecological Modeling. Some of which have gone under the related headings of chaos theory or nonlinear theory. The dynamical systems have two types' continuous and discrete dynamical systems. If the time in the equation is implicit then is called an autonomous equation. Further, if the time is explicit then it is called non-autonomous. Mathematics has always benefitted from its involvement with developing sciences. Each successive interaction revitalizes and enhances the field. Biomedical sciences are the premier science of the near future. Mathematical biology is a fast-growing well recognized and the most exciting modern application of mathematics. The increasing use of mathematics in biology is inevitable as biology becomes quantitative.

The biological mathematical model becomes the theme for the dynamical systems. Therefore, the biological model was studied theoretically and numerically. The complexity of biological sciences makes interdisciplinary involvement essential. For the mathematician, biology opens up new and exciting branches while for the biologists' mathematical modelling offers another research tool.

Eli and Abanum (2020) presented a comparison between the Analytical and Numerical results of the stability analysis of a dynamical system. They formulated the system of ordinary Differential Equations involving Sickle Cells, HIV and T-Cells with the aid of a biological mathematical model. The eigenvalues were obtained to test for the trivial steady-state solution or points using a characteristic equation which is analytical.

Finally, they carried out a numerical simulation to test the level of reliability of the result. Solomonovich *et al.*(1998) studied the stability analysis problem for a new class of discrete-time recurrent neural networks with mixed time delays. The mixed time delays that consist of both the discrete and distributed time delays are addressed, for the first time, when analyzing the asymptotic stability for discrete-time neural networks. The activation functions are not required to be differentiable or strictly monotonic. The existence of the equilibrium point was first proved under mild conditions by constructing a new Lyapnuov-Krasovskii functional, a linear matrix inequality (LMI) approach is developed to establish sufficient conditions for the discrete-time neural networks to be globally asymptotically stable. As an extension, they further consider the stability analysis problem for the same class of neural networks but with state-dependent stochastic disturbances. All the conditions obtained are expressed in terms of LMIs whose feasibility can be easily checked by using the numerically efficient Matlab LMI Toolbox. A simulation example is presented to show the usefulness of the derived LMI-based stability condition.

Eli and Ekakaa, (2021) studied the effect of discrete time delays on the stability of a dynamical system using ODE45 numerical simulation techniques. It was shown from the result that the dynamical system is dominantly unstable.

Edward and Ford, (2003) studied the boundness and stability of differential equations. Their paper discusses the qualitative behaviour of solutions to different equations, focusing on the boundedness and stability of solutions. Examples demonstrate how the use of Lipschintz constants can provide insights into the qualitative behaviour of solutions to some nonlinear problems. Manchester Centre for Computational Mathematicsheir findings, and their conclusions. Chellaboina *et al*., (2003), discussed a dissipative dynamical systems approach to stability analysis of time-delay systems. In their paper, the concepts of dissipativity and exponential dissipativity are used to provide sufficient conditions for guaranteeing the asymptotic stability of a time delay dynamical system. Specifically, representing a time delay dynamical system as a negative feedback interconnection of a finite-dimensional linear dynamical system and an infinite-dimensional time delay operator, they show that the time delay operator is dissipative concerning a quadratic supply rate and with a storage functional involving an integral term identical to the integral term appearing in standard Lyapunov–Krasovskii functionals. Finally, using the stability of feedback interconnection results for dissipative systems, they develop enough conditions for asymptotic stability of time delay dynamical systems. The overall approach provides a dissipativity theoretic interpretation of Lyapunov–Krasovskiifunctionals for asymptotically stable dynamical systems with arbitrary time delay.

Rajendra (2021) analysed the standard approach of dynamical systems towards biomedical science. The purpose of their study is to meet the current and future needs for the interaction between various science and technology areas on the one hand and dynamical systems on the other hand. They discuss various models for interacting populations like Predator–Prey model, Lotka – Volterra system, the RealisticPredator–Prey model, Continuous Population models for single species, Discrete Population models for single species etc. They develop models which capture the essence of various interactions allowing the outcomes to be more fully understood. It has a very broad perspective to analyse biomedical issues using one of the most important branches of mathematics. The main objective of the study is to elaborate dynamical tools and this type of study will act as the bridge between biomedical sciences and mathematical sciences. Anand and Melba Mary (2016), also investigated an Improved Dynamic Response of DC to DC Converter Using Hybrid PSO Tuned Fuzzy Sliding Mode Controller. According to them, DC/DC switching converters are widely used in numerous appliances in modern existence. In their paper, the dynamic and transient response of phase shift series resonant DC/DC converter is improved using hybrid particle swarm optimization tuned fuzzy sliding mode controller under starting and load step change conditions. The aim of the control is to regulate the output voltage beneath the load change. The model of the hybrid particle swarm optimization tuned fuzzy sliding mode controller is implemented using the Sim Power Systems toolbox of MATLAB SIMULINK. The performance of the proposed dynamic novel control under step load change conditions is investigated.

Theory of Dynamic Interactions: Innovations was examined by Alejandro Álvarez (2016). The theory of dynamic interactions suggests a new paradigm of mechanics and initiates them into a new area of knowledge, hitherto undeveloped. In their paper, they describe the innovations that this theory brings to physics, and in particular, the ideas expressed in a new book by Doctor Barceló: New Paradigm in Physics. It is necessary to analyse the incorporation into mechanics not only of knowledge about bodies with inertial movement but also that of non-inertial systems. It is necessary for a new structure of knowledge that can incorporate both inertial and accelerated systems. In this paper, we referred to the main innovations and novel ideas proposed by Doctor Barceló in his new book, concerning the rotational dynamics

1.1 Model assumptions

For this study, we shall consider the following assumptions:

(i) The growth of yeast species 1 and yeast species 2 depends on the difference between the survival rate and the death rate.

(ii) The growth of these two species can also be affected by the interaction within each species otherwise called the self-interaction coefficient of the intra-competition process.

(iii) The growth of these two species can also be affected by the inter-competition coefficient which specified the contribution of each species to inhibit the growth of each species.

(iv) The growth of these two competing species can also be affected by the initial data values of yeast species 1 and yeast species 2 when under the length of the growing season in the unit of weeks.

1.2 Mathematical formulations

For this study, we have considered the following multi-parameter continuous dynamical system of a nonlinear first Order Ordinary Differential Equation from Eli and Ekakaa, 2021.

$$\frac{dx}{dt} = \propto_1 x - \beta_1 x^2 - \gamma_1 xy \qquad (1)$$

$$\frac{dy}{dt} = \propto_2 y - \beta_2 y^2 - \gamma_2 xy \qquad (2)$$

$x(t)$denotes the biomass of yeast specy 1 (candida albican) at time t in the unit of weeks.

$y(t)$ denotes the biomass of yeast specy 2 (candida parapsilosis) at time t in the unit of weeks.

$\propto_1$and $\propto_2$ specifies the growth rate of yeast species 1 and 2 respectively.

$\beta_1$andβ_2 specifies the intra-competition coefficient of yeast species 1 and 2 respectively.

$\gamma_1$andγ_2 denotes the competition of yeast species 1 and yeast 2 respectively where γ_1 is the contribution of the yeast species to inhibit the growth of species 2 as γ_2 is the contribution of the yeast specy 2 to inhibit the growth of the specy 1

2.0 Linearization of the Dynamical System

To obtain different eigenvalues to test for the stability of the system, it is important to linearize the system by letting the function F_1 and F_2 represent equations (1) and (2) respectively as

$$F_1 \quad \propto_1 x - \beta_1 x^2 - \gamma_1 xy \qquad (3)$$

$$F_2 = \propto_2 x - \beta_2 y^2 - \gamma_2 xy \qquad (4)$$

Differentiating (3) and (4) partially $w.r.tx$ and y we get

$$J_{11} = \frac{\delta F_1}{\delta x} = \propto_1 - 2\beta_1 x - \gamma_1 y \qquad (5)$$

$$J_{12} = \frac{\delta F_1}{\delta y} = -\gamma_1 x \qquad (6)$$

$$J_{21} = \frac{\delta F_2}{\delta x} = -\gamma_2 y \qquad (7)$$

$$J_{22} = \frac{2F_2}{2y} = \propto_2 - 2\beta_2 y - \gamma_2 x \qquad (8)$$

With the following model parameters values, in Eli and Ekakaa, 2021.

$\propto_1 = 0.1$, $\propto_1 = 0.08$, $\beta_1 = 0.0014$, $\beta_2 = 0.001$, $\gamma_1 = 0.0012$, $\gamma_2 = 0.0009$,

3.0 Results and Discussion

3.1 Results

Given the characteristics equation

$|J - \lambda \mathrm{I}|$

Applying the ODE 45 numerical method where I is the identity matrix of order 2 × 2 matrix and λ is a scalar.

Here a numerical method is applied to obtain the eigenvalues using the Matlab ODE 45. The possible steady state is the point (x, y) = (0, 0), applying the numerical method, we have obtained two eigenvalues $\lambda 1$ = 0.0800, $\lambda 2$ = 0.100.

To check for the stability type of this steady-state solution for this interacting problem, a Jacobian Matrix was defined from which two eigenvalues were calculated, the characteristics equation being $|J - \lambda \mathrm{I}|$ =0.

Table 1: Quantifying the effect of decreasing the model parameter α_1 = 0.1 and α_2 = 0.08 on the type of stability of a trivial steady state solution using Matlab Algorithm.

α_1&α_2 Variation	α_1	α_2	x	y	λ_1	λ_2	TOS
100%	0.1000	0.0800	0	0	0.0800	0.1000	Unstable
10%	0.0100	0.0080	0	0	0.0080	0.0100	Unstable
20%	0.0200	0.0160	0	0	0.0160	0.0200	Unstable
90%	0.0900	0.0720	0	0	0.0720	0.0900	Unstable
99%	0.0990	0.0792	0	0	0.0792	0.0990	Unstable
101%	0.1010	0.0808	0	0	0.0808	0.1010	Unstable
110%	0.1100	0.0880	0	0	0.880	0.1100	Unstable
120%	0.1200	0.0960	0	0	0.0960	0.1200	Unstable
130%	0.1300	0.1040	0	0	0.1040	0.1300	Unstable
140%	0.1400	0.1120	0	0	0.1120	0.1400	Unstable
150%	0.1500	0.1200	0	0	0.1200	0.1500	Unstable

****TOS = Type of Stability**

Table 2: Quantifying the effect of decreasing the α_1 and α_2 together by 10% on the biodiversity loss: using ODE44

LGS	$\dot{x}(old)$	$\dot{x}(new)$	BL	$\dot{y}(old)$	$\dot{y}(old)$	BL
1	20.0000	20.0000	0	30.0000	30.0000	0
8	29.9745	16.6859	44.3329	38.3574	27.5848	28.0849
15	41.5737	14.1675	65.9220	48.3189	25.7551	46.6976

22	52.8941	12.1988	76.9373	58.4827	24.3292	58.3993
29	62.0864	10.6148	82.9032	67.4179	23.1850	65.6100
36	68.1802	9.3161	86.3360	74.2993	22.2493	70.0545
43	71.2927	8.2356	88.4482	79.1076	21.4726	72.8564
50	72.1270	7.3237	89.8462	82.2717	20.8186	74.6953
57	71.4373	6.5459	90.8369	84.2948	20.2619	75.9630
64	69.7796	5.8762	91.5789	85.5525	19.7835	76.8756
71	67.5880	5.2947	92.1662	86.3374	19.3689	77.5661
78	65.1155	4.7863	92.6495	86.8294	19.0070	78.1100
85	62.5255	4.3391	93.0603	87.1433	18.6891	78.5536
92	59.9118	3.9436	93.4177	87.3469	18.4084	78.9250
99	57.3254	3.5920	93.7339	87.4793	18.1592	79.2417
106	54.8017	3.2783	94.0178	87.5682	54.8017	37.4183
113	52.3567	2.9973	94.2752	87.6282	17.7385	79.7571
120	49.9996	2.7447	94.5106	87.6693	17.5601	79.9701
127	47.7351	2.5170	94.7272	87.6981	17.3994	80.1599
134	45.5637	2.3111	94.9278	87.7184	17.2543	80.3299
141	43.4849	2.1245	95.1144	87.7329	17.1229	80.4830

Table 3: Quantifying the effect of decreasing the α_1 and α_2 together by 50% on the biodiversity loss: using ODE45

LGS	$\dot{x}(old)$	$\dot{x}(new)$	BL	$\dot{y}(old)$	$\dot{y}(old)$	BL
1	20.0000	20.0000	0	30.0000	30.0000	0
8	29.9745	21.6928	27.6293	38.3574	31.6419	17.5077
15	41.5737	23.2156	44.1580	48.3189	33.1851	31.3207
22	52.8941	24.5379	53.6093	58.4827	34.6034	40.8315
29	62.0864	25.6433	58.6973	67.4179	35.8806	46.7788
36	68.1802	26.5290	61.0899	74.2993	37.0108	50.1868
43	71.2927	27.2026	61.8439	79.1076	37.9959	51.9694
50	72.1270	27.6795	61.6239	82.2717	38.8437	52.7861
57	71.4373	27.9800	60.8328	84.2948	39.5661	53.0623
64	69.7796	28.1261	59.6930	85.5525	40.1766	53.0387
71	67.5880	28.1402	58.3652	86.3374	40.6895	52.8715
78	65.1155	28.0433	56.9330	86.8294	41.1184	52.6446
85	62.5255	27.8548	55.4504	87.1433	41.4761	52.4047
92	59.9118	27.5918	53.9459	87.3469	41.7738	52.1748
99	57.3254	27.2689	52.4313	87.4793	42.0213	51.9643
106	54.8017	26.8987	50.9163	87.5682	54.8017	37.4183
113	52.3567	26.4916	49.4017	87.6282	42.3984	51.6156
120	49.9996	26.0564	47.8868	87.6693	42.5410	51.4756
127	47.7351	25.6003	46.3700	87.6981	42.6600	51.3558
134	45.5637	25.1292	44.8482	87.7184	42.7595	51.2537
141	43.4849	24.6479	43.3185	87.7329	42.8428	51.1668

Table 4: Quantifying the effect of decreasing the α_1 and α_2 together by 90% on the biodiversity loss: using ODE45.

LGS	$\dot{x}(old)$	$\dot{x}(new)$	BL	$\dot{y}(old)$	$\dot{y}(old)$	BL
1	20.0000	20.0000	0	30.0000	30.0000	0
8	29.9745	28.1100	6.2204	38.3574	36.8449	3.9431
15	41.5737	37.1350	10.6768	48.3189	44.6518	7.5895
22	52.8941	45.8815	13.2578	58.4827	52.5360	10.1684
29	62.0864	53.2195	14.2816	67.4179	59.6228	11.5624
36	68.1802	58.4830	14.2230	74.2993	65.3549	12.0384
43	71.2927	61.6247	13.5611	79.1076	69.6293	11.9815
50	72.1270	62.9925	12.6645	82.2717	72.6429	11.7037
57	71.4373	63.0556	11.7330	84.2948	74.7001	11.3824
64	69.7796	62.2176	10.8369	85.5525	76.0697	11.0842
71	67.5880	60.8160	10.0195	86.3374	76.9785	10.8399
78	65.1155	59.0715	9.2819	86.8294	77.5802	10.6522
85	62.5255	57.1376	8.6170	87.1433	77.9810	10.5140
92	59.9118	55.1155	8.0056	87.3469	78.2521	10.4123
99	57.3254	53.0630	7.4354	87.4793	78.4356	10.3381
106	54.8017	51.0219	6.8972	87.5682	54.8017	37.4183
113	52.3567	49.0158	6.3812	87.6282	78.6506	10.2451
120	49.9996	47.0587	5.8819	87.6693	78.7127	10.2164
127	47.7351	45.1601	5.3944	87.6981	78.7572	10.1951
134	45.5637	43.3242	4.9152	87.7184	78.7892	10.1794
141	43.4849	41.5533	4.4419	87.7329	78.8126	10.1676

Table 5: Quantifying the effect of decreasing the α_1 and α_2 together by 95% on the biodiversity loss: using ODE45

LGS	$\dot{x}(old)$	$\dot{x}(new)$	BL	$\dot{y}(old)$	$\dot{y}(old)$	BL
1	20.0000	20.0000	0	30.0000	30.0000	0
8	29.9745	29.0281	3.1573	38.3574	37.5897	2.0015
15	41.5737	39.3015	5.4656	48.3189	46.4410	3.8865
22	52.8941	49.3011	6.7928	58.4827	55.4322	5.2162
29	62.0864	57.5671	7.2790	67.4179	63.4338	5.9096
36	68.1802	63.2806	7.1863	74.2993	69.7562	6.1146
43	71.2927	66.4562	6.7840	79.1076	74.3244	6.0464
50	72.1270	67.5988	6.2781	82.2717	77.4369	5.8767
57	71.4373	67.3144	5.7714	84.2948	79.4929	5.6966
64	69.7796	66.0850	5.2946	85.5525	80.8160	5.5364
71	67.5880	64.2990	4.8662	86.3374	81.6674	5.4090
78	65.1155	62.1954	4.4846	86.8294	82.2157	5.3136
85	62.5255	59.9341	4.1446	87.1433	82.5721	5.2456
92	59.9118	57.6161	3.8318	87.3469	82.8084	5.1960
99	57.3254	55.2956	3.5407	87.4793	82.9650	5.1604
106	54.8017	53.0116	3.2665	87.5682	54.8017	37.4183

113	52.3567	50.7843	3.0033	87.6282	83.1445	5.1167
120	49.9996	48.6252	2.7488	87.6693	83.1952	5.1035
127	47.7351	46.5415	2.5003	87.6981	83.2310	5.0937
134	45.5637	44.5357	2.2562	87.7184	83.2565	5.0866
141	43.4849	42.6085	2.0152	87.7329	83.2749	5.0813

Table 6: Quantifying the effect of decreasing the α_1 and α_2 together by 99.9% on the biodiversity loss: using ODE45

LGS	$\dot{x}(old)$	$\dot{x}(new)$	BL	$\dot{y}(old)$	$\dot{y}(old)$	BL
1	20.0000	20.0000	0	30.0000	30.0000	0
8	29.9745	29.9553	0.0641	38.3574	38.3418	0.0406
15	41.5737	41.5274	0.1115	48.3189	48.2806	0.0793
22	52.8941	52.8204	0.1393	58.4827	58.4201	0.1071
29	62.0864	61.9945	0.1480	67.4179	67.3367	0.1205
36	68.1802	68.0814	0.1449	74.2993	74.2072	0.1240
43	71.2927	71.1963	0.1352	79.1076	79.0113	0.1217
50	72.1270	72.0373	0.1244	82.2717	82.1747	0.1179
57	71.4373	71.3562	0.1135	84.2948	84.1987	0.1140
64	69.7796	69.7075	0.1034	85.5525	85.4580	0.1105
71	67.5880	67.5241	0.0946	86.3374	86.2442	0.1079
78	65.1155	65.0591	0.0867	86.8294	86.7374	0.1060
85	62.5255	62.4756	0.0798	87.1433	87.0520	0.1047
92	59.9118	59.8678	0.0734	87.3469	87.2563	0.1037
99	57.3254	57.2867	0.0675	87.4793	87.3892	0.1031
106	54.8017	54.7678	0.0619	87.5682	54.8017	37.4183
113	52.3567	52.3271	0.0566	87.6282	87.5387	0.1022
120	49.9996	49.9739	0.0514	87.6693	87.5799	0.1020
127	47.7351	47.7130	0.0464	87.6981	87.6088	0.1018
134	45.5637	45.5449	0.0414	87.7184	87.6292	0.1017
141	43.4849	43.4690	0.0365	87.7329	87.6438	0.1016

Table 7: Quantifying the effect of increasing the α_1 and α_2 together by 101% on the biodiversity gain: using ODE45

LGS	$\dot{x}(old)$	$\dot{x}(new)$	BG	$\dot{y}(old)$	$\dot{y}(old)$	BG
1	20.0000	20.0000	0	30.0000	30.0000	0
8	29.9745	30.1671	0.6427	38.3574	38.5137	0.4074
15	41.5737	42.0397	1.1209	48.3189	48.7043	0.7976
22	52.8941	53.6346	1.4000	58.4827	59.1123	1.0765
29	62.0864	63.0087	1.4856	67.4179	68.2341	1.2106
36	68.1802	69.1698	1.4514	74.2993	75.2232	1.2435
43	71.2927	72.2561	1.3513	79.1076	80.0715	1.2185
50	72.1270	73.0219	1.2408	82.2717	83.2423	1.1797
57	71.4373	72.2448	1.1304	84.2948	85.2556	1.1398
64	69.7796	70.4971	1.0283	85.5525	86.4979	1.1050
71	67.5880	68.2228	0.9393	86.3374	87.2689	1.0789

78	65.1155	65.6757	0.8602	86.8294	87.7495	1.0597
85	62.5255	63.0202	0.7912	87.1433	88.0553	1.0466
92	59.9118	60.3474	0.7270	87.3469	88.2527	1.0370
99	57.3254	57.7081	0.6676	87.4793	88.3806	1.0302
106	54.8017	55.1370	0.6118	87.5682	54.8017	37.4183
113	52.3567	52.6489	0.5580	87.6282	88.5239	1.0222
120	49.9996	50.2527	0.5062	87.6693	88.5633	1.0198
127	47.7351	47.9525	0.4555	87.6981	88.5909	1.0180
134	45.5637	45.7486	0.4058	87.7184	88.6102	1.0167
141	43.4849	43.6400	0.3567	87.7329	88.6241	1.0158

Table 8: Quantifying the effect of increasing the α_1 and α_2 together by 105% on the biodiversity gain: using ODE45

LGS	$\dot{x}(old)$	$\dot{x}(new)$	BG	$\dot{y}(old)$	$\dot{y}(old)$	BG
1	20.0000	20.0000	0	30.0000	30.0000	0
8	29.9745	30.9493	3.2522	38.3574	39.1483	2.0619
15	41.5737	43.9474	5.7095	48.3189	50.2827	4.0642
22	52.8941	56.6660	7.1310	58.4827	61.6931	5.4894
29	62.0864	66.7579	7.5242	67.4179	71.5625	6.1475
36	68.1802	73.1522	7.2924	74.2993	78.9642	6.2786
43	71.2927	76.0943	6.7351	79.1076	83.9497	6.1210
50	72.1270	76.5603	6.1465	82.2717	87.1326	5.9084
57	71.4373	75.4136	5.5662	84.2948	89.0967	5.6965
64	69.7796	73.2958	5.0391	85.5525	90.2723	5.5168
71	67.5880	70.6855	4.5829	86.3374	90.9860	5.3843
78	65.1155	67.8394	4.1832	86.8294	91.4221	5.2893
85	62.5255	64.9220	3.8329	87.1433	91.6966	5.2251
92	59.9118	62.0136	3.5081	87.3469	91.8698	5.1781
99	57.3254	59.1647	3.2087	87.4793	91.9810	5.1460
106	54.8017	56.4052	2.9260	87.5682	54.8017	-37.4183
113	52.3567	53.7464	2.6542	87.6282	92.1037	5.1074
120	49.9996	51.1957	2.3921	87.6693	92.1370	5.0961
127	47.7351	48.7547	2.1360	87.6981	92.1601	5.0879
134	45.5637	46.4225	1.8848	87.7184	92.1761	5.0819
141	43.4849	44.1969	1.6375	87.7329	92.1876	5.0776

Table 9: Quantifying the effect of increasing the α_1 and α_2 together by 110% on the biodiversity gain: using ODE45

LGS	$\dot{x}(old)$	$\dot{x}(new)$	BG	$\dot{y}(old)$	$\dot{y}(old)$	BG
1	20.0000	20.0000	0	30.0000	30.0000	0
8	29.9745	31.9537	6.6031	38.3574	39.9633	4.1867
15	41.5737	46.4320	11.6859	48.3189	52.3401	8.3221
22	52.8941	60.6086	14.5848	58.4827	65.0583	11.2437
29	62.0864	71.5689	15.2732	67.4179	75.8600	12.5219

36	68.1802	78.1682	14.6493	74.2993	83.7312	12.6944
43	71.2927	80.8459	13.3998	79.1076	88.8396	12.3022
50	72.1270	80.8818	12.1381	82.2717	92.0076	11.8339
57	71.4373	79.2319	10.9110	84.2948	93.8886	11.3812
64	69.7796	76.6322	9.8204	85.5525	94.9719	11.0100
71	67.5880	73.5943	8.8866	86.3374	95.6123	10.7427
78	65.1155	70.3762	8.0790	86.8294	95.9960	10.5570
85	62.5255	67.1317	7.3669	87.1433	96.2333	10.4311
92	59.9118	63.9315	6.7093	87.3469	96.3790	10.3405
99	57.3254	60.8249	6.1047	87.4793	96.4723	10.2801
106	54.8017	57.8332	5.5318	87.5682	54.8017	-37.4183
113	52.3567	54.9652	4.9821	87.6282	96.5728	10.2073
120	49.9996	52.2255	4.4517	87.6693	96.5997	10.1864
127	47.7351	49.6129	3.9338	87.6981	96.6181	10.1712
134	45.5637	47.1249	3.4263	87.7184	96.6308	10.1603
141	43.4849	44.7577	2.9270	87.7329	96.6398	10.1523

Table 10: Quantifying the effect of increasing the α_1 and α_2 together by 120% on the biodiversity gain: using ODE45

LGS	$\dot{x}(old)$	$\dot{x}(new)$	BG	$\dot{y}(old)$	$\dot{y}(old)$	BG
1	20.0000	20.0000	0	30.0000	30.0000	0
8	29.9745	34.0545	13.6117	38.3574	41.6683	8.6317
15	41.5737	51.7457	24.4674	48.3189	56.7464	17.4413
22	52.8941	68.9815	30.4144	58.4827	72.2405	23.5245
29	62.0864	81.5480	31.3461	67.4179	84.8743	25.8928
36	68.1802	88.2206	29.3932	74.2993	93.4924	25.8321
43	71.2927	90.1036	26.3854	79.1076	98.6929	24.7579
50	72.1270	88.9970	23.3894	82.2717	101.6643	23.5714
57	71.4373	86.2511	20.7367	84.2948	103.3194	22.5692
64	69.7796	82.7078	18.5273	85.5525	104.2420	21.8457
71	67.5880	78.8502	16.6630	86.3374	104.7697	21.3492
78	65.1155	74.9115	15.0440	86.8294	105.0717	21.0094
85	62.5255	71.0265	13.5961	87.1433	105.2484	20.7763
92	59.9118	67.2684	12.2791	87.3469	105.3576	20.6196
99	57.3254	63.6609	11.0520	87.4793	105.4239	20.5130
106	54.8017	60.2206	9.8882	87.5682	54.8017	-37.4183
113	52.3567	56.9501	8.7732	87.6282	105.4939	20.3880
120	49.9996	53.8467	7.6943	87.6693	105.5120	20.3522
127	47.7351	50.9063	6.6434	87.6981	105.5242	20.3267
134	45.5637	48.1223	5.6154	87.7184	105.5326	20.3085
141	43.4849	45.4878	4.6062	87.7329	105.5385	20.2952

Table 11: Quantifying the effect of increasing the α_1 and α_2 together by 130% on the biodiversity gain: using ODE45

LGS	$\dot{x}(old)$	$\dot{x}(new)$	BG	$\dot{y}(old)$	$\dot{y}(old)$	BG
1	20.0000	20.0000	0	30.0000	30.0000	0
8	29.9745	36.2833	21.0474	38.3574	43.4777	13.3490
15	41.5737	57.5359	38.3950	48.3189	61.5577	27.3987
22	52.8941	77.9715	47.4106	58.4827	80.0090	36.8080
29	62.0864	91.8830	47.9922	67.4179	94.3666	39.9726
36	68.1802	98.1581	43.9686	74.2993	103.4549	39.2407
43	71.2927	98.9256	38.7597	79.1076	108.5634	37.2352
50	72.1270	96.5533	33.8657	82.2717	111.2748	35.2528
57	71.4373	92.6643	29.7141	84.2948	112.6998	33.6973
64	69.7796	88.1519	26.3290	85.5525	113.4614	32.6219
71	67.5880	83.4533	23.4736	86.3374	113.8720	31.8919
78	65.1155	78.7983	21.0130	86.8294	114.1003	31.4074
85	62.5255	74.2973	18.8272	87.1433	114.2332	31.0867
92	59.9118	69.9885	16.8192	87.3469	114.3090	30.8677
99	57.3254	65.8988	14.9558	87.4793	114.3561	30.7236
106	54.8017	62.0287	13.1875	87.5682	54.8017	-37.4183
113	52.3567	58.3740	11.4929	87.6282	114.4039	30.5559
120	49.9996	54.9281	9.8570	87.6693	114.4161	30.5087
127	47.7351	51.6811	8.2666	87.6981	114.4242	30.4751
134	45.5637	48.6234	6.7152	87.7184	114.4296	30.4512
141	43.4849	45.7448	5.1972	87.7329	114.4335	30.4339

Table 12: Quantifying the effect of increasing the α_1 and α_2 together by 140% on the biodiversity gain: using ODE45

LGS	$\dot{x}(old)$	$\dot{x}(new)$	BG	$\dot{y}(old)$	$\dot{y}(old)$	BG
1	20.0000	20.0000	0	30.0000	30.0000	0
8	29.9745	38.6468	28.9324	38.3574	45.3971	18.3528
15	41.5737	63.8197	53.5097	48.3189	66.7910	38.2294
22	52.8941	87.5347	65.4906	58.4827	88.3405	51.0540
29	62.0864	102.4011	64.9333	67.4179	104.2176	54.5844
36	68.1802	107.8518	58.1863	74.2993	113.5240	52.7929
43	71.2927	107.2154	50.3875	79.1076	118.3732	49.6358
50	72.1270	103.4914	43.4850	82.2717	120.7869	46.8146
57	71.4373	98.4433	37.8038	84.2948	121.9870	44.7147
64	69.7796	92.9624	33.2229	85.5525	122.5996	43.3033
71	67.5880	87.4485	29.3847	86.3374	122.9161	42.3672
78	65.1155	82.1058	26.0926	86.8294	123.0914	41.7623
85	62.5255	76.9963	23.1438	87.1433	123.1846	41.3587
92	59.9118	72.1645	20.4513	87.3469	123.2402	41.0927
99	57.3254	67.6118	17.9439	87.4793	123.2734	40.9172
106	54.8017	63.3317	15.5651	87.5682	54.8017	-37.4183

113	52.3567	59.3151	13.2904	87.6282	123.3062	40.7151
120	49.9996	55.5484	11.0976	87.6693	123.3144	40.6586
127	47.7351	52.0179	8.9721	87.6981	123.3198	40.6186
134	45.5637	48.7101	6.9054	87.7184	123.3235	40.5903
141	43.4849	45.6114	4.8902	87.7329	123.3260	40.5699

Table 13: Quantifying the effect of increasing the α_1 and α_2 together by 150% on the biodiversity gain: using ODE45

LGS	$\dot{x}(old)$	$\dot{x}(new)$	BG	$\dot{y}(old)$	$\dot{y}(old)$	BG
1	20.0000	20.0000	0	30.0000	30.0000	0
8	29.9745	41.1519	37.2897	38.3574	47.4320	23.6581
15	41.5737	70.6097	69.8421	48.3189	72.4603	49.9626
22	52.8941	97.6041	84.5274	58.4827	97.1934	66.1917
29	62.0864	112.9470	81.9192	67.4179	114.3188	69.5674
36	68.1802	117.1731	71.8578	74.2993	123.6055	66.3616
43	71.2927	114.9486	61.2347	79.1076	128.0916	61.9208
50	72.1270	109.8164	52.2542	82.2717	130.1902	58.2442
57	71.4373	103.6292	45.0631	84.2948	131.1913	55.6339
64	69.7796	97.1923	39.2846	85.5525	131.6714	53.9071
71	67.5880	90.9004	34.4920	86.3374	131.9211	52.7973
78	65.1155	84.8762	30.3471	86.8294	132.0471	52.0764
85	62.5255	79.1915	26.6547	87.1433	132.1175	51.6096
92	59.9118	73.8549	23.2728	87.3469	132.1578	51.3021
99	57.3254	68.8587	20.1192	87.4793	132.1805	51.0992
106	54.8017	64.1919	17.1349	87.5682	54.8017	-37.4183
113	52.3567	59.8355	14.2843	87.6282	132.2037	50.8688
120	49.9996	55.7715	11.5438	87.6693	132.2093	50.8045
127	47.7351	51.9815	8.8958	87.6981	132.2130	50.7593
134	45.5637	48.4478	6.3297	87.7184	132.2155	50.7273
141	43.4849	45.1535	3.8374	87.7329	132.2172	50.7043

LGS = Length of growing season, x(old) = Measures the biomass of yeast specie 1 when all model parameters are fixed at 100%, y(new) = Measures the biomass of yeast specie 2 when α_1 and α_2 only are varied, BL = Biodiversity Loss, BG = Biodiversity Gain

4.2 Discussion of Results

Table 2 to Table 13 are the results of the effect of decreasing and increasing α_1 and α_2 together. Table 2 shows the extent of the percentage of biodiversity loss (BL) due to the low environmental perturbation value by 10% (0.10) concerning yeast species. A close look at the first-row result of example 1, we observed that the numerically simulated data of yeast species 1 and 2 when the model parameters are fixed at 100% and when α_1 and $\alpha_2$together are varied at 10% that is *x*(old) and *x*(new), *y*(old) and *y*(new) gives the same value of 20kg and 30kg which is consistent with the notion of population modelling and prediction which shows that the population size of the growing population does not change. Applying the method of ODE45 numerical simulation, we observed that in the same first row in example 1, the estimated population is approximately 0.00 because the *x*(old) and *x*(new), *y*(old) and *y*(new) are equal. Observing from example 2 to example 21 of the yeast species 1 and 2 when model parameter values are fixed at 100% and α_1 and α_2 together only are varied at 10%, the solution map or solution trajectories follow a random pattern, from this prediction *x*(old) and *y*(old) in table 2 to table 3 remain the same because all the model parameters are fixed at 100% while *x*(new) and *y*(new) varies because of the variation of α_1 and α_2 together.

Now the biomass of yeast species 1 and 2, *x*(new) and *y*(new) in the unit of the kilogram (kg) has lost biomass over that of *x*(old) and *y*(old) showing evidence of biodiversity loss.

From Table 2 to Table 13, we varied α_1 and α_2 together called the environmental perturbation from 10% to 150% in (new) and (new) while in (old) and (old) all the model parameters are fixed at 100%. As we increased the decrease from 10% to 99.99%, the value of the first scenario is less the value of the second scenario and third is less than the fourth.

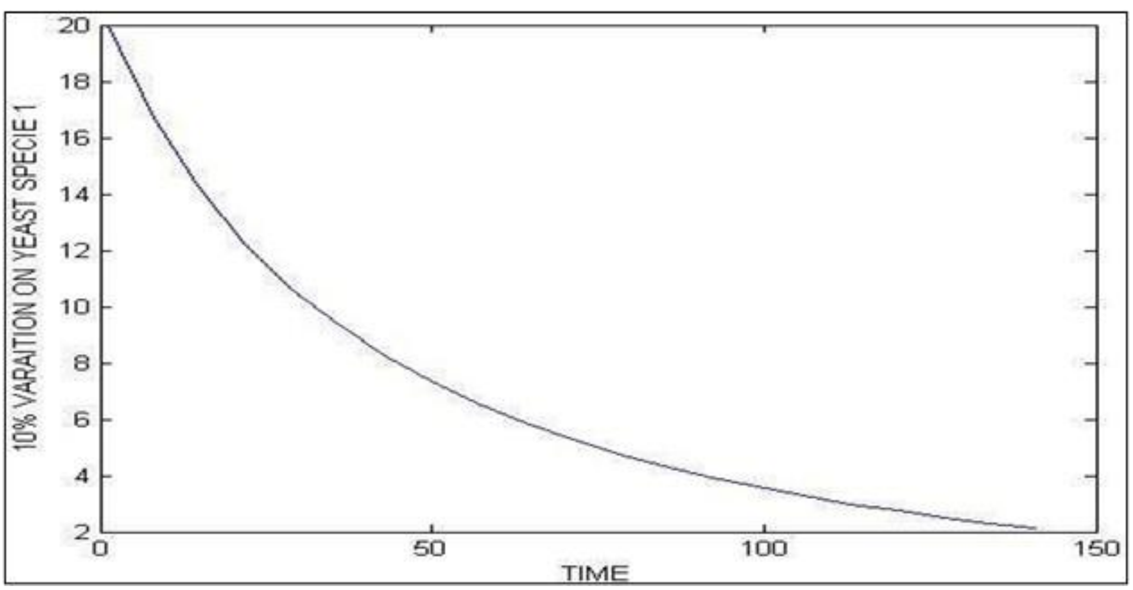

Fig. 1. Graphical Simulation on the effects of decreasing α_1 and α_2 together by 10% on biodiversity loss of yeast specie 1

As the percentage increased, there was an improved biomass of yeast species 1 and 2 called x(new) and y(new) which gets closer to the biomass of yeast species 1 and 2 at fixed model parameters called x(old) and y(old). The result obtained shows that as the percentage increases, the loss in biodiversity from 80 (approximately) to 0.00.

From example 1 to example 21, we consistently observed an increase on yeast species 1 and 2, and the remains after varying together α_1 and $\alpha_2$from 101% to 150% are greater than that is when all the model parameters are fixed at 100%.

The values of the first scenario are less than the values of the second scenario and the second is less than the third and so on as we increased α_1 and α_2 together, the values get bigger which means the biomass of yeast species 1 and 2 in the unit of a kilogram (kg) has maintained an improved (new) and (new) over that of (old) and (old) showing evidence of biodiversity gain.

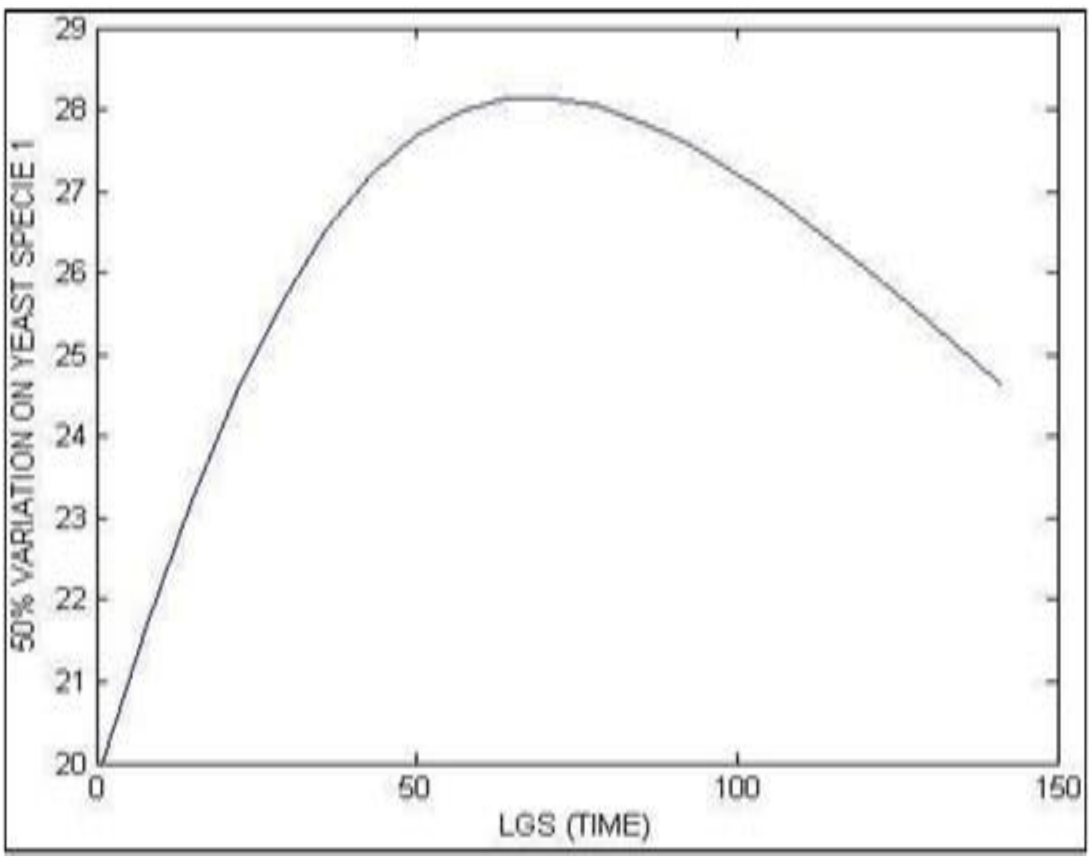

Fig 2. Graphical Simulation on the effects of decreasing α_1 and α_2 together by 50% on biodiversity loss of yeast specie 1.

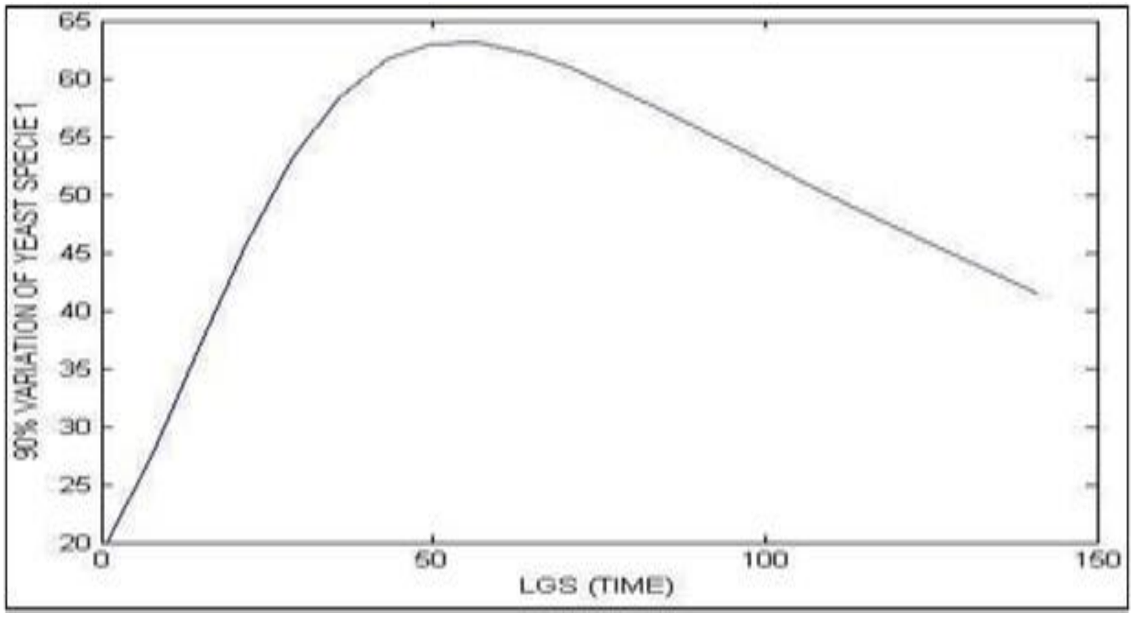

Fig. 3. Graphical Simulation on the effects of decreasing α_1 and α_2 together by 90% on biodiversity loss of yeast specie 1.

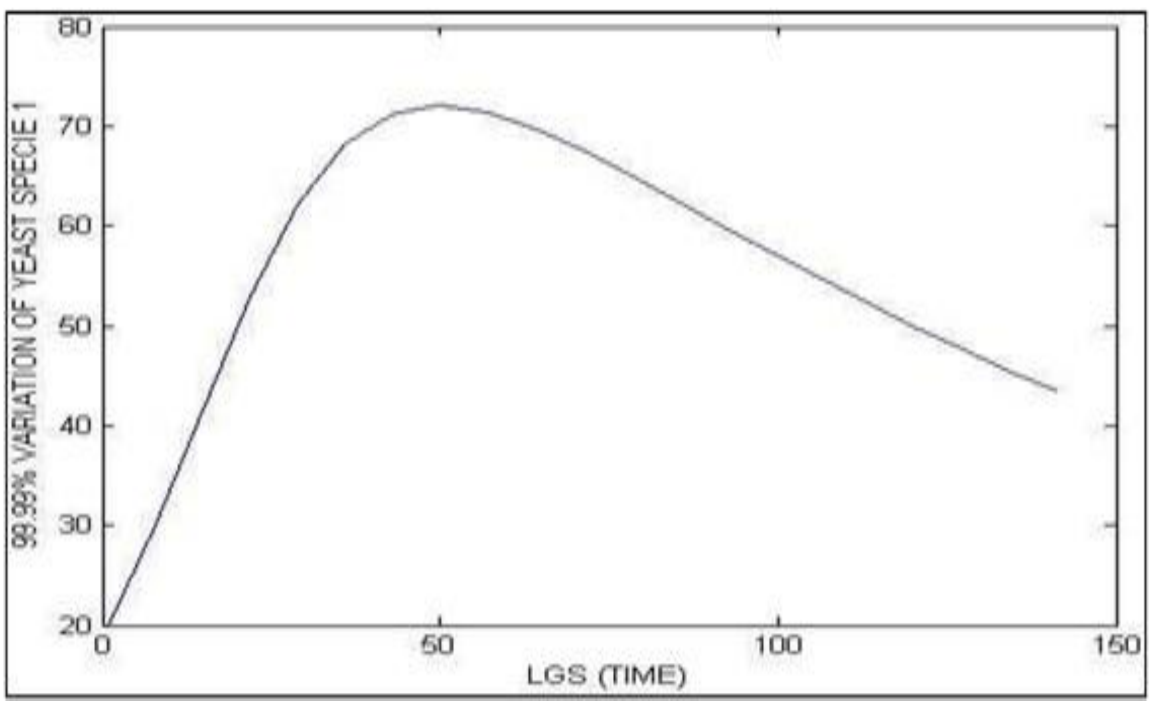

Fig. 4. Graphical Simulation on the effects of decreasing α_1 and α_2 together by 99.99% on biodiversity loss of yeast specie 1.

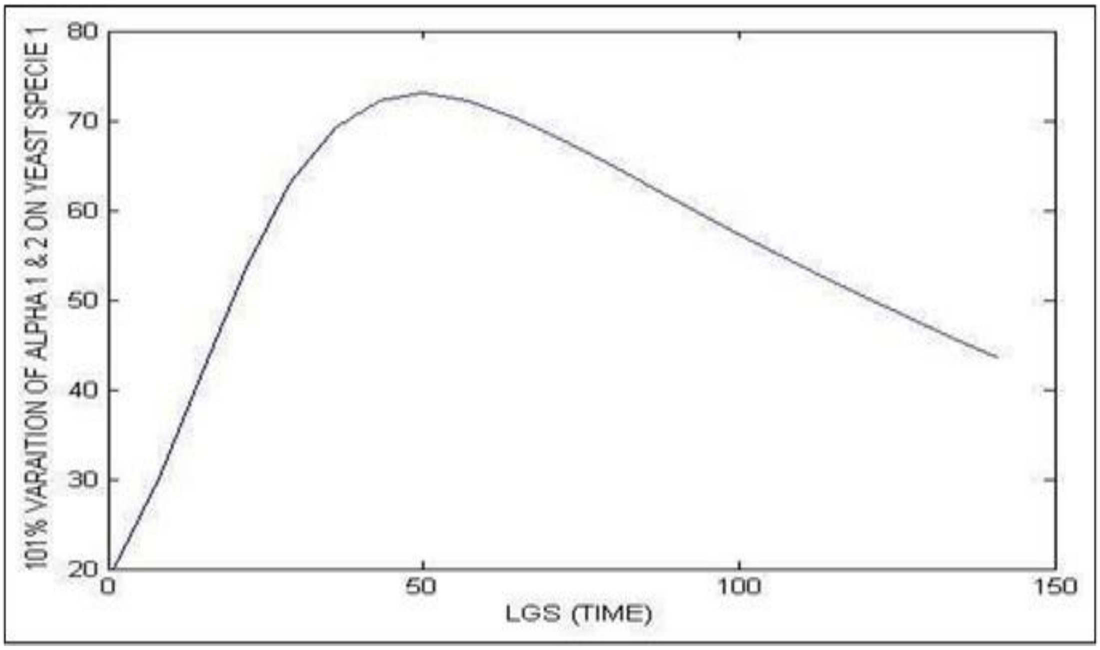

Fig. 5. Graphical Simulation on the effects of increasing α_1 and $\alpha_2$together by 101% on biodiversity loss of yeast specie 1.

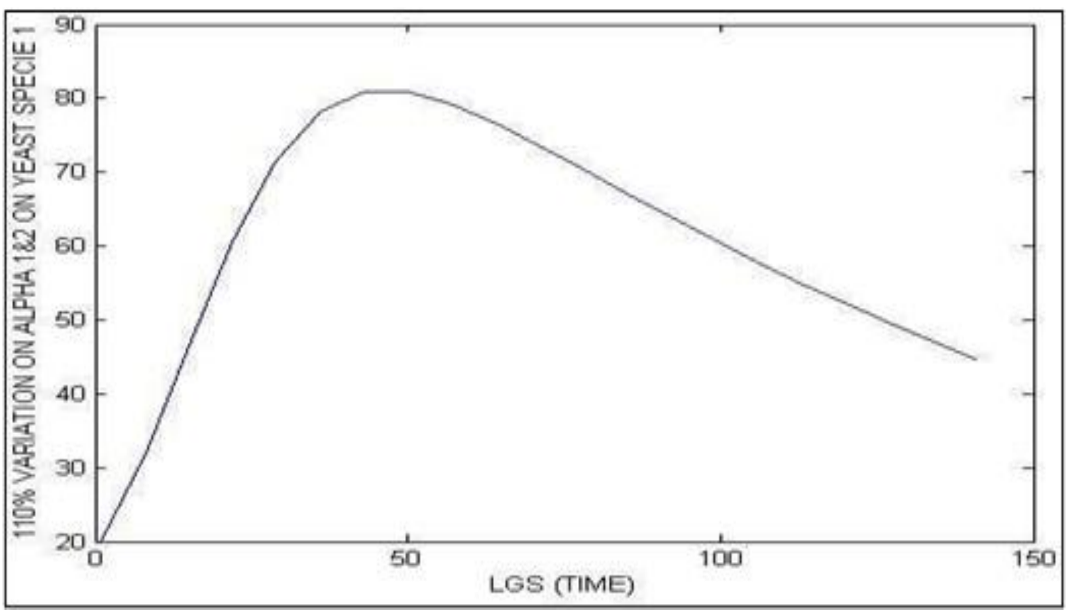

Fig. 6. Graphical Simulation on the effects of increasing α_1 and $\alpha_2$together by 110% on biodiversity loss of yeast specie 1.

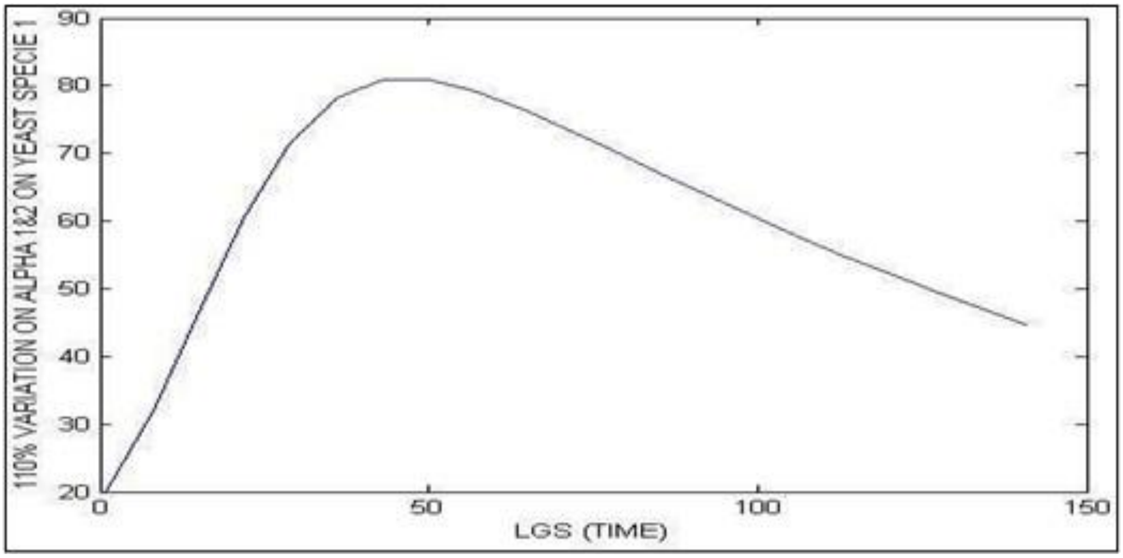

Fig. 7. Graphical Simulation on the effects of increasing α_1 and $\alpha_2$together by 150% on biodiversity loss of yeast specie 1

4.0 Conclusion

On the implementation of ODE45 computational approach, a region of instability was found. From example 1 to example 21, we consistently observed an increase in yeast species 1 and 2. The remains after varying α_1 and α_2 together from 101% to 150% are greater the remains when all model parameters are fixed at 100%. It was also found that the values of the first scenario are less than the values of the second scenario and the second is less than the third and so on as we increased α_1 and α_2 together, the values get bigger which means the biomass of yeast species 1 and 2 in the unit of a kilogram (Kg) has maintained an improved x(new) and y(new) over that of x(old) and y(old) showing evidence of biodiversity gain.

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Declarations

The authors declare that they have no conflict of interest.

Data availability

All data used in this study will be readily available to the public.

Consent for publication

Not Applicable

Availability of data and materials

The publisher has the right to make the data Public.

Competing interests

The authors declared no conflict of interest.

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Authors' contributions

Both authors contributed equally to the work