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# GARCH Modelling of Nigeria Stock Exchange Returns with Odd Generalized Exponential Laplace Distribution

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**Abstract:** *The modelling of volatility of asset returns plays an important role in risk assessment and decision-making processes for both investors and financial institutions. In this paper, we have modelled the volatility in Nigeria Stock Exchange (NSE) returns using the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and some of its variants with an Odd Generalized Exponential Laplace Distribution (OGELAD) due to its ability to capture the time-varying and nonlinear nature of financial time series. Fitting the different models indicates the new error distribution outperforms other error distributions for all volatility models. The majority of the parameters for all fitted models and error distributions are significant at 5%, 1%, and 0.1% level of significance. The diagnostic check of the fitted models shows they have been adequately specified. Furthermore, the forecasting performance of the fitted models shows that the new error distribution outperformed existing error distributions in out-sample forecasts. While GARCH (1,1) with an OGELAD is selected for fitting the volatility, the GJR-GARCH (1,1) model with an OGELAD is preferred for forecasting the volatility of NSE returns. Thus, GARCH models with a non-normal error distribution provide a robust distribution for modelling volatility.*

**Keywords:** *Volatility, heteroscedasticity, returns, stylized facts, innovations*

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## 1. 0 Introduction

Following the empirical research into the volatility of asset returns pioneered by Cowles (1933), Working (1934), and Cowles and Jones (1937), the changes in the price of asset returns like stocks are described by a random walk process. Further research into the uncertainty of price changes led to the development of the Efficient Market Hypothesis by Fama (1965, 1970); this supported the unpredictability of asset returns. However, empirical evidence during these periods found the inappropriateness of the random walk model for modelling price changes, particularly the assumption of independence, identically distributed and normality of errors (Kendall and Hill, 1953). The weaknesses of these earliest models led to the development of the seminal paper by Engle in 1982. The popularly accepted Engle's Autoregressive Conditional Heteroscedastic (Engle, 1982) volatility model which captures the basic characteristics of asset returns has been used in modelling the volatility of asset returns. Even though Engle's (1982) model has gained acceptability it has been challenged because of its weaknesses which vary from assuming equal effect for both positive and negative shocks on volatility to over-predicting the volatility because they respond slowly to large isolated shocks of the asset returns series. These challenges have led to the development of other univariate volatility models like the: Random Coefficient Autoregressive (RCA) model of Nicholls and Quinn (1982); Generalized Autoregressive Conditional Heteroscedastic (GARCH) model of Bollerslev (1986); Conditional Heteroscedastic Autoregressive Moving Average (CHARMA) model of Tsay

(1987); Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) model of Nelson (1991); Stochastic Volatility (SV) models of Melino and Turnbull (1990), and Harvey *et al*. (1994) among others. As popularly reported in empirical studies, the marginal distribution of asset returns is known to have heavier tails and a high peak than the normal distribution. Even when suitable models are fitted to asset returns series, the resulting residuals still exhibit excess kurtosis ( > 3) indicating that the standard assumption that asset returns follow normal distribution is invalid. In addition to this empirical evidence, other regularities ("stylized facts") that describe asset returns series have been identified. The most common of these stylized facts include volatility clustering and its persistence, leverage effect, thick tails, bell-shaped symmetry, and co-movements in volatilities. According to Mills and Markellos (2008), these stylized facts are indications of the extension to time series models which incorporate error distributions to model outlier activity and time-varying unconditional variances would be very useful.

As a result of the inability of the normal GARCH model to explain the leptokurtic nature of asset returns, two popular distributions – Student's t-distribution and Generalized Error Distribution (GED) are commonly used in economic modelling because they can capture leptokurtic and heavy-tailed behaviours (Fan *et al*., 2008; Chkili *et al*., 2012; Mabrouk and Saadi, 2012). Though these distributions and their variants such as skewed student's t-distribution and skewed GED have found wider acceptability in the literature, they also have their limitations. For instance, Yang and Wade (1993) revealed that the tail behaviour of GARCH model remains too short even with Student's t-distributed error terms. Uyaebo *et al.,* (2015) examined the asymmetric GARCH models with endogenous break dummy on two innovation assumptions (Student's and GED) using daily all share index of Nigeria, Kenya, United States, Germany, South Africa and China. The results indicated that volatility of Nigeria and Kenya stock returns react to market shock faster when compared to other countries and the absence of leverage effect was also confirmed in Nigeria and Kenya stock returns. Al-Najjar (2016) used symmetric and asymmetric GARCH models in modelling and estimating volatility in Jordan's stock market. The study found that the symmetric (ARCH/GARCH) model with normal distribution provided evidence for volatility clustering and leptokurtosis while the asymmetric (EGARCH) model with Student's-*t* distribution does not provide evidence of leverage effect in the stock returns at Amman Stock Exchange. Asemota and Ekejiuba (2017) examined the volatility of banks' equity returns for six banks using GARCH models. The EGARCH (1,1) and CGARCH (1,1) models with Student's-*t* distribution were found to outperform other GARCH models for two of the six banks which reveal the presence of ARCH effects. In their study, they recommend modelling stock market volatility using variants of GARCH models and alternative error distribution to achieve robustness of results. Feng and Shi (2017) using a tempered stable distribution argued that the specified distribution could be a useful tool in the modelling of financial volatility using GARCH-type specification models. The development of GARCH model with fat-tailed distributions in practical finance research to accurately forecast financial volatility is gaining wider acceptability as it provides the opportunity for investors to gain knowledge of the prediction of expected returns (Feng and Shi, 2017; and Altun *et al*., 2017). Dikko and Agboola (2018) compared two new error distributions (exponentiated generalized student's-*t* distribution – EGSSTD and exponentiated student's-*t* distribution – ESSTD) with other error distributions using symmetric (GARCH) and asymmetric (GJR-GARCH, EGARCH, TGARCH, APARCH) models. The results obtained show that GARCH model with EGSSTD error distribution outperformed other models. In terms of forecasting performance, the GARCH model with ESSTD error distribution outperformed other volatility models and error distributions. This paper therefore, modelled the volatility in the Nigeria Stock Exchange (NSE) index with a new conditional error distribution to capture the stylized facts and basic characteristics of the returns' series.

## 2.0 Materials and Methods

### *2.1 Odd Generalized Exponential Laplace Distribution*

Recently, Obalowu and David (2021) proposed a four-parameter distribution called Odd Generalized Exponential Laplace Distribution (OGELaD). The cdf and pdf of the OGELaD are respectively:

$$F(x;\Phi)=\left[1-exp\left\{-\alpha\left(\frac{\frac{1}{2}+\frac{1}{2}\frac{|x-\mu|}{(x-\mu)}\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}\frac{|x-\mu|}{(x-\mu)}\left[1-exp\left(-\frac{|x-\mu|}{x-\mu}\right)\right]\right)}\right)\right\}\right]^{\beta} \tag{7}$$

and

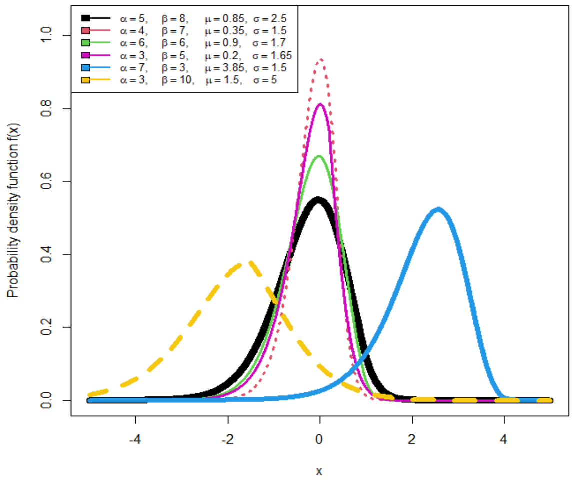
$$f(x;\Phi)=\frac{\alpha\beta exp\left(-\frac{|x-\mu|}{\sigma}\right)exp\left(-\alpha\left[\frac{\frac{1}{2}+\frac{1}{2}\frac{|x-\mu|}{(x-\mu)}\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}\frac{|x-\mu|}{(x-\mu)}\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]\right)}\right]\right)}{2\sigma\left(1-\left[\frac{1}{2}+\frac{1}{2}\frac{|x-\mu|}{(x-\mu)}\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]\right]\right)^{2}} \tag{8}$$

$$\times\left[1-exp\left(-\alpha\left[\frac{\frac{1}{2}+\frac{1}{2}\frac{|x-\mu|}{(x-\mu)}\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}\frac{|x-\mu|}{(x-\mu)}\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]\right)}\right]\right)\right]^{\beta-1}$$

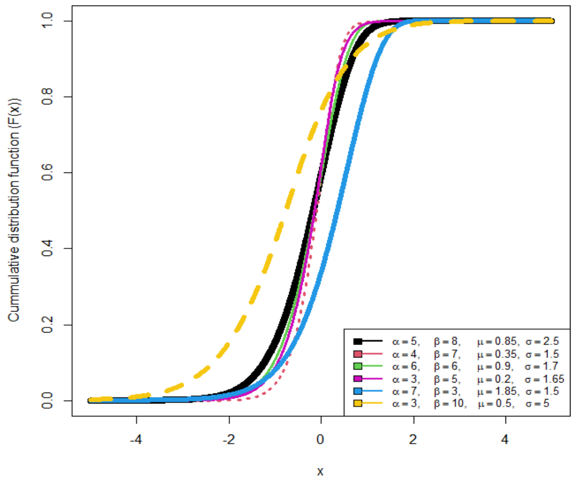
where $\Phi = (\alpha, \beta, \mu, \sigma)$, and $\alpha, \beta, \sigma > 0$, $-\infty < \mu, x > \infty$.

The pdf of the OGELaD has four parameters; $\alpha$ and $\beta$ are shape parameters, $\sigma$ is a scale parameter and $\mu$ is a location parameter.

The pdf and cdf plots of the OGELAD are given below




(a) Pdf Plot




(b) Cdf Plot

**Fig. 1: Density and cdf plots of the OGELaD**

The density plot (a) in Fig. 1 is indicative of the fact that the OGELaD can be right-skewed, symmetric, or left-skewed depending on the values of the parameters considered as such its suitability for data sets with different shapes. Furthermore, the cdf plot (b) shows with an increase in *x*, the distribution converges to 1.

### 2.2 GARCH Modelling

The (G)ARCH model specified by Engle (1982) and Bollerslev (1986) consist of the mean equation and the variance (volatility) equation.

(i) Mean Equation

The mean equation requires specifying the distribution of the innovation. One popular specification of the mean equation of returns is the ARMA (1,1) given by

$$r_t = \theta + \tau r_{t\text{-}1} + \eta\varepsilon_{t\text{-}1} + \varepsilon_t \qquad (9)$$

where, $\theta$ is a constant, $\tau$ and $\eta$ are the coefficients of the AR and MA terms, and $\varepsilon_t$ is the residual series.

(ii) Variance Equation

The volatility equation used in this study includes:

(a) GARCH (1,1) Model

$$\sigma_t^2 = \kappa + a_1\varepsilon_{t-1}^2 + b_1\sigma_{t-1}^2 \qquad (10)$$

where, $\kappa > 0$, $a_1 > 0$, and $b_1 > 0$

(b) EGARCH (1,1) Model

$$\sigma_t^2 = \kappa + a_1\left\{\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}}\right\} + \gamma_1\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + b_1\ln\left(\sigma_{t-1}^2\right) \qquad (11)$$

where, $\gamma_1$ measures the symmetry or the leverage effect. Here, $\gamma_1 \neq 0$

(c) GJR-GARCH (1,1) Model

$$\sigma_t^2 = \kappa + \gamma_1\varepsilon_{t-1}^2 S_{t-1}^- + a_1\varepsilon_{t-1}^2 + b_1\sigma_{t-1}^2 \qquad (12)$$

where, $\kappa > 0$, $a_1 > 0$, $b_1 > 0$, $\gamma_1 \neq 0$, and $S_{t-1}^-$ is a dummy variable equal to one (1) when $\varepsilon_{t\text{-}1} < 0$ and zero (0) otherwise.

(d) TGARCH (1,1) Model

$$\sigma_t = \kappa + a_1\left(\left|\varepsilon_{t-1}\right| - \gamma_1\varepsilon_{t-1}\right) + b_1\sigma_{t-1} \qquad (13)$$

where, $\kappa > 0$, $a_1 > 0$, $b_1 > 0$, $\left|\gamma_1\right| \leq 1$

### *2.3 Conditional distribution of Volatility Models*

The conditional distribution of the innovation (error term) of the volatility models is often specified in the following form:

$$\varepsilon_t = \sigma_t v_t \qquad (14)$$

where $v_t \overset{iid}{\sim} N(0,1)$ i.e. $v_t$ is an independent and identically distributed process with zero mean and unit variance. Several distributions have been proposed as the distribution of the innovation. Engle (1982) and Bollerslev (1986) assumed the distribution to follow a normal distribution with mean, $\mu$ and variance, $\sigma^2$. The conditional distributions of innovation used in this paper include:

(i) Normal Distribution

$$f\left(x;\mu,\sigma\right) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{\left(x-\mu\right)^2}{2\sigma^2}\right) \qquad (15)$$

(ii) Skewed Normal Distribution

$$f\left(x;\alpha,\sigma,\mu\right) = \frac{2}{\sigma\sqrt{\pi}} exp\left(-\frac{\left(x-\mu\right)^2}{2\sigma^2}\right)\int_{-\infty}^{\alpha\left(\frac{[x-\mu]}{\sigma}\right)} \frac{1}{2\pi} exp\left(-\frac{t^2}{2}\right)dt \qquad (16)$$

(iii) Skewed Student's t-Distribution

$$f\left(x;\lambda,v\right)=b\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi\left(v-2\right)\Gamma\left(\frac{v}{2}\right)}}\left(1+\frac{\zeta^{2}}{v-2}\right) \tag{17}$$

where, $\zeta=\begin{cases}\left(\frac{bx+a}{1-\lambda}\right); & \text{if } x<-\frac{a}{b}\\ \left(\frac{bx+a}{1-\lambda}\right); & \text{if } x\geq-\frac{a}{b}\end{cases}$

Also, the constants $a=4\lambda c\frac{v-2}{v-1}$, $b=1+3\lambda^{2}-a^{2}$, and $c=\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi\left(v-2\right)}\Gamma\left(\frac{v}{2}\right)}$

(iv) Skewed GED

$$f\left(x;\mu,\sigma,\eta,\lambda\right)=\frac{\eta}{2\sigma\sqrt{\Gamma\left(\frac{1}{\eta}\right)}}\left(\frac{\left|x-\mu+\delta\sigma\right|^{\eta}}{\left[1-sgn\left(x-\mu+\delta\sigma\right)\lambda\right]^{\eta}\theta^{\eta}\sigma^{\eta}}\right) \tag{18}$$

where, $\theta=\Gamma\left(\frac{1}{\eta}\right)^{0.5}\Gamma\left(\frac{3}{\eta}\right)^{-0.5}S\left(\lambda\right)^{-1}$,

$\delta=2\lambda AS\left(\lambda\right)^{-1}$,

$S\left(\lambda\right)=\sqrt{1+3\lambda^{2}-4A^{2}\lambda^{2}}$

$A=\Gamma\left(\frac{2}{\eta}\right)^{0.5}\Gamma\left(\frac{1}{\eta}\right)^{-0.5}\Gamma\left(\frac{3}{\eta}\right)^{-0.5}$,

with the constraints, $\theta,\eta>0,\ -1<\lambda<1,\ -\infty<x<\infty$

(v) Normal Inverse Gaussian Distribution

$$f\left(x;\mu,\alpha,\beta,\delta,\gamma\right)=\frac{\alpha\delta}{\pi\sqrt{\delta^{2}+\left(x-\mu\right)^{2}}}exp\left(\delta\gamma+\beta\left(x-\mu\right)K_{1}\left(\alpha\sqrt{\delta^{2}+\left(x-\mu\right)^{2}}\right)\right) \tag{19}$$

where, $\gamma=\sqrt{\alpha^{2}-\beta^{2}}$, $\mu$ is a location parameter, $\alpha$ and $\beta$ are shape parameters that control the heaviness of density, $\delta$ is a scale parameter, $K_1$ is the modified Bessel function of the second kind of order 1.

$$f\left(x;\alpha,\beta,\mu,\sigma\right)=\frac{\alpha\beta exp\left(-\frac{|x-\mu|}{\sigma}\right)exp\left(-\alpha\left[\frac{\frac{1}{2}+\frac{1}{2}sgn(x-\mu)\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}sgn(x-\mu)\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]\right)}\right]\right)}{2\sigma\left(1-\left[\frac{1}{2}+\frac{1}{2}sgn(x-\mu)\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]\right]\right)^{2}} \tag{20}$$

$$\times\left[1-exp\left(-\alpha\left[\frac{\frac{1}{2}+\frac{1}{2}sgn(x-\mu)\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}sgn(x-\mu)\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]\right)}\right]\right)\right]^{\beta-1}$$

The above conditional distributions are compared with the new Odd Generalized Exponential Laplace Error Distribution given above

## 3.0 Results and Discussion

### *3.1 Data*

The data set is 2,456 daily observations of the Nigerian Stock Exchange (NSE) All Share Index measured in points from January 31, 2012 to December 31, 2021. The NSE All Share Index is the stock index which tracks the performance of all stocks listed on the Nigerian Stock Exchange. In 2020, it was named the best-performing stock market among the 93 equity indexes tracked by Bloomberg across the world. This data has been obtained from https://ng.investing.com/indices

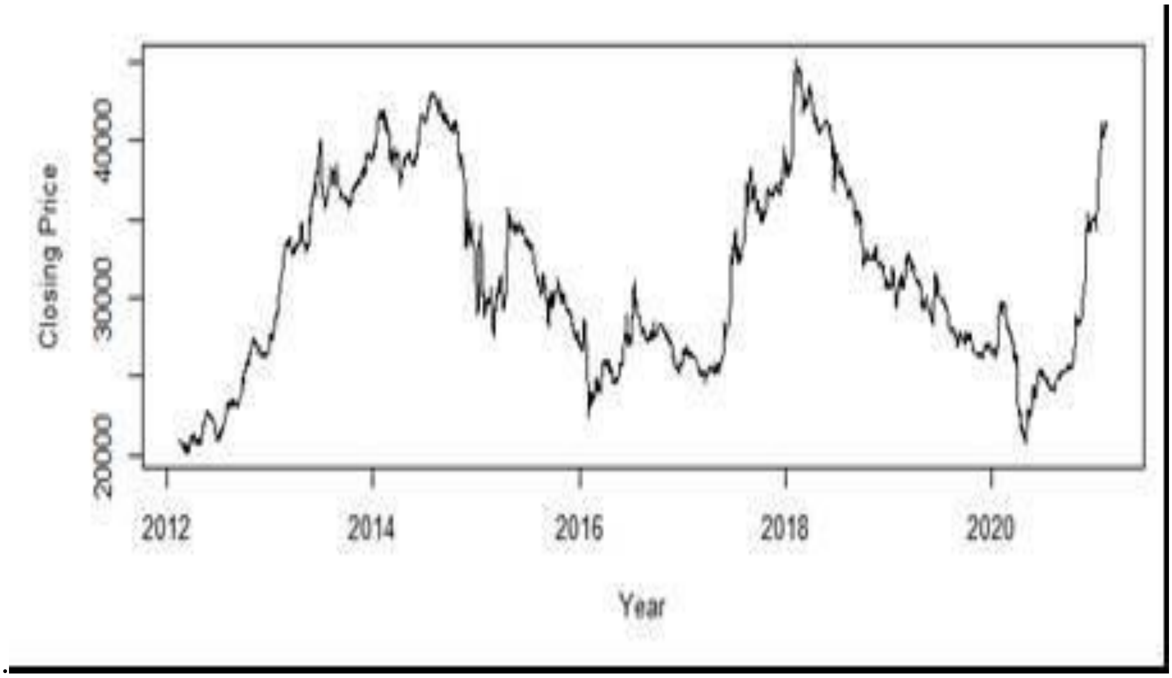



**Fig. 2: Time Plot of NSE Index**

The plot of the NSE index is given in Fig. 2. The index trended upwards reaching a high of about 31,000 points sometime in early 2013. Thereafter, a decline was observed before a period of recovery till sometime in mid-2014. After this time, the index experienced a decline again as a result of falling oil prices and political uncertainty leading to the 2015 general election in Nigeria. Between 2016 and 2018, the index continued to experience growth. However, from mid-2019 to early 2020, the index plummeted as concerns over the global economy and COVID-19 pandemic continued to grow. Since mid-2020, the index has continued to grow showing recovery from the impact of the COVID-19 pandemic.

**Table 1: Descriptive statistics of NSE Returns**

| Statistic | Minimum | Maximum | Mean | Standard Deviation | Skewness | Kurtosis | Jarque-Bera Test (p-value) | N |
|---|---|---|---|---|---|---|---|---|
| NSE | -5.0329 | 7.9848 | 0.0294 | 0.9875 | 0.3381 | 8.5954 | 0.00001 | 2,456 |

The descriptive statistics of the NSE returns series in Table 1 show that the average return is 0.029 with a volatility of 0.988. Though the returns series indicate a positive skewness (0.338), there is high kurtosis (8.595) indicating consistent fluctuation away from the average returns of the index. Some of these fluctuations are positive while others are negative. Fig. 3 gives the plot of the returns series.

### *3.2 Stationarity and Heteroscedasticity Test*

The returns plot for the NSE shown in Fig. 3 indicates the returns plot is stationary and trendless with varying amplitude over time, in contrast to the original series. Additionally, Volatility clustering can be seen in the returns plot, where periods of low volatility are followed by periods of low volatility and periods of high volatility are followed by periods of high volatility. Table 3 gives the result of the hypothesis, there is no ARCH effect in the residuals of returns. The NSE returns show evidence for rejecting the null hypothesis of no ARCH effect in the residuals of the returns. Furthermore, the ADF test in Table 2 rejects the null hypothesis of a unit root in the returns at both the 1% and 5% levels

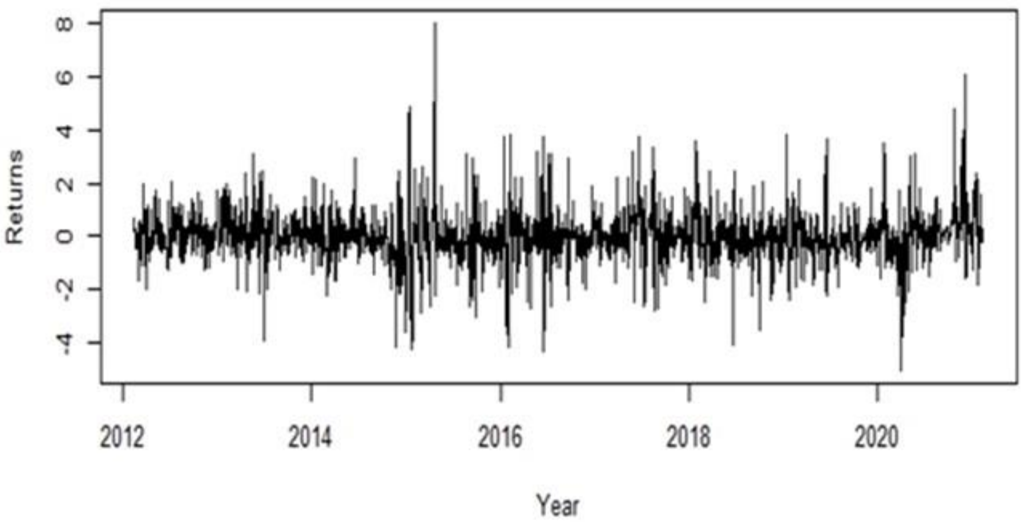



**Fig. 3: Returns Plot of NSE**

### *3.3 Estimation of Parameter*

The parameter estimates of the fitted models for the different error innovations are given in Table 4. For the fitted GARCH (1,1) models, all the parameters under the specified error distributions are significant at least a 5% level of significance except for the skew parameters of the NIG and OGELAD error innovations.

**Table 2: Unit Root Test (ADF test statistic)**

| Returns | t-statistic | p-value | Comment |
|---|---|---|---|
| **NSE** | -12.466 | 0.001 | Stationary |

**Table 3: Testing for ARCH Effects**

| Returns | ARCH-LM statistic | p-value |
|---|---|---|
| NSE | 325.19 | 0.0001 |

**Table 4: Estimates of Various GARCH Models for Different Error Distributions**

| Model | Error Distribution | Estimates | | | | | |
|---|---|---|---|---|---|---|---|
| | | $\kappa$ | $a_1$ | $b_1$ | $\gamma_1$ | Skew | Shape |
| **GARCH (1,1)** | NORM | 0.110801* | 0.21025** | 0.667605*** | | | |
| | SNORM | 0.102930*** | 0.207739*** | 0.683114*** | | 1.066158*** | |
| | SSTD | 0.120838*** | 0.339410*** | 0.623252*** | | 1.015054*** | 3.372594*** |
| | SGED | 0.106625*** | 0.260569*** | 0.638683*** | | 1.041345*** | 1.052773*** |
| | NIG | 0.109017*** | 0.294980*** | 0.624023*** | | 0.036500 | 0.667647*** |
| | OGELAD | 0.2004*** | 0.329800*** | 0.600200*** | | 0.0000003674 | 0.000011*** |
| **EGARCH (1,1)** | NORM | -0.012355 | 0.043210* | 0.866329*** | 0.357130*** | | |
| | SNORM | -0.010951 | 0.037110* | 0.872315*** | 0.353660*** | 1.056503*** | |
| | SSTD | -0.018526 | 0.020716 | 0.880817*** | 0.474405*** | 1.015162*** | 3.385366*** |
| | SGED | -0.040868* | 0.025095 | 0.876785*** | 0.407701*** | 1.045136*** | 1.054198*** |
| | NIG | -0.037608* | 0.022843 | 0.877137*** | 0.440486*** | 0.035587 | 0.670913*** |
| | OGELAD | 0.070000*** | 0.08000*** | 0.950000*** | 0.320000*** | 0.0000751*** | -0.000075*** |
| **TGARCH (1,1)** | NORM | 0.126904*** | 0.208451*** | 0.706635*** | -0.147118** | | |
| | SNORM | 0.120798*** | 0.204913*** | 0.716067*** | -0.129959** | 1.050347*** | |
| | SSTD | 0.120634*** | 0.286669*** | 0.684453*** | 0.070823 | 1.010221*** | 3.399325*** |
| | SGED | 0.116881*** | 0.242554*** | 0.694418*** | -0.088258 | 1.040300*** | 1.056288*** |
| | NIG | 0.116737*** | 0.264461*** | 0.684973*** | -0.080055 | 0.027305 | 0.673575*** |
| | OGELAD | 0.140000*** | 0.220000*** | 0.660000*** | -0.070000*** | 0.00000033 | 0.00000986*** |
| **GJR-GARCH (1,1)** | NORM | 0.113949*** | 0.256156*** | 0.662266*** | -0.08078* | | |
| | SNORM | 0.105991*** | 0.242239*** | 0.678123*** | -0.067871* | 1.060687*** | |
| | SSTD | 0.122486*** | 0.362517*** | 0.620472*** | -0.046151 | 1.015391*** | 3.375902*** |
| | SGED | 0.109191*** | 0.287503*** | 0.633855*** | -0.051823 | 1.042368*** | 1.054332*** |
| | NIG | 0.110651*** | 0.319227*** | 0.621099*** | -0.048418 | 0.035733 | 0.669221*** |
| | OGELAD | 0.012400*** | 0.063040*** | 0.821300*** | 0.2550*** | 0.006170*** | 0.003469 |

The EGARCH (1,1) estimates of the NSE indicates statistical significance of the persistence and leverage effect parameters at 0.1%, 1%, and 5% level of significance. Nonetheless, negative shocks do not have more impact on volatility than positive shocks of the same magnitude. For the TGARCH (1,1), the constant term ($\kappa$), the ARCH term ($a_1$), and GARCH term ($b_1$) are statistically significant for all error innovations at the the specified level of significance. More so, the leverage parameter is negative for all error innovations, it is only significant for the NORM, SNORM, and OGELAD error innovations. The parameters of the GJR-GARCH (1,1) are mostly statistically significant. In this model, the leverage effect parameter ($\gamma_1$) is only significant for the NORM, SNORM, and OGELAD error innovations.

### *3.4 Model Selection*

The log-likelihood, AIC, BIC, and HQIC for the fitted volatility models are given in Table 5. The result indicates the OGELAD error innovation fitted best the volatility of the NSE returns as it produces the maximum log-likelihood and least values of the information criteria among various error innovations. Particularly, the GARCH (1,1) model outperforms other volatility models in fitting the volatility of NSE returns.

**Table 5: Volatility Model Selection for NSE Returns**

| **Model** | **Error Distribution** | **Log-likelihood** | **AIC** | **BIC** | **HQIC** |
|---|---|---:|---:|---:|---:|
| **GARCH (1,1)** | NORM | -3099.657 | 2.5290 | 2.5432 | 2.5342 |
| | SNORM | -3095.034 | 2.5261 | 2.5426 | 2.5321 |
| | SSTD | -2931.01 | 2.3933 | 2.4122 | 2.4002 |
| | SGED | -2931.289 | 2.3936 | 2.4125 | 2.4004 |
| | NIG | -2924.588 | 2.3881 | 2.4070 | 2.3950 |
| | OGELAD | 22447.49 | -18.2732 | -18.2542 | -18.2663 |
| **EGARCH (1,1)** | NORM | -3094.197 | 2.5254 | 2.5420 | 2.5314 |
| | SNORM | -3090.849 | 2.5235 | 2.5424 | 2.5304 |
| | SSTD | -2929.073 | 2.3926 | 2.4138 | 2.4003 |
| | SGED | -2928.959 | 2.3925 | 2.4138 | 2.4002 |
| | NIG | -2922.746 | 2.3874 | 2.4087 | 2.3951 |
| | OGELAD | **1287.257** | **-1.0417** | **-1.0228** | **-1.0349** |
| **TGARCH (1,1)** | NORM | -3092.033 | 2.5236 | 2.5402 | 2.5297 |
| | SNORM | -3089.386 | 2.5223 | 2.5412 | 2.5292 |
| | SSTD | -2928.332 | 2.3920 | 2.4132 | 2.3997 |
| | SGED | -2928.11 | 2.3918 | 2.4131 | 2.3995 |
| | NIG | -2921.999 | 2.3868 | 2.4081 | 2.3945 |
| | OGELAD | 1095.657 | -0.8857 | -0.8668 | -0.8788 |
| **GJR-GARCH (1,1)** | NORM | -3096.569 | 2.5273 | 2.5439 | 2.5333 |
| | SNORM | -3092.716 | 2.5250 | 2.5439 | 2.5319 |
| | SSTD | -2930.746 | 2.3939 | 2.4152 | 2.4017 |
| | SGED | -2930.754 | 2.3939 | 2.4152 | 2.4017 |
| | NIG | -2924.217 | 2.3886 | 2.4099 | 2.3963 |
| | OGELAD | 26106.14 | -13.8268 | -13.8136 | -13.8221 |

The absence of ARCH effect in the residuals and squared standardized residuals for the fitted models indicate the fitted models under the different error distributions have been properly specified. For these different models and error innovations, the *p*-value approaches 1 for the standardized squared residuals indicating no heteroscedasticity in the standardized squared residuals of the fitted models (see Table 6).

**Table 6: Heteroscedasticity Test for Volatility Models for NSE Returns**

| **Model** | **Error Distribution** | **Standardized Residuals** | | **Standardized Squared Residuals** | |
|---|---|---|---|---|---|
| | | Statistic | p-value | Statistic | p-value |
| **GARCH (1,1)** | NORM | 11.991 | 0.4464 | 2.0940 | 0.9992 |
| | SNORM | 12.135 | 0.4349 | 2.0112 | 0.9994 |
| | SSTD | 16.175 | 0.1834 | 3.0439 | 0.9952 |
| | SGED | 14.202 | 0.2880 | 2.7782 | 0.9969 |
| | NIG | 15.695 | 0.2056 | 2.9924 | 0.9956 |
| | OGELAD | 13.826 | 0.3217 | 2.1453 | 0.9986 |
| **EGARCH (1,1)** | NORM | 12.090 | 0.4385 | 2.5536 | 0.9980 |
| | SNORM | 12.202 | 0.4296 | 2.5021 | 0.9982 |
| | SSTD | 14.308 | 0.2814 | 2.6693 | 0.9975 |
| | SGED | 12.488 | 0.4074 | 2.6829 | 0.9974 |
| | NIG | 13.618 | 0.3257 | 2.7194 | 0.9972 |
| | OGELAD | 14.536 | 0.2601 | 2.6501 | 0.9978 |
| **TGARCH (1,1)** | NORM | 13.270 | 0.3497 | 2.9360 | 0.9960 |
| | SNORM | 13.307 | 0.3471 | 2.8608 | 0.9964 |
| | SSTD | 14.623 | 0.2627 | 3.2071 | 0.9939 |
| | SGED | 13.118 | 0.3605 | 3.0714 | 0.9950 |
| | NIG | 14.067 | 0.2964 | 3.2117 | 0.9939 |
| | OGELAD | 14.321 | 0.2811 | 3.1485 | 0.9924 |
| **GJR-GARCH (1,1)** | NORM | 13.009 | 0.3684 | 2.6876 | 0.9974 |
| | SNORM | 12.836 | 0.3811 | 2.5159 | 0.9981 |
| | SSTD | 16.797 | 0.1574 | 3.2630 | 0.9934 |
| | SGED | 14.941 | 0.2447 | 3.1141 | 0.9947 |
| | NIG | 16.405 | 0.1734 | 3.2594 | 0.9934 |
| | OGELAD | 16.825 | 0.1483 | 3.3143 | 0.9919 |

Fig. 4 plots the GARCH volatility of the selected model – GARCH (1,1) with the new error distribution (OGELAD). The plot shows the fitted GARCH (1,1) captures the different spikes demonstrated in the realized volatilities. The most notable periods in the lot are the first quarter of 2015 which coincides with the 2015 Nigeria general election; a period of political instability and early 2021 which can be attributed to the high exchange rate volatility of the Nigerian currency.

### *3.5 Forecasting of Volatility Using Fitted Model for NSE Returns*

The examination of the different specified models for their forecasting ability and performance is presented in Table 7. The MAE and RMSE forecast adequacy measures indicate GARCH models with OGELAD error innovation have better forecasting performance for GARCH (1,1) and GJR-GARCH (1,1) compared to models with other error innovations; the only exception is EGARCH (1,1) and TGARCH (1,1) with SGED. Generally, the GJR-GARCH (1,1) with OGELAD error innovation provide the best out-of-sample forecast of the volatility of the NSE returns for 30 days.

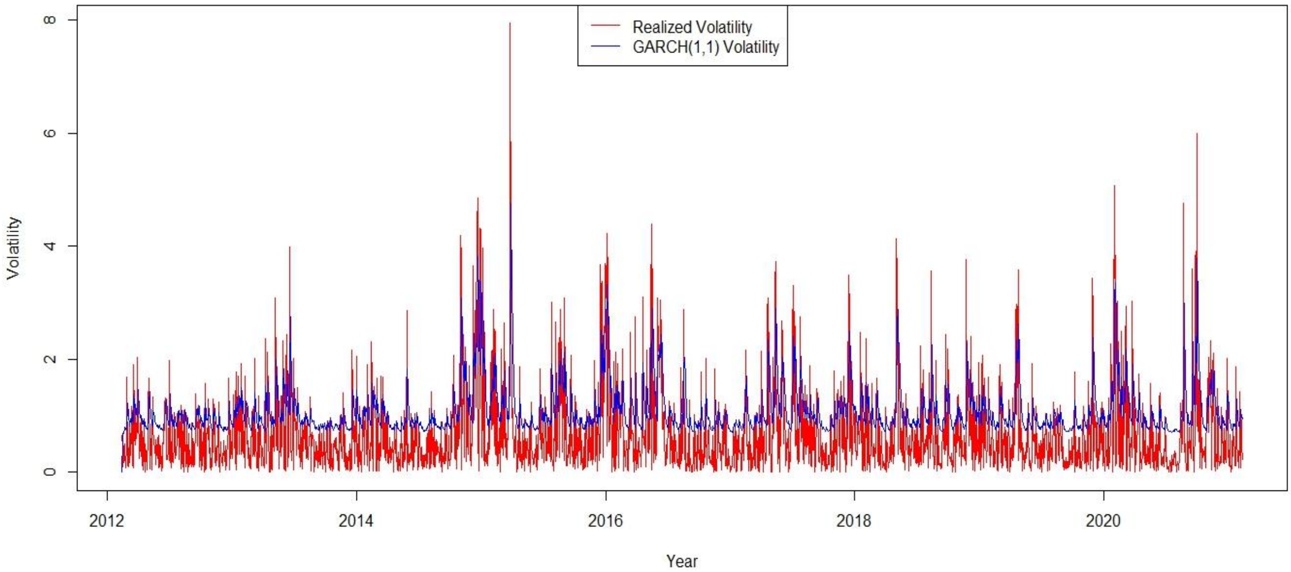



**Fig. 4: Plot of Realized and GARCH(1,1) Volatility for NSE Returns**

**Table 7: Forecasting Performance of Fitted Models for NSE Returns**

| Model | Error Distribution | MAE | RMSE |
|---|---|---|---|
| **GARCH(1,1)** | NORM | 0.6240 | 0.6957 |
| | SNORM | 0.6193 | 0.6910 |
| | SSTD | 1.0776 | 1.1448 |
| | SGED | 0.6706 | 0.7448 |
| | NIG | 0.7613 | 0.8427 |
| | OGELAD | 0.6184 | 0.6855 |
| **EGARCH(1,1)** | NORM | 0.5951 | 0.6639 |
| | SNORM | 0.5942 | 0.6635 |
| | SSTD | 0.6139 | 0.6829 |
| | SGED | 0.5528 | 0.6151 |
| | NIG | 0.5675 | 0.6310 |
| | OGELAD | 1.8274 | 1.9450 |
| **TGARCH(1,1)** | NORM | 0.6116 | 0.6833 |
| | SNORM | 0.6110 | 0.6831 |
| | SSTD | 0.6366 | 0.7083 |
| | SGED | 0.5626 | 0.6272 |
| | NIG | 0.5822 | 0.6493 |
| | OGELAD | 0.5621 | 0.6311 |

| | | | |
|---|---|---|---|
| **GJR-GARCH(1,1)** | NORM | 0.6320 | 0.7041 |
| | SNORM | 0.6274 | 0.6993 |
| | SSTD | 1.0767 | 1.1635 |
| | SGED | 0.6740 | 0.7488 |
| | NIG | 0.7643 | 0.8464 |
| | OGELAD | **0.3652** | **0.4693** |

## 4.0 Conclusion

In this paper, the observed volatility in NSE returns has been modelled using a new error distribution for GARCH models. The error distribution used succeeded in eliminating all traces of statistically significant ARCH effect from the residual of the fitted models. For the fitted model, volatility is quite persistent ($a_1 + b_1 < 1$) and significant for specified error distributions. Similarly, the asymmetry parameter for the selected models is positive and significant indicating negative shocks do not have a greater effect on volatility than positive shocks of the same magnitude. Both the symmetric and asymmetric models have successfully modelled the volatility of the NSE returns considered for the period under study. Overall, the OGELAD error distribution provided an improvement over existing error distributions in modelling and forecasting of volatility, thus its suitability to real world application by investors who trade indexes.

## 5.0 References

Altun, E., Alizadeh, M., Ozel, G., Tatlidil, H., & Maksayi, N. (2017). Forecasting value-at-risk with two-step Method: GARCH Exponentiated Odd Lod-Logistic Normal Model. *Romanian Journal of Economic Forecasting*, 20, 4, pp.97-115.

Asemota, O. J. & Ekejiuba, U. C. (2017). An application of asymmetric GARCH models on volatility of banks equity in nigeria's stock market. *CBN Journal of Applied Statistics*, 8, 1, pp.73-99.

Chkili, W., Aloui, C. & Nguyen, D. K. (2012). Asymmetric effects and long memory in dynamic volatility relationships between stock returns and exchange rates. *Journal of International Financial Markets, Institutions and Money*, 22, pp.738-757

Cowles, A. (1933). *Can stock market forecasters forecast? Econometrica*, pp.309-324.

Cowles, A. & Jones, H. E. (1937). Some a posteriori probabilities in stock market action. *Econometrica*, pp.280-294.

Dikko, H. G. & Agboola, S. (2018). On a new exponentiated error innovation distributions: *evidence of Nigeria Stock Exchange. Journal of Statistics Applications & Probability*, 7, 2, pp. 321 -331

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, pp.987-1007.

Fama, E. F. (1965). The behavior of stock-market prices. *The Journal of Business*, 38, 1, pp.34-105.

Fama, E. F. (1970). Efficient capital markets a review of theory and empirical work. Journal of Finance, pp.383-417.

Fan, Y., Zhang, Y.-J., Tsai, H.-T., & Wei, Y.-M. (2008). Estimating 'value at risk'of crude oil price and its spillover effect using the ged-garch approach. Energy Economics, 30(6), pp.3156-3171.

Feng, L. & Shi, Y. (2017). A simulation study on the distributions of disturbances in the Garch model.

*Cogent Economics & Finance*, 5, 1, 1355503.

Harvey, A., Ruiz, E., & Shephard, N. (1994). Multivariate stochastic variance models. *The Review of Economic Studies*, 61, 2, pp.247-264.

Kendall, M. G. & Hill, A. B. (1953). The Analysis of Economic Time Series Part I: Prices. *Journal of the Royal Statistical Society*, 116, 1, pp.11-34.

Mabrouk, S. & Saadi, S. (2012). Parametric value-at-risk analysis: Evidence from stock indices. *The Quarterly Review of Economics and Finance*, 52, 3, pp.305-321.

Melino, A., & Turnbull, S. (1990). "Pricing Foreign Currency Options with Stochastic Volatility," *Journal of Econometrics*, 45, pp.7-39.

Mills, T. C. & Markellos, T. N. (2008). *The econometric modelling of financial time series*. Cambridge. Cambridge University Press.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, pp.347-370.

Nicholls, D. F. & Quinn, B. G. (1982). Lecture notes in Statistics. Random coefficient autoregressive models: *An Introduction*, 11, pp.139-149.

Obalowu, J. & David, R. O. (2021). The Odd *Generalized exponential Laplace distribution*. 39th Annual Conference of the Nigerian Mathematical Society, April 20-24, 2021.

Uyaebo, S. O., Atoi, V. N. & Usman, F. (2015). Nigeria stock market volatility in comparison with some countries: application of asymmetric GARCH Models. CBN *Journal of Applied Statistics*, 6, 2, pp.133-160.

Working, H. (1934). A random-difference series for use in the analysis of time series. *Journal of the American Statistical Association*, 29, 185, pp.11-24.

Yang, S.-R. & Wade, B. B. (1993). Nonlinear dynamics of daily futures prices: conditional heteroskedasticity or chaos? *The Journal of Futures Markets*, 13(2), pp.175.

**Declarations**

The authors declare that they have no conflict of interest.

**Data availability**

All data used in this study will be readily available to the public.

**Consent for publication**

Not Applicable

**Availability of data and materials**

The publisher has the right to make the data Public.

**Competing interests**

The authors declared no conflict of interest.

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Reuben Oluwabukunmi David conceived the idea. Both Job Obalowu and Reuben Oluwabukunmi David wrote different sections of the initial draft. The final manuscript was revised by both authors.