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A New Family of Smooth Transition Autoregressive (STAR) Models: Properties and Application of its Symmetric Version to Exchange Rates

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Abstract: *A good number of economic variables undergo the process of regime shifts. In modeling such variables, it is necessary to consider a model that has provision for the regime form of nonstationarity. The smooth transition autoregressive (STAR) model is a choice model for time series with regime shifts. Given the role of transition functions in the performance of STAR models, this study introduced a family of transition functions by modifying the conventional logistic function. This new family, called the power logistic transition function, has the symmetric transition function and asymmetric transition function as special cases, making it useful in constructing both symmetric and asymmetric STAR models. The symmetric form of the family and the associated STAR model are extensively explained. The performance of the symmetric version of the power logistic smooth transition autoregressive model was illustrated with a monthly exchange rate of naira to United States dollar and African Financial Community Franc spanning from January 2004 to April 2021, which were extracted from Central Bank of Nigeria statistical bulletin. The numerical results obtained show that the symmetric power logistic smooth transition autoregressive model outperforms the linear autoregressive model and other existing symmetric smooth transition autoregressive models.*

Keywords: Regime shifts, Time series, Nonstationarity, Power logistic function, smooth transition autoregressive model

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1.0 Introduction

Time series analysts have made appreciable efforts to introduce a rich class of regime-switching models in which different time series are allowed to undergo regime shifts. Chan and Tong (1986) suggested the generalization of the nonlinear two-regime univariate self-exciting threshold autoregressive (SETAR) model introduced by Tong (1983) to smooth threshold autoregressive (STAR) model to make the transition from one regime to the other smooth. Also, Terasvirta (1994) made a slight generalization of the exponential autoregressive (EAR) model of Haggan and Ozaki (1981) and the SETAR model of Tong (1983) into a single family of models called smooth transition autoregressive (STAR) models. The exponential STAR (ESTAR) is the modification of EAR, while the logistic STAR (LSTAR) family contains, as a special case, the SETAR model. However, the idea of smooth transition autoregression is traced back to Bacon and Watts (1971). The smooth transition autoregressive (STAR) model is a special class of nonlinear models that has a widespread application to exchange rates. Several empirical studies have shown that STAR models account for the dynamics of exchange rates (Taylor and Sarno, 1998; Sarantis, 1999; Taylor and Peel, 2000; Sarno, 2000; Baum *et al.,* 2001; Liew *et al.,* 2003; Tong and Lim, 1980).

Different analysts have made diverse modifications to STAR models to capture different properties of time series in the areas that the STAR models are found to be deficient. For instance, Anderson (1997) modified ESTAR to asymmetric ESTAR to capture possible asymmetry in the investors' response to situations when the bill of shorter maturity is either overpriced or underpriced relative to the bill of longer maturity. Liew *et al.* (2003) modified the LSTAR model to absolute logistic STAR (ALSTAR) to capture the dynamics of two Yen-based nominal exchange rates. Shangodoyin *et al.* (2009) proposed an alternative representation of the LSTAR model called the error logistic smooth transition regression (ELSTR) model with the asymmetric transition function to model the United States of America and Nigerian inflation series separately. Silverstone (2005) generalized a two-regime second-order logistic transition function (LSTR2) to a B-parameter smooth transition autoregressive (BSTAR) model with an asymmetric transition function for modeling the annual growth rates of Italian industrial production. Ajmi and El-Montasser (2012) generalized the BSTAR model to Seasonal Bi- parameter Smooth Transition autoregressive (SEA-BSTAR) model by the introduction of seasonal dummies into the model to account for the seasonal effect in the United Kingdom industrial production index. Yaya and Shittu (2016) generalized the LSTAR model to two symmetric STAR models, namely, the absolute error logistic STAR (AELSTAR) model and the quadratic logistic STAR (QLSTAR) model, to account for the symmetric properties of economic time series.

The choice of transition function is fundamental to the forecast performance of a STAR model. Thus, when we empirically compare symmetric or asymmetric models built with different transition functions, their performance ranking differs. A time series analyst using STAR models often desire the most efficient of the models. The relative efficiency of each of the models varies across data. In a variety of situations, it is necessary to introduce a new STAR model by modifying the existing one to improve fit. One way of modifying a STAR model is to replace its transition function with a new one. The purpose of this work is to introduce a new family of transition functions called the power logistic (PL) function, paying attention to its symmetric version and the related STAR model for empirical illustrations. This paper also compares the forecasting performance of the symmetric Power Logistic STAR (SPLSTAR) model with other existing symmetric STAR models together with the linear autoregressive (AR) model.

This paper is further organized as follows: Section 2 covers the materials and methods.

Results are presented in Section 3. Discussion of results is carried out in Section 4, while Section 5 concludes the study.

2.0 Materials and Method

2.1 Source of Data

The monthly exchange rate of naira to United States dollar (USD) and African Financial Community (CFA) Franc spanning from January 2004 to April 2021, which were extracted from the Central Bank of Nigeria statistical bulletin, will be used for empirical analyses.

2.2 The Proposed Family of Transition Functions and its Properties

The proposed family has the following representation:

$$F(y_{t-d};\delta,\lambda) = \left\{1 + 0.5\exp[-\delta(y_{t-d}^{i} - \lambda)]\right\}^{-2} - \frac{1}{1.5^2}, \delta > 0,\ \ i = 1,2, \quad (1)$$

where δ is the scale parameter and λ is the location parameter. $\frac{1}{1.5^2}$ is subtracted from (1) to ensure that $F(y_{t-d};\delta = 0,\lambda) = 0$ and it is useful in performing linearity tests. 0.5 is attached to (1) to ensure that the derivative of order $(2s + 1), s \geq 0$ exists and that $\frac{d^k}{d\delta^k}F(y_{t-d};\delta = 0,\lambda) \neq 0$, for odd k and $1 \leq k \leq 2s + 1$. According to Luukkonem *et al.* (1988), a transition function must satisfy the following conditions:

(I) The transition function, $F(\delta)$, is a continuous function possessing a non-zero derivative of order $(2s + 1), s \geq 0$.

(II) $F(0) = 0$ and $\frac{d^k}{d\delta^k}F(\delta)\Big|_{\delta=0} \neq 0$, for odd k and $1 \leq k \leq 2s + 1$. The condition $F(0) = 0$ is not restrictive as it is only used for performing the linearity test. The Taylor series approximation of $F(y_{t-d};\delta,\lambda)$ is generally used to test the null hypothesis of linearity against the alternative of STAR-type nonlinearity due to the problem of unidentified nuisance parameters under the null hypothesis. It will be used to verify the requisite properties of the power logistic function. If f is defined in the interval containing "x_0" and its derivatives of all orders exist at $\delta = x_0$, then by Taylor series expansion,

$$f(x) \cong f(x_0) + \frac{f'(x_0)(x - x_0)}{1!} + \frac{f''(x_0)(x - x_0)^2}{2!} + \frac{f'''(x_0)(x - x_0)^3}{3!} + \cdots, \quad (2)$$

where $f^{j}(x_0)$ denotes the jth derivative of f evaluated at the point x_0. The Taylor series third-order approximation to the power logistic function (1) is

$$F(y_{t-d};\delta,\lambda) \cong F(y_{t-d};\delta = x_0,\lambda) + \sum_{j=1}^{3}\frac{F^{j}(y_{t-d};\delta = x_0,\lambda)(\delta - x_0)^{j}}{j!}. \quad (3)$$

From (1), we obtain

$$F'(y_{t-d};\delta = x_0,\lambda) = \frac{(y_{t-d}^{i} - \lambda)\exp[-x_0(y_{t-d}^{i} - \lambda)]}{\{1 + 0.5\exp[-x_0(y_{t-d}^{i} - \lambda)]\}^{3}}, \quad (4)$$

$$F''(y_{t-d};\delta = x_0,\lambda) = \frac{-\left(y_{t-d}^{i}-\lambda\right)^{2}\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\left\{1+0.5\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right\} + 1.5\left(y_{t-d}^{i}-\lambda\right)^{2}\left\{\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right\}^{2}}{\left\{1+0.5\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right\}^{4}}. \quad (5)$$

$$F'''(y_{t-d};\delta = x_0,\lambda) = \frac{q.r.s.t}{u}, \quad (6)$$

where

$q = \left(y_{t-d}^{i}-\lambda\right)^{3}\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]$,
$r = \left[\left\{1+0.5\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right\} - 1.5\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right]$,
$s = \left\{1+0.5\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right\} + 3\left(y_{t-d}^{i}-\lambda\right)^{3}\left\{\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right\},^{2}$
$t = \left[\left\{\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right\} - \left\{1+0.5\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right\}\right]$,
$u = \left\{1+0.5\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right\}^{5}$.

Substituting (4), (5) and (6) into (3), we have

$$F(y_{t-d};\delta,\lambda) = a + b\times\frac{(\delta - x_0)}{1!} + \frac{c+d}{u}\times\frac{(\delta - x_0)^2}{2!} + \frac{q.r.s.t}{u}\times\frac{(\delta - x_0)^3}{3!}, \quad (7)$$

where

$a = \left\{1+0.5\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right\}^{-2} - \frac{1}{1.5^2}$,

$b = \frac{\left(y_{t-d}^{i}-\lambda\right)\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]}{\left\{1+0.5\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right\}^{3}}$,

$c = -\left(y_{t-d}^{i}-\lambda\right)^{2}\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\left\{1+0.5\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right\}$,
$d = 1.5\left(y_{t-d}^{i}-\lambda\right)^{2}\left\{\exp\left[-x_0\left(y_{t-d}^{i}-\lambda\right)\right]\right\}^{2}$.

When $x_0 = 0$ in (7), we have

$$F(y_{t-d};\delta,\lambda) \cong \frac{\left(y_{t-d}^{i}-\lambda\right)\delta}{\{1.5\}^3} - \frac{(3)(0.5)\left(y_{t-d}^{i}-\lambda\right)^{3}\delta^3}{1.5^5\times 3!},$$

where $F''(y_{t-d};\delta = 0,\lambda) = 0$.

$$\therefore F(y_{t-d};\delta,\lambda) \cong 0.2963\left(y_{t-d}^{i}-\lambda\right)\delta - 0.0329\left(y_{t-d}^{i}-\lambda\right)^{3}\delta^3. \quad (8)$$

(8) proves that (1) is a continuous function possessing a non-zero derivative of order $(2s+1), s = 0, 1$. Also, $F(y_{t-d};\delta = 0,\lambda) = 0$ and $\frac{d^k}{d\delta^k}F(\delta)\Big|_{\delta=0} \neq 0$, for $k = 1,3$ and $1 \leq k \leq 2s+1$.

The advantage of power logistic function (1) over the existing transition functions is that it accounts for both asymmetric and symmetric properties of time series.

When $i = 1$ in (1), the asymmetric power logistic (APL) function is obtained and is given by

$$F(y_{t-d};\delta,\lambda) = \{1 + 0.5\exp[-\delta(y_{t-d} - \lambda)]\}^{-2} - \frac{1}{1.5^2}, \delta > 0. \quad (9)$$

At $\delta = 1$, APL possesses an elongated S-shape, while at $\delta = 5$, APL exhibits normal S-shape but changes to Z- shape at $\delta = 15,\ 50$. Hence, at large δ, the shape of APL changes from S to Z (Figure 1). Both S and Z shapes signify the asymmetric property of APL.

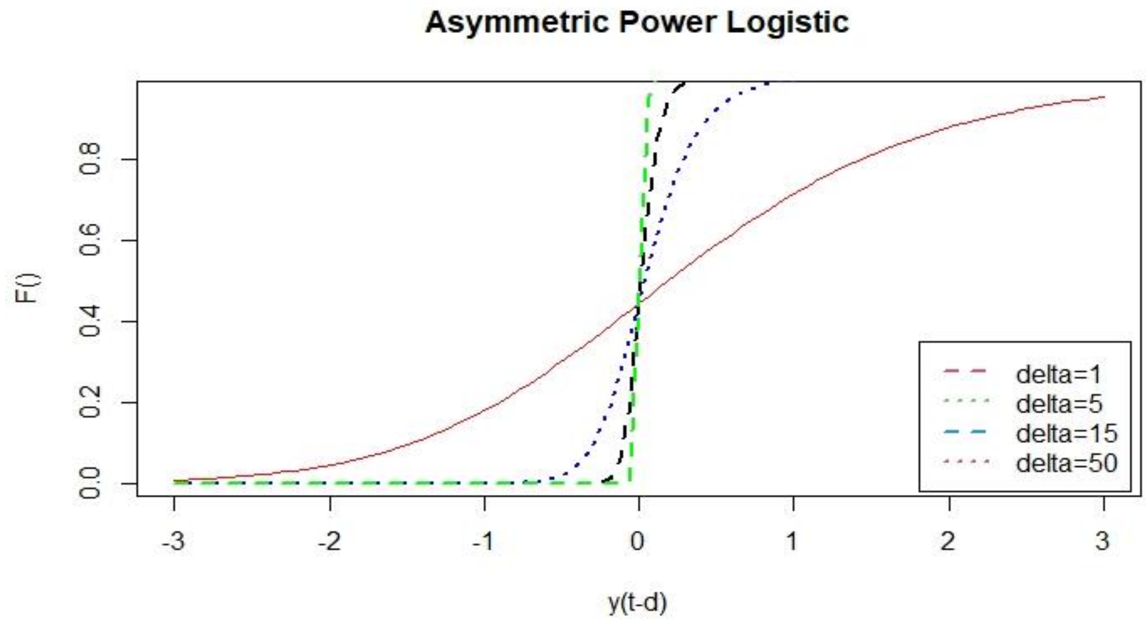

Fig 1: Asymmetric Power Logistic Function with Different Values of Delta (δ).

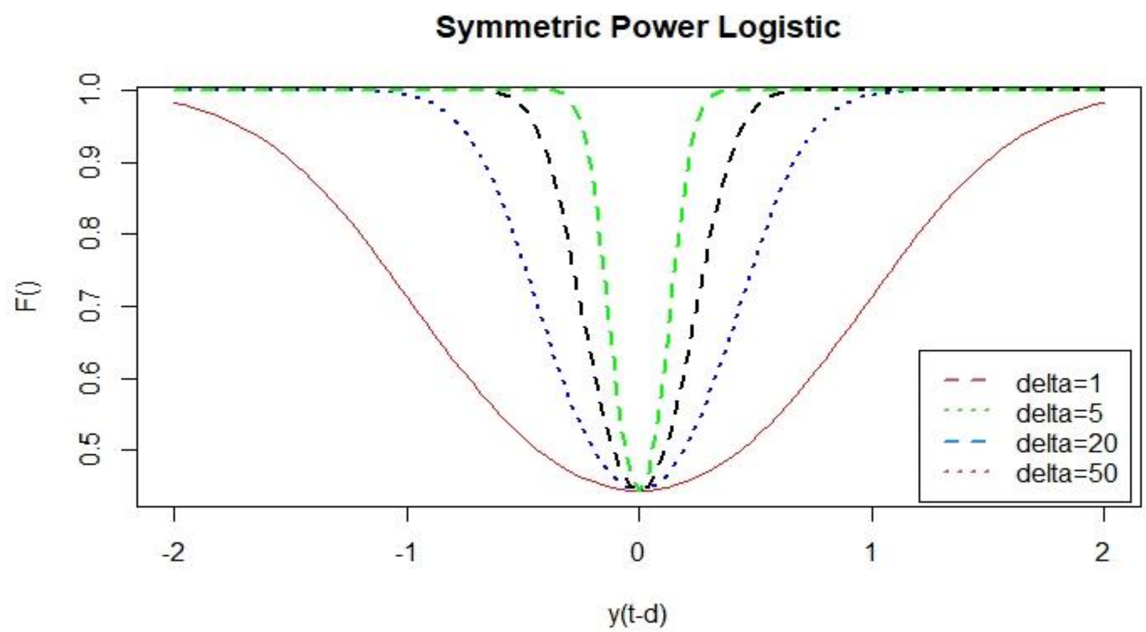

Fig 2: Symmetric Power Logistic Function with Different Values of Delta (δ)

When $i = 2$ in (1), the symmetric power logistic (SPL) function is obtained and is given by

$$F(y_{t-d}; \delta, \lambda) = \left\{1 + 0.5\exp[-\delta(y_{t-d}^2 - \lambda)]\right\}^{-2} - \frac{1}{1.5^2}, \delta > 0, \qquad (10)$$

Following Figure 2, the SPL transition is a V-shaped transition function with a broad base at $\delta = 1$ similar to the inverted bell shape of a normal distribution. The V- shape gets thinner as the δparameter increases.

2.3 STAR Model Specification, Estimation and Corresponding Hypothesis Testing

According to Dijk *et al.* (2002), a two-regime STAR model for a univariate time series y_t, which is observed at $t = 1-p, 1-(1-p), \ldots, -1,0,1, \ldots, T-1, T$ is given by

$$y_t = (\Phi_{1,0} + \Phi_{1,1}y_{t-1} + \cdots + \Phi_{1,\mathrm{p}}y_{t-p})(1 - \mathrm{F}(y_{t-d}; \delta, \lambda)) + (\Phi_{2,0} + \Phi_{2,1}y_{t-1} + \cdots + \Phi_{2,\mathrm{p}}y_{t-p})\mathrm{F}(y_{t-d}; \delta, \lambda)) + \nu_{t.} \quad (11)$$

(11) can be rewritten as

$$y_t = \Phi_1' w_t(1 - \mathrm{F}(y_{t-d}; \delta, \lambda)) + \Phi_2' w_t \mathrm{F}(y_{t-d}; \delta, \lambda) + \boldsymbol{\nu}_{t,} \quad (12)$$

where $\Phi_\mathrm{i} = (\Phi_{\mathrm{i},0}, \Phi_{\mathrm{i},1}, \cdots, \Phi_{\mathrm{i,p}})'$ for $i = 1,2$, $w_t = (1, y_{t-1}, \ldots, y_{t-p})'$. $\mathrm{F}(y_{t-d}; \delta, \lambda)$ is the transition function. The $\nu_t{}'s$ are assumed to be a martingale difference sequence concerning the history of the time series up to time $t-1$, which is donated as $\Omega_{t-1} = \{y_{t-1}, y_{t-2}, \ldots, y_{1-(p-1),}\, y_{1-p}\}$, $E[\nu_t/\Omega_{t-1}] = 0$, and $E[\nu_t^2/\Omega_{t-1}] = \sigma^2$.

Model (9) with (12) is called the asymmetric Power logistic STAR (APLSTAR) model, while model (10) with (12) is called the symmetric Power logistic STAR (SPLSTAR) model.

The first-order logistic function (LSTR1) proposed by Terasvirta (1994) given by

$$\mathrm{F}(y_{t-d}; \delta, \lambda) = (1 + \exp[-\delta(y_{t-d} - \lambda)])^{-1}, \ \delta > 0, \quad (13)$$

and the STAR model (13) with (12) is called the logistic STAR (LSTAR) model.

The second-order logistic (LSTR2) function given by

$$\mathrm{F}(y_{t-d}; \delta, \lambda) = (1 + \exp[-\delta(y_{t-d} - \lambda_1)(y_{t-d} - \lambda_2)])^{-1}, \lambda_1 \leq \lambda_{2,}\ \delta > 0, \quad (14)$$

where $\lambda = (\lambda_1, \lambda_2)'$, as proposed by Jansen and Terasvirta (1996).

Another common choice of $\mathrm{F}(y_{t-d}; \delta, \lambda)$ is the exponential function proposed by Terasvita (1994) given by

$$\mathrm{F}(y_{t-d}; \delta, \lambda) = 1 - \exp[-\delta(y_{t-d} - \lambda)^2], \ \delta > 0. \quad (15)$$

Model (15) with (12) is called the exponential STAR (ESTAR) model.

In this work, we employ a new version of (13) by choosing the symmetric power logistic transition function in (10). Before fitting a STAR model into time series, it is necessary to establish the appropriateness of the STAR model by carrying out a suitable nonlinearity test on the series. Upon the confirmation of the suitability of the model in modeling the time series, the optimal lag length of an AR model for each regime in the model is sought using the Akaike information criterion (AIC)

The parameters in the STAR model (12) can be estimated by nonlinear least squares (NLS).

Let $\mathrm{F}(\mathrm{w_t}, \xi) = \Phi_1' w_t\big(1 - \mathrm{F}(\mathrm{s_t}: \delta, \lambda)\big) + \Phi_1' w_t \mathrm{F}(\mathrm{s_t}: \delta, \lambda)$, where $\xi = \{\Phi_1', \Phi_2', \delta, \lambda\}$, then (12) becomes

$$y_t = \mathrm{F}(\mathrm{w_t}, \xi) + \nu_t \quad (16)$$

The parameters $\xi = \{\Phi_1', \Phi_2', \delta, \lambda\}$ can be estimated using NLS as follows:

$$\hat{\xi} = \underset{\xi}{\operatorname{argmin}}\, \mathrm{Q_T}(\xi) = \underset{\xi}{\operatorname{argmin}} \sum_{t=1}^{T} (y_t - \mathrm{F}(\mathrm{w_t}, \xi))^2 \quad (17)$$

The nonlinear least squares estimate is the value of $\hat{\xi}$ That minimizes (17).

If ν_t is assumed to be normally distributed, then NLS is equivalent to maximum likelihood. The NLS estimates are consistent and asymptotically normal (Tong, 1990).

In testing linearity against (12), the null hypotheses, such as $\mathrm{H_{01}}: \Phi_1 = \Phi_2$ or $\mathrm{H_{02}}: \delta = 0$ will reduce (12) to a linear AR model and whichever formulation of the null hypothesis is used, the model contains unidentified parameters. Where $\mathrm{H_{01}}$ is used, δ and λ in (12) are the unidentified nuisance parameters. Where $\mathrm{H_{02}}$ is used, λ, Φ_1 and Φ_2 represent the unidentified nuisance parameters. The solution to unidentified nuisance parameters under the null hypothesis is the third Taylor series approximation of $F(y_{t-d};\delta,\lambda)$ proposed by Luukkonen (1988) leads to the following auxiliary regression equation assuming that d is known:

$$y_t = \beta_0' w_t + \beta_1' w_t y_{t-d} + \beta_2' w_t y_{t-d}^2 + \beta_3' w_t y_{t-d}^3 + \varepsilon_t \quad (19)$$

The null hypothesis of linearity within the auxiliary regression (19) to be tested against alternative STAR models is $H_{03}: \beta_1 = \beta_2 = \beta_3 = 0$.

The F test statistic is given by

$$L_3M = \frac{(SSR_0 - SSR_1)/3(p+1).}{SSR_1/(T - 4(p+1))} \quad (20)$$

which is approximately F distributed with $3(p+1)$ and $T - 4(P+1)$ degrees of freedom.

The delay parameter, $d, 1 \le d \le p$, is the one with the smallest value of the residual sum of squares (RSS) of the estimated regression model (19) based on the nonlinear time series.

The choice between LSTAR (Asymmetric) and ESTAR (Symmetric) models after establishing the existence of nonlinearity in a series. Based on this, Terasvirta (1994) suggested that the choice of transition function be based on a sequence of tests within (19) as follows:

$H_{04}: \beta_3 = 0$

$H_{05}: \beta_2 = 0|\beta_3 = 0$

$H_{06}: \beta_1 = 0|\beta_2 = \beta_3 = 0$

If H_{04} is rejected implies that the model is LSTAR and that the ETAR family of the model is rejected. If H_{05} is rejected, it is evidence that the true model is the ESTAR model. If the true model is a LSTAR model, then H_{06} is rejected.

According to Terasvirta (1994), if H_{06} is rejected while H_{05} was accepted, it favours LSTAR model. Also, if H_{06} is accepted while H_{05} was rejected, pointing at an ESTAR model. Hence, if the model is a LSTAR model, H_{04} and H_{06} are strongly rejected than H_{05}, otherwise, ESTAR is the true model

Escribano and Jorda (2001) suggest that a second-order Taylor approximation is necessary to capture the two inflexion points of the exponential function, yielding the auxiliary regression.

$$y_t = \beta_0' w_t + \beta_1' w_t s_t + \beta_2' w_t y_{t-d}^2 + \beta_3' w_t y_{t-d}^3 + \beta_4' w_t y_{t-d}^4 + \varepsilon_t \quad (21)$$

The null hypothesis to be tested is $H_{07}: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$. The resultant LM test statistic denoted by L_4M, has an asymptotic χ^2 distribution with $4(p+1)$ degrees of freedom under the null hypothesis.

We adopted Escribano and Jorda procedure (EJP) based on the following two hypotheses within the auxiliary regression (21):

$H_{0L}: \beta_2 = \beta_4 = 0$ with an F-test (F_L)

$H_{0L}: \beta_1 = \beta_3 = 0$ with an F-test (F_E)

If the minimum p-value corresponds to F_E, select the LSTAR model; otherwise, select the ESTAR model.

2.4 Evaluation of Forecasting Performance

The root mean square error (RMSE) is used to evaluate the forecast performance of the overall in-sample observations. The ratio of RMSE of the STAR model to that of the corresponding benchmark AR model chosen by AIC will be obtained to determine the efficiency of the new model.

If $y_t, t = 1, 2, \dots, h$ are the actual values of the observations used in the estimation of the model, $f_t, t = 1, 2, \dots, h$ are the forecast values, then, $e_t = y_t - f_t$, $t = 1, 2, \dots, h$ are the forecast error. The RMSE of the in-sample forecast is the square root of the mean square error (MSE) given by

$$\text{RMSE} = \sqrt{\frac{1}{P}\sum_{i=1}^{h} e_t^2} \quad (34)$$

3.0 Results and Discussion

3.1. Time Series Plots

The time series plot of the exchange rate of naira to USD shown in Figure 3 indicates that the naira gains value in the second quarter of 2006 to the third quarter of 2008 when compared to USD. It started to depreciate in the first quarter of 2009 and get worsened in the first quarter of 2016 and the first quarter of 2020 due to the economic recession and the covid-19 pandemic. The time series plot of the exchange rate of naira to CFA Franc shown in Figure 4 indicates that CFA Franc gradually gains value from 2004 to 2005, where it depreciates and thereafter gains value from the third quarter of 2006 to 2016 and appreciates more from the second quarter of 2016 to 2020 and beyond when compare to naira.

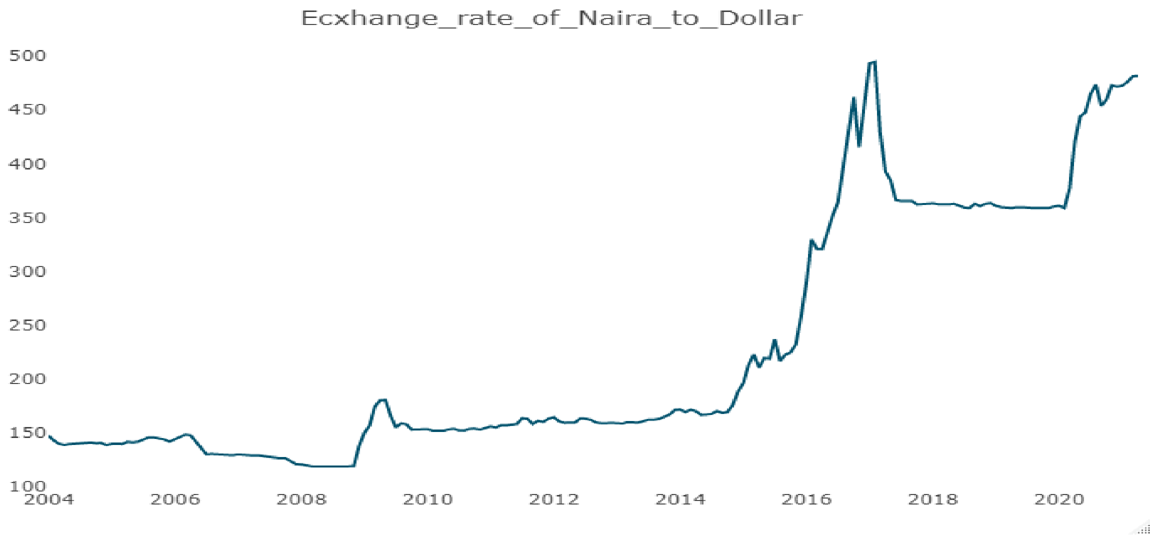

Fig 3: Time series plot of exchange rate of naira to USD

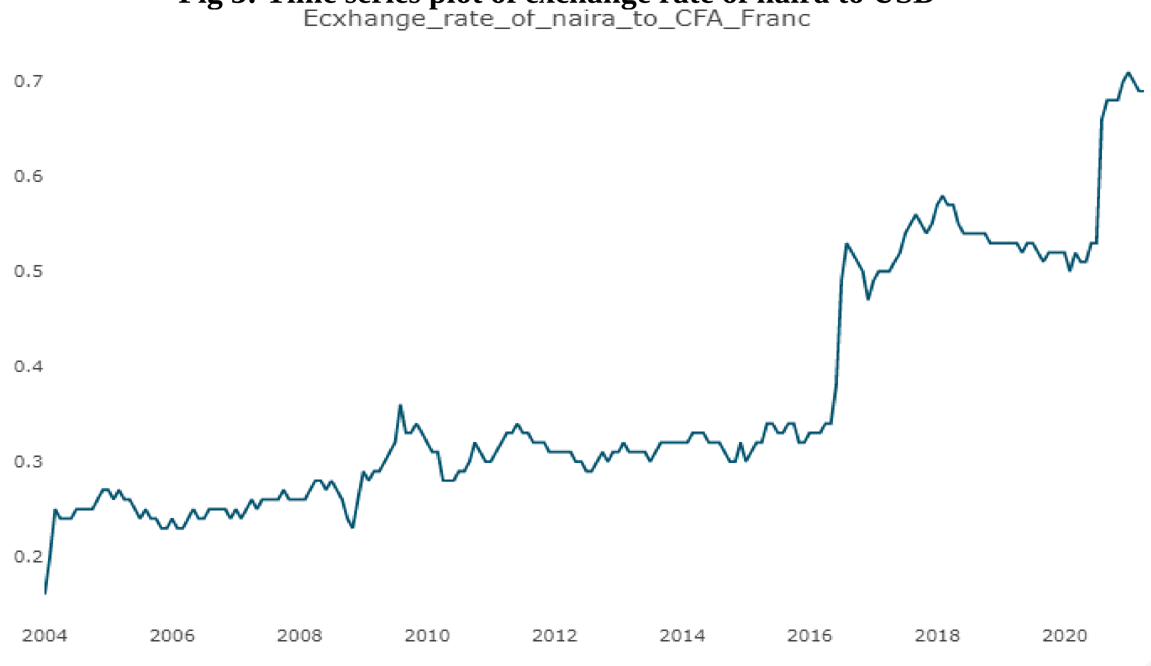

Fig 4: Time series plot of exchange rate of naira to CFA Franc.

The autocorrelation function (ACF) of the two series shown in Figures 5 and 6 decay very slowly, indicating the presence of a trend in the series. A formal test for the stationarity of the series is explained below.

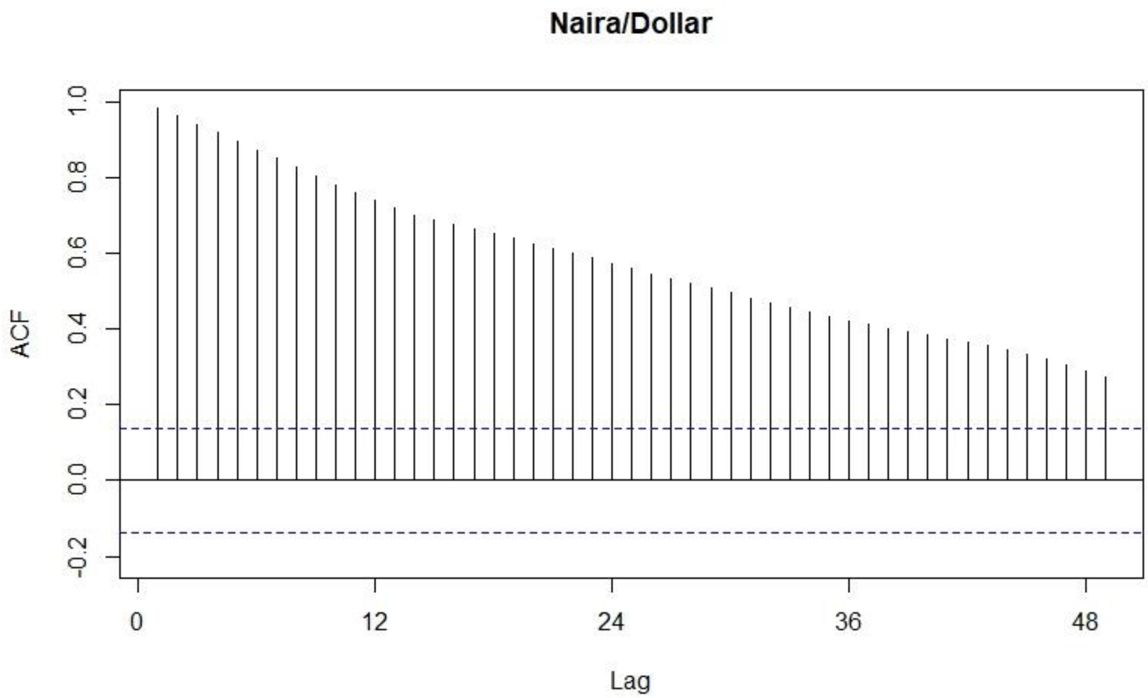

Fig 5: ACF of exchange rate of naira to USD

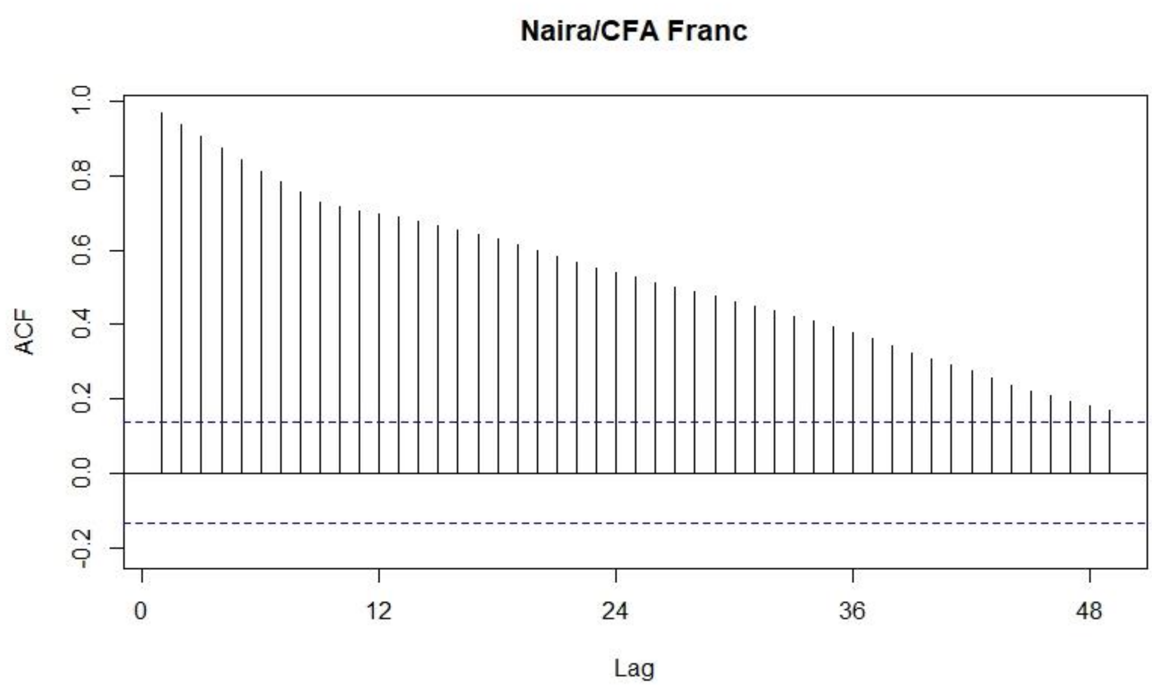

Fig 6: ACF of the exchange rate of naira to CFA Franc

3.2 Stationarity Test

From Table 1, the augmented Dickey-Fuller test confirmed the presence of unit roots in the two series since the p-value$= 0.597 > 0.05$ and p-value $= 0.6693 > 0.05$ for the exchange rate of naira to USD and CFA Franc, respectively. However,the two series are stationary at the first difference since the p-value $= 0.01 < 0.05$ for each of the two series.

Table 1: Results of Augmented Dickey-Fuller Tests

Variable	Original Series (Level)			First Differenced Series	
	Dickey Fuller Statistic	ρ- Value	Lag Order	Dickey Fuller Statistic	ρ- Value
Naira to Dollar	-1.95	0.597	5	-5.5688	<0.01
Naira to CFA Franc	-1.777	0.6693	5	-6.3449	<0.01

3.3 Optimal Lag Length for the Linear and STAR Models

The optimal lag length for the linear stationary autoregressive (AR) model representing the conditional mean model under the null hypothesis for the two series is determined to carry out the linearity test. Terasvirta (1994) suggested that the order could be determined by the Akaike information criterion (AIC). Hence, ARIMA (4,1,0) and ARIMA(1,1,0) models for the exchange rate of naira to USD and CFA Franc, respectively, are chosen based on the value of AIC shown in Table 2 to represent the conditional mean model under the null hypothesis.

Table 2: Determination of Order of Linear Models Fitted to Exchange Rates

Model	**AIC**	
	Naira to USD	Naira to CFA Franc
ARIMA(1,1,0)	1565.48	***-1109.93***
ARIMA(2,1,0)	1565.89	-1108.02
ARIMA(3,1,0)	1566.2	-1106.03
ARIMA(4,1,0)	***1558.29***	-1104.11
ARIMA(5,1,0)	1565.2	-1102.42

Based on Table 3, the estimated residuals from ARIMA(4,1,0) and ARIMA(1,1,0) models are free from serial correlation since the p-value of Box-Pierce statistic $= 0.7211 > 0.05$ and $0.82855 > 0.05$ for Naira to USD and CFA Franc exchange rate, respectively. Hence, the null hypothesis of no serial correlation could not be rejected at a 5% level of significance.

The test of the null hypothesis of linearity against alternative STAR models using the Lagrange multiplier (LM) test proposed by Terasvirta (1994) is carried out. In Table 4, the p-value$= 0.0000 < 0.05$ for the exchange rate of naira to USD leads to the rejection of the null hypothesis of linearity, but we could not reject the null hypothesis of linearity for the exchange rate of naira to CFA Franc since its p-value = $0.4622 > 0.05$. Hence, the exchange rate of naira to USD is a nonlinear time series and is modelled with STAR model, while the exchange rate of naira to CFA Franc is classified as a linear process. The delay parameter, $d, 1 \leq d \leq 4$ is the smallest value of the residual sum of squares (RSS) of the estimated regression model (29). In Table 4, d=2 is the smallest value of RSS; we slelct $d = 2$ for the exchange rate of naira to USD.

Based on Table 5, the specification of the transition function is based on the Terasvita procedure (TP) and Escribano and Jorda procedure (EJP). For TP, $\rho(H_{04}) = 0.1997 > 0.05$ (H_{04} is not rejected) implies that $\beta_3 = 0$ in (29), suggesting the ESTAR model with an exponential function. Also, since $\rho(H_{05}) = 0.000 < 0.05$, H_{05} is rejected and $\beta_2 \neq 0$ in (29) and H_{06} is accepted while H_{05} is rejected, confirming the ESTAR model with exponential function for the modeling exchange rate of naira to USD. For EJP, $\rho(H_{0L}) = 0.0000 < \rho(H_{0E}) = 0.0203$ suggests the ESTAR model. Both TP and EJP suggest an ESTAR model with an exponential function (symmetric) for modeling the exchange rate of naira to USD. Consequently, we fit symmetric STAR models to an exchange rate of naira to USD and determine the best model at the evaluation stage using forecast evaluation measures and residual standard error for each of the estimated STAR models.

Table 3: Parameter Estimates of Linear Models Fitted to Exchange Rates

Exchange Rate	Model	Parameter	Estimate	Box-Pierce Statistic
Naira to USD	ARIMA (4,1,0)	ϕ_1	0.34140 (0.00411)	0.12741 (0.7211)
		ϕ_2	-0.092975 (0.19309)	
		ϕ_3	0.011949 (0.86740)	
		ϕ_4	0.214275 (0.00143)	
Naira to CFA Franc	ARIMA (1,1,0)	ϕ_1	0.210593 (0.002182)	0.046946 (0.8285)

****The values in the parentheses are p-values of estimated parameters.**

Table 4: Outcomes of Linearity Test

Variable	F- Statistic	ρ-Value	RSS of the Delay Length (d)			
			1	2	3	4
Naira to USD	7.583956	0.0000	17815.47	**12930.62**	14911.41	18043.03
Naira to CFA Franc	0.904751	0.4622	-	-	-	-

**** RSS is the residual sum of squares**

Table 5: Selection of Transition Function for Exchange Rate of Naira to USD

TP			EJP		
Null Hypothesis	F- Statistic	Transition Function	Null Hypothesis	F- Statistic	Transition Function
H_{04}	1.56368 (0.1997)		H_{0L}	7.86431 (0.0000)	Exponential function
H_{05}	11.16604 (0.0000)	Exponential function	H_{0E}	3.34480 (0.0203)	
H_{06}	6.76766 (0.0000)				

****In Table 5, the values in the parentheses are p values of estimated parameters. $p(H_{04})$, $p(H_{05})$ and $p(H_{06})$ are the p values of the test corresponding to H_{04}, H_{05} and H_{06}, respectively. $p(H_{0L})$and $p(H_{0E})$ are the p values of the test that corresponds to H_{0L} and H_{0E}, respectively**.

3.4 Estimation of STAR Models Fitted to Exchange Rate of Naira to Dollar

Five symmetric STAR (ESTAR, quadratic STAR (QLSTAR), absolute error logistic STAR (AELSTAR) and absolute logistic STAR (ALSTAR) models were fitted to the exchange rate of naira to USD. The parameter estimates of ESTAR, QLSTAR, AELSTAR and ALSTAR using the nonlinear least squares method are shown in Table 6.

The residuals from the estimated STAR models were tested for serial correlation using the portmanteau lack of fit test. The results revealed that the residuals are uncorrelated since the p-value of each of the estimated models > 0.05 as shown in Table 7. Consequently, the null hypothesis of no serial correlation for each of the estimated models (SPLSTAR, ESTAR, ALSTAR, QLLSTAR and AELSTAR) is not rejected. For model evaluation, RMSE and standard error of residuals were computed for each of the estimated STAR models. The results also show that the SPLSTAR model has both the lowest standard error of residuals and root mean square error (RMSE) relative to the other model under comparison. All the symmetric STAR models considered except the ESTAR model outperformed its linear counterpart.

Table 6: Parameter Estimates of Symmetric STAR Models Fitted to Exchange Rate of Naira to USD.

Parameters	Estimated Models				
	SPLSTAR	ESTAR	ALSTAR	QLSTAR	AELSTAR
Φ_{10}	-0.20245	-1.89188	-2.59640	-0.15256	-0.16697
	(0.8948)	(0.1751)	(0.2751)	(0.1949)	(0.8193)
Φ_{11}	0.45740	-0.20245	0.33482	0.1046	0.31480
	(0.0000)	(0.894)	(0.0000)	(0.1592)	(0.0000)
Φ_{12}	-0.60134	0.47365	-2.20197	-0.59375	-0.12307
	(0.0158)	(0.808)	(0.688)	(0.01058)	(0.36054)
Φ_{13}	-0.35026	0.35678	-1.76451	-0.33566	0.30168
	(0.09347)	(0.836)	(0.714)	(0.06208)	(0.01035)
Φ_{14}	0.49532	-0.50493	2.00034	0.48098	-0.18191

	(0.00902)	(0.736)	(0.679)	(0.00274)	(0.15807)
Φ_{20}	-0.55280	45.90488	74.98879	-0.60737	-410.59058
	(0.0000)	(0.8181)	(0.8181)	(0.1940)	(0.0058)
Φ_{21}	0.33904	0.34608	0.33302	0.33960	0.07386
	(4.37e-07)	(1.79e-06)	(1.02e-06)	(4.38e-07)	(0.34337)
Φ_{22}	0.56129	-0.57036	2.17373	0.55311	0.32375
	(0.03100)	(0.771)	(0.692)	(0.02315)	(0.06796)
Φ_{23}	0.55089	-0.34857	1.97291	0.53635	-1.00402
	((0.01273))	(0.841)	(0.681)	(0.00581)	(1.78e-05)
Φ_{24}	-0.48963	0.73524	-1.99253	-0.47498	0.51834
	(0.01615)	(0.626)	(0.680)	(0.00741)	(0.00873)
λ	6.33657	-3.01734	-1.44328	10.59869	12.97391
	(0.28353)	(0.022)	(0.843)	(0.21070)	(< 2e-16)
δ	110.7481	74.00304	1124.0001	120.74821	1547.78986
	(0.3999)	(0.9133)	(0.9953)	(0.4999)	(0.9990)

****The values in the parentheses are p-values of estimated parameters.**

Table 7: Estimates of Box-Pierce and Lagrange Multiplier Statistics for Exchange Rate of Naira to USD.

Model	Box-Pierce Test		$\sigma_{\hat{v}}$	RMSE
	$Q_{\hat{v}_t}(K)$	p-value		
SPLSTAR	2.1015	0.1472	9.625	9.432221
ESTAR	0.11191	0.738	10.48	10.27395
ALSTAR	1.9036	0.1677	9.779	9.583539
QLSTAR	2.1296	0.1445	9.633	9.440546
AELSTAR	0.062136	0.8032	9.841	9.643963
AR(4)	0.12741	0.7211	10.140	10.1520

**** $Q_{\hat{v}_t}(K)$) is the Box-Pierce Statistic, $\sigma_{\hat{v}}$ is the standard error of residuals and RMSE is the root mean square error for the conditional mean model.**

3.5 Discussion of Findings

The Power logistic transition function is being proposed. The major advantage of this function over all the transition functions in common use is that it is relatively more flexible**.** As a consequence, it is useful in constructing both symmetric and asymmetric STAR models. In this work, considerable attention has been paid to the symmetric power logistic transition function and its associated STAR model in this study**.**

The findings of this study are in line with the works of Liew *et al.* (2003), Anderson (1997), Shangodoyin *et al.* (2009), Siliverstovs (2005) and Yaya and Shittu (2016) in which STAR models are modified to capture different properties of time series in the areas that the STAR models are found to be deficient. On the other hand, the uniqueness of this study is the modification of the LSTAR model to the PLSTAR model, which possesses both symmetric and asymmetric transition functions.

Symmetric STAR models are justifiably fitted to a monthly exchange rate of naira to United States dollar over the period January 2004 to April 2021. The fit of the SPLSTAR model is compared with the fits of the competing (comparable) models. So far, only the symmetric aspect of the modified model has been applied to capture the nonlinear properties of the exchange rates. It is therefore recommended that in subsequent studies, the asymmetric power logistic STAR model should be considered and its performance should be compared with that of asymmetric STAR models.

4.0 Conclusion

In this study, the well-known logistic function is modified, leading to a power logistic family of transition functions. Two special cases of this family, namely, the symmetric power logistic function and asymmetric power logistic function, are defined. The power logistic function is approximated by using the Taylor series expansion up to the third order. Certain properties of the functions are determined following the approximations. The two functions are used to formulate the symmetric and asymmetric STAR models. Only the symmetric power logistic transition function and its associated STAR model are considered in this study**.** It is observed that the symmetric power logistic STAR model is the most suitable for the exchange rate of naira to the United States dollar.

5.0 Refereces

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