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A Unique Generalization of Einstein Field Equation; Pathway for Continuous Generation of Gravitational Waves

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Abstract: *In this theoretical exploration, we introduce a novel extension to the Einstein field Equations by incorporating a newly defined metric tensor, termed the "Golden Metric Tensor". This approach aims to complement and expand upon the well-established Einstein field equations devoid of its initial incompleteness thereby offering a fresh perspective on the nature of gravity and its interplay with spacetime. Our result is found to be mathematically most elegant, physically most natural, and satisfactory for application to a sinusoidal time distribution of mass within a spheroidal body to generate gravitational waves.*

Keywords: Gravitational waves, spheroidal body, special relativity, golden metric tensor, field equation

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1.0 Introduction

Spherically symmetric solutions in general relativity are crucial for understanding the gravitational effects around objects with spherical symmetry, such as stars and black holes (Kim *et al.*, 2018). One of the most renowned examples is the Schwarzschild metric (Antoci and Liebscher, 2003) which characterizes the geometry around a non-rotating uncharged black hole. This metric reveals fascinating phenomena, such as the existence of an event horizon beyond which escape is impossible, and the presence of a singularity at the center. The Schwarzschild solution has played a pivotal role in astrophysics, providing the theoretical foundation for the study of black holes and their observable effects on nearby matter and light. Another notable example is the Friedmann–Lemaitre–Robertson–Walker (FLRW) metric, which defines how its scale factor changes with cosmic time, and characterizes a homogeneous and isotropic universe (Lematre, 1931). The Reissner-Nordström metric, which describes a charged black hole, is another example worthy of note (Giorgi, 2020; Rincóni *et al*.., 2019). This solution introduces an electric charge term, alongside the mass term present in the Schwarzschild metric. The presence of charge influences the behaviour of the electromagnetic field near the black hole, and it introduces additional intriguing features, such as the possibility of an inner Cauchy horizon. Understanding charged black holes is essential for comprehending their interactions with charged particles and magnetic fields in astrophysical environments. Although there is still considerable disagreement about its physical interpretation (Kaloper *et al*., 2010; Lake & Abdelqader, 2011), McVittie's combination of the Schwarzschild and FLRW metrics resulted in a new spherically-symmetric solution that depicts a point mass contained in an expanding universe. The fact that a metric tensor known as the golden metric tensor for all gravitational fields in nature has been created is intriguing. This metric tensor holds for all four spacetime coordinates, regular geometries found in nature, and regular mass distributions. It reduces to the well-known Euclidean metric tensor for all spacetimes in gravitational fields in nature in the limit of c^0, in complete accordance with the equivalency principles of physics and mathematics (Koffa, *et al*.. 2016).

Beyond individual black holes, spherically symmetric solutions have been instrumental in modelling more complex systems, such as binary black hole systems and the cosmological effects of massive spherical bodies. These solutions not only provide insights into the behaviour of gravity in extreme conditions but also serve as a foundation for testing the predictions of general relativity through observations and experiments. Overall, spherically symmetric solutions constitute a cornerstone of our understanding of the gravitational universe and play a vital role in contemporary astrophysical research.

Furthermore, spherically symmetric solutions are foundational in astrophysical applications. They allow scientists to model the dynamics of binary black hole systems, where two compact objects orbit around each other. This modelling is crucial in predicting the gravitational waves emitted during their coalescence, a phenomenon that has been directly observed by experiments like LIGO and Virgo (Abbott, 2016; Acernese, 2014). Additionally, spherically symmetric solutions play a crucial role in understanding the evolution and behaviour of stars. By applying these solutions to models of stellar structure, scientists can make predictions about the life cycles of stars, including their eventual fate as black holes or other compact remnants.

The field of General Relativity, conceived by Albert Einstein in the early 20th century, stands as one of the cornerstones of modern theoretical physics. At its heart lies the Einstein Field Equations (EFE); a set of mathematical expressions that elegantly encapsulate the fundamental relationship between spacetime curvature and the distribution of matter and energy within the universe. The search for new, precise analytical solutions to Einstein's field equations is one of the major issues facing general relativity. After the discovery of general relativity by Einstein in 1915, many potent techniques have been developed for the derivation of new solutions to the gravitational field equations (Turimov, et al.., 2018; MacCallum, 2006; Rincón et al., 2019). An array of fascinating exact solutions to the Einstein field equations can be found in the literature (Contopoulos, *et al*.., 2016; Frutos-Alfaro *et al*., 2018; Gibbons and Volkov, 2017; MacCallum, 2006; Rincón *et al*., 2019) and (Stephani *et al*., 2003; Basu and Ray, 1998, Stephani *et al.,* 2009). Under the conformal field theory approach, some approximate static solutions of the Einstein equations are shown in (Fan, Chen, and Lü, 2016; Fan and Lü, 2015; Jyoti and Kumar, 2020).

However, as our understanding of the cosmos has deepened, so too has the desire to refine and extend the foundations laid by Einstein. This pursuit has led us to consider novel approaches, including the introduction of a new metric tensor, hereby referred to as the "Golden Metric Tensor," alongside the incorporation of a complementary Laplacian operator.

This research endeavours to usher in a new era of gravitational physics by synthesizing the classical framework of General Relativity with this novel metric tensor and Laplacian operator. The proposed extensions are poised to enrich our comprehension of gravity's intricacies and unlock new avenues for exploration.

The well-known Newton's gravitational field equation in the spherical coordinates system is given by (Obaboye, 2016);

$$\nabla^2 f(\underline{r}, t) = 4\pi G \rho_0(\underline{r}, t) \qquad (1)$$

Where all the symbols have their usual meaning.

Equation (1) will have to be generalized and transformed into our oblate spheroidal coordinates system and the Laplacian replaced by the Riemannian Laplacian. This is necessary because the Euclidean Laplacian cannot account for the variation of the density of the proper mass in the equation with time coordinates. This variation of proper mass with the time coordinate is what gives rise to radiation energy in the form of gravitational waves as predicted by Einstein in his theory of General Relativity.

The golden metric tensor for all gravitational fields in nature in spherical polar coordinates (r, θ, ϕ, x^0) is given as

2.0 Theory

$$g_{00} = -\left(1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right) \qquad (2)$$

$$g_{11} = \left(1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right)^{-1} \qquad (3)$$

$$g_{22} = r^2 \left(1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right)^{-1} \qquad (4)$$

$$g_{33} = r^2 \sin^2\theta \left(1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right)^{-1} \qquad (5)$$

$$g_{\mu v} = 0; otherwise \qquad (6)$$

The Cartesian coordinates are related the spherical polar coordinates as:

$$r = [x^2 + y^2 + z^2]^{\frac{1}{2}} \qquad (7)$$

$$\theta = \cos^{-1}\left\{\frac{z}{[x^2 + y^2 + z^2]^{\frac{1}{2}}}\right\} \qquad (8)$$

and

$$\phi = \tan^{-1}\frac{y}{x} \qquad (9)$$

It may be note that the Cartesian coordinates (x, y, z) are related to the Oblate Spheroidal coordinates (η, ξ, ϕ) as

$$x = a(1 - \eta^2)^{\frac{1}{2}}(1 + \xi^2)^{\frac{1}{2}} \cos\phi \qquad (10)$$

$$y = a(1 - \eta^2)^{\frac{1}{2}}(1 + \xi^2)^{\frac{1}{2}} \sin\phi \qquad (11)$$

$$z = a\eta\xi \qquad (12)$$

From the well-known transformation equation given by the covariant tensor (Koffa et al..,2016 ; Ogunleye, et al., 2016)

$$\bar{g}_{\mu\upsilon} = \frac{\partial x^q}{\partial \bar{x}^\mu}\frac{\partial x^s}{\partial \bar{x}^\upsilon} g_{qs} \tag{13}$$

Consequently, upon transformation of (2)-(5) using equation (13), we obtain the golden metric tensor for all gravitational fields in Oblate Spheroidal Coordinates as:

$$g_{00} = -\left(1 + \frac{2}{c^2} f(\eta, \xi, \phi, x^0)\right) \tag{14}$$

$$g_{11} = \frac{a^2(\eta^2 + \xi^2)}{(1 - \eta^2)}\left(1 + \frac{2}{c^2} f(\eta, \xi, \phi, x^0)\right)^{-1} \tag{15}$$

$$g_{22} = \frac{a^2(\eta^2 + \xi^2)}{(1 + \xi^2)}\left(1 + \frac{2}{c^2} f(\eta, \xi, \phi, x^0)\right)^{-1} \tag{16}$$

$$g_{33} = a^2(1 - \eta^2)(\eta^2 + \xi^2)\left(1 + \frac{2}{c^2} f(\eta, \xi, \phi, x^0)\right)^{-1} \tag{17}$$

$$g_{\mu\nu} = 0; \text{otherwise} \tag{18}$$

It is easy to see that the contravariant metric tensor for the gravitational field obtained by using the Quotient Theorem of tensor analysis becomes:

$$g^{00} = -\left(1 + \frac{2}{c^2} f(\eta, \xi, \phi, x^0)\right)^{-1} \tag{19}$$

$$g^{11} = \frac{(1 - \eta^2)}{a^2(\eta^2 + \xi^2)}\left(1 + \frac{2}{c^2} f(\eta, \xi, \phi, x^0)\right) \tag{20}$$

$$g^{22} = \frac{(1 + \xi^2)}{a^2(\eta^2 + \xi^2)}\left(1 + \frac{2}{c^2} f(\eta, \xi, \phi, x^0)\right) \tag{21}$$

$$g^{33} = \frac{\left(1 + \frac{2}{c^2} f(\eta, \xi, \phi, x^0)\right)}{a^2(1 - \eta^2)(1 + \xi^2)} \tag{22}$$

$$g^{\mu\nu} = 0, \text{otherwise} \tag{23}$$

Next, we define the covariant metric tensor in a 4 by 4 metric as follows:

$$g_{\mu\upsilon} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \tag{24}$$

Then the determinant of the metric tensor $g_{\mu\upsilon}$, denoted as g can be determined as:

$$g = \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix} \tag{25}$$

Hence,

$$g = -a^6(\eta^2 + \xi^2)^2\left(1 + \frac{2}{c^2} f\right)^{-2} \tag{26}$$

According to the theory of Tensor Analysis, the Laplacian operator ∇_R^2, is given in all gravitational fields and all orthogonal curvilinear coordinates x^μ by

$$\nabla_R^2 = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^\alpha}\left[\sqrt{g}\, g^{\alpha\beta}\frac{\partial}{\partial x^\beta}\right] \tag{27}$$

Where g is the determinant of the metric tensor $g_{\mu\nu}$ and ∇_R^2 is called Riemannian Laplacian operator.

Based upon the golden metric tensor in oblate spheroidal coordinates, $\sqrt{g}$ is given as :

$$\sqrt{g} = ia^3(\eta^2+\xi^2)\ \left(1+\frac{2}{c^2}f\right)^{-1} \tag{28}$$

Hence, the Riemannian Laplacian operator is given in Oblate Spheroidal Coordinates as:

$$\nabla_R^2 = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^0}\left[\sqrt{g}g^{00}\frac{\partial}{\partial x^0}\right] + \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^1}\left[\sqrt{g}g^{11}\frac{\partial}{\partial x^1}\right] + \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^2}\left[\sqrt{g}g^{22}\frac{\partial}{\partial x^2}\right] + \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^3}\left[\sqrt{g}g^{33}\frac{\partial}{\partial x^3}\right] \tag{29}$$

So that by substituting (19) – (23) and (28) in (29) we have,

$$\nabla_R^2 = -\frac{1}{c^2\left(1+\frac{2}{c^2}f\right)}\frac{\partial^2}{\partial t^2} + \frac{1}{a^2(\eta^2+\xi^2)\ \left(1+\frac{2}{c^2}f\right)^{-1}}\frac{\partial}{\partial \eta}\left[(1-\eta^2)\frac{\partial}{\partial \eta}\right] + \frac{1}{a^2(\eta^2+\xi^2)\ \left(1+\frac{2}{c^2}f\right)^{-1}}\frac{\partial}{\partial \xi}\left[(1+\xi^2)\frac{\partial}{\partial \xi}\right] + \frac{1}{a^2(\eta^2+\xi^2)\ \left(1+\frac{2}{c^2}f\right)^{-1}}\frac{\partial^2}{\partial \phi^2} \tag{30}$$

Equation (30) is the generalized Riemann's Laplacian operator in oblate spheroidal coordinate based on the golden metric tensor. The Riemannian Laplacian operator implies appropriate corresponding generalizations of all the fundamental physical quantities of today (which have been based upon Euclidean Geometry):

- Riemannian generalization of Newtonian Gravitational Potential equation,
- Riemannian generalization of Maxwell's Electric field Equations
- Riemannian generalization of Schrodinger's Dynamical Quantum Mechanical Wave Equation,
- And so on

2.1 Generalized Gravitational Wave Equation Based Upon the Golden Metric Tensor

Consider a sinusoidal time-varying distribution of mass with density given by

$$\rho_0 + \rho_e e^{iwt} \tag{31}$$

where;

ρ_0 is the constant density of the spheroid at the beginning.

ρ_e is the density of valence electrons in the spheroid.

The generalized Newton's gravitational field equation based upon Riemann geometry and the golden metric tensor is therefore given as

$$\nabla_R^2 f(\xi,t) = 4\pi G\left[\rho_0 + \rho_e e^{iwt}\right] \tag{32}$$

∇_R^2 is the Riemannian Laplacian based on the golden metric tensor for all gravitational fields in nature and all other symbols have their usual meanings. It should be noted that Euclidean Laplacian has been replaced with Riemannian Laplacian based on the golden metric tensor in the oblate spheroidal coordinates system.

Using (30) and (31), it implies that the general gravitation wave equation (32) becomes;

$$\begin{aligned}&\frac{1}{a^2(\eta^2+\xi^2)\ \left(1+\frac{2}{c^2}f\right)^{-1}}\frac{\partial}{\partial \xi}\left[(1+\xi^2)\frac{\partial}{\partial \xi}\right]f(\xi,t)\\ &\quad + \frac{1}{a^2(\eta^2+\xi^2)\ \left(1+\frac{2}{c^2}f\right)^{-1}}\frac{\partial}{\partial \eta}\left[(1-\eta^2)\frac{\partial}{\partial \eta}\right]f(\xi,t)\\ &\quad + \frac{1}{a^2(\eta^2+\xi^2)\ \left(1+\frac{2}{c^2}f\right)^{-1}}\frac{\partial^2}{\partial \phi^2}f(\xi,t) - \frac{1}{c^2\left(1+\frac{2}{c^2}f\right)}\frac{\partial^2}{\partial t^2}f(\xi,t)\\ &\quad = 4\pi G\left[\rho_0+\rho_e e^{iwt}\right]\end{aligned} \tag{33}$$

3.0 Results and Discussion

Equation (33) is therefore the generalized wave equation which resolves the incompleteness stated in Newton's gravitational field equation. This equation is mathematically most elegant, physically most natural and satisfactory for application to a sinusoidal time distribution of mass within a spheroidal body to generate gravitational waves. It is also important to note that in the limit of c^0, the general gravitational field equation reduces to the pure Newton's gravitational equation as required by the principles of equivalence of physics. Therefore, the gravitational field equation contains post-Newton correction terms of order c^{-2} and all degrees of nonlinearity in the gravitational scalar potential and its derivatives.

It is important to state that this equation contains:

i. The $\left(1 + \frac{2}{c^2} f\right)$ term which is not found in Newton's gravitational field equation. The consequence of this is that it predicts correction terms to the gravitational field of all massive bodies.

ii. The time component which predicts the existence of gravitational waves with velocity which is equal to the speed of light in vacuo.

The construction of this new generalized Laplacian operator to extend Einstein's wave equation marks a significant advancement in our understanding of gravitational wave generation within spheroidal bodies. This novel operator introduces terms that, when applied to a sinusoidal time distribution of mass, contribute to the production of gravitational waves, complementing Einstein's established wave equation.

One of the key implications of this result is its potential to shed light on the behaviour of gravitational waves in non-uniform mass distributions. While Einstein's wave equation has been invaluable in describing gravitational wave propagation in symmetric systems, this new generalized Laplacian offers a refined framework for scenarios where mass distribution varies with time and exhibits spheroidal symmetry. This is particularly relevant in astrophysical contexts, such as rotating neutron stars or binary systems undergoing orbital evolution.

The introduction of the sinusoidal time distribution of mass is a crucial aspect of this result. It suggests that dynamic changes in mass within a spheroidal body, exhibiting oscillatory behaviour, can serve as a source for the generation of gravitational waves. This introduces a rich avenue for further exploration into the astrophysical phenomena that may give rise to such conditions. For instance, this could have implications for our understanding of compact binary systems, where objects orbit each other in elliptical paths.

The complementary nature of this new generalized form of Einstein's wave equation is noteworthy. It implies that both frameworks can coexist harmoniously, with each offering unique insights into different facets of gravitational wave dynamics. This duality not only enriches our theoretical arsenal but also opens avenues for cross-validation of results obtained through distinct mathematical approaches.

The incorporation of the generalized Laplacian operator underscores the profound influence of geometric considerations on gravitational wave generation. This aligns with the broader framework of General Relativity, where geometry plays a central role in shaping the behaviour of gravitational fields. The interplay between geometry and mass distribution, as encapsulated in this result, further reinforces the elegance and coherence of Einstein's gravitational theory.

4.0 Conclusion

While the theoretical construct presented in this research is highly promising, it naturally invites empirical validation. Experimental tests, involving precise measurements of gravitational waves emanating from non-uniform mass distributions, will be instrumental in affirming the predictive power of this new framework. Additionally, collaborations with observational astronomers and astrophysicists will be crucial in identifying suitable celestial objects or events for targeted observations.

The development of this new generalized Laplacian operator represents a significant step forward in the theoretical understanding of gravitational wave generation within spheroidal bodies. Its complementarity with Einstein's wave equation and its reliance on a sinusoidal time distribution of mass introduces a fresh perspective on the complex interplay between mass, geometry, and gravitational waves. This result not only expands the theoretical foundation of General Relativity but also beckons towards a new era of precision astrophysical observations**.**

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Compliance with Ethical Standards Declarations

The authors declare that they have no conflict of interest.

Data availability

All data used in this study will be readily available to the public.

Consent for publication

Not Applicable

Availability of data and materials

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Authors' contributions

Koffa Jude, Jacob Omonile and Taofiq Ibrahim developed the conceptualization. Enock Oladimeji, Helen Edogbanya and Obaje Vivian resolved the equations. Eghaghe did the proof reading in addition to interpreting the solution.