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Graphical Solution of Eigenstate of an Electron in a Finite Quantum Well

Akwuegbu Ozochi Chinyere, Oriaku Chijioke Innocent, Dinneya Obinna Christian, Nkpoku Emmanuel Chidiebere, Nwaehiodo Immaculate Ihechi

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Abstract: This study explores the eigenstates of an electron in a finite quantum well using the Schrödinger wave equation. Quantum mechanics, a fundamental theory in physics, describes the properties of molecules, atoms, and subatomic particles through quantization of energy and wave-particle duality. A quantum well, a nanometer-thin layer in semiconductor materials, confines electrons to a two-dimensional layer, resulting in quantized energy spectra essential for various electronic and optoelectronic devices. Unlike the infinite potential well, the finite potential well allows for the probability of finding particles outside the well, necessitating accurate calculations of bound states. This research employs a graphical method using MATLAB to solve for the eigenstates and eigenenergies of electrons in a finite quantum well. By deriving the time-independent Schrödinger equation, applying boundary conditions, and utilizing transcendental equations, we determine the energy levels and eigenfunctions of the system. The study highlights the practical applications of quantum wells in modern electronic devices and underscores the importance of understanding quantum confinement in developing advanced technologies.

Keywords:*Eigenstates, quantum, Schrodinger equation, mechanics*

Akwuegbu, Ozochi Chinyere
Department of Physics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria
Email:ozochia@gmail.com
Orcid id:0009-0002-7131-9855

Oriaku, Chijioke Innocent
Department of Physics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria
Email: oriakuc29@gmail.com

Dinneya, Obinna Christian
Department of Physics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria
Email: dinneya.obinna@mouau.edu.ng

Nkpoku, Emmanuel Chidiebere
Department of Physics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria
Email: nkpokuemmanuel@gmail.com

Nwaehiodo, Immaculate Ihechi
Department of Physics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria
Email: immaculatenwaehiodo@gmail.com

1.0 Introduction

Quantum is the smallest possible and therefore indivisible unit of a given quantity or quantifiable phenomenon. Quantum mechanics is a branch of mechanics that deals with the mathematical description of particles, incorporating the concept of quantization of energy wave-particle, the uncertainty principle and the corresponding principle. It is used to describe properties of molecules, atoms and subatomic particles. In quantum mechanics, physical problems are solved by algebraic and graphic methods. By applying the one-dimensional Schrodinger time-independent wave equation, we can obtain the eigenenergy values and eigenfunction of a particle in a square well potential of finite height.

Quantum mechanics can be thought of roughly as the study of physics on very small length scale; although there is also certain macroscopic system it directly applies to. The descriptor "quantum" arises because in contrast with classical mechanics certain quantities take on only discrete value. In quantum mechanics particles have wavelike properties and a particular wave equation, the Schrodinger equation, governs how these waves behave. (Satya, 2016)

A quantum well is a region of reduced dimensionality in a semiconductor material, where electrons are confined in the three-dimensional bulk material to a two-dimensional layer. This confinement results in the electrons having a quantized energy spectrum. Quantum wells are used in semiconductor devices such as quantum dot lasers and can also be used in electronic and optoelectronic devices like transistors, solar cells, and LEDs.

Quantum Well is a nanometer-thin layer which can confine (quasi) particles (typically electrons or holes) in the dimension perpendicular to the layer surface, whereas the movement in the other dimension is not restricted.(Makino & Zory, 1993). This confinement is a quantum effect. A Quantum well is a potential well with only discrete energy values. They are formed by sandwiching a very thin layer of a small-band – gap material.

The Finite Potential Well also known as the finite square well is a concept from quantum mechanics. It is an extension of the infinite potential well, in which a particle is confined to a "box", but one which has finite potential "Well". Unlike the infinite potential well, there is a probability associated with particles being found outside the box. The quantum mechanical interpretation is unlike the classical interpretation, where if the total energy of the particle is less than the potential energy barrier of the wells it cannot be found outside the box. In the quantum interpretation, there is a non–zero probability of the particle being outside the box even then the energy of the particle is less than the potential energy barrier of the wells.(Chiani & Williams, 2016)

Eigenstate quantum is a state of a quantized dynamic system (such as an atom, molecule or crystal) in which one of the variables defining the state (such as energy or angular momentum has a determinate fixed value. Eigenstate of quantum mechanics is the state in which the system is an eigenstate of the observable, which means that the value of the observable is exactly known.In quantum mechanics, a finite quantum well is a potential well that confines particles in a finite region of space. The study of electrons in a finite quantum well is important in the development of electronic devices such as transistors and lasers. One of the key properties of electrons in a quantum well is their energy levels which can be found using the Schrodinger equation. In this project, we will use a graphical method to solve for the eigenstate of the electron in a finite quantum well. Quantum mechanics is mathematics is usually introduced through physical systems described by one dimensional well. (Griffiths & Serway, 2005)

In an infinite well model, the eigenvalue of an electron in a quantum well is approximated in excess but in the realistic quantum well where there is a barrier there is a need to calculate an accurate bound state. In this case, we will be using the graphical solution to approximate the eigen energy.

We aim to determine the Graphical Solution of eigenstateelectron In A Finite Quantum Well by; understanding the concept of a finite quantum well and its potential energy function, deriving the Schrodinger equation for a particle in a finite quantum well, obtaining the graphical solution using Math lab, applying the graphical method to solve for the eigenstates and aretheydepended on the good parameter, and finally comparing the graphical method with the infinite quantum well methods of solution.

Quantum well devices have been the objects of intensive research during the last two decades. Some of the devices have matured into commercially useful products and form part of modern electronic circuits. Some others require further development but have the promise of being useful commercially shortly. The study of the devices is, therefore, gradually becoming compulsory for electronics specialists. The functioning of the devices, however, involves aspects of physics which are Scanty in the literature (Nag, 2006)

When the size of the confining structure is comparable with the wavelength of the particle the electronic and optical properties are changed. Quantum confining can be done in three different ways such as three dimensional (3D) when confined in a quantum dot, two-dimensional (2D) when confined in a quantum wire and one-dimensional (1D) when confined in the quantum well.A potential well having only discrete energy values is known as a quantum well (QW). 1D confinement is possible in QW. When the QW thickness is comparable to the carrier wavelength only then the confinement is possible. The allowed energies as functions of the barrier height can be found numerically by solving a transcendental equation (Murphy & Phillips, 1976). Quantum well is a nanometer-thin layer which can confine (quasi-)particles (typically electrons or holes) in the dimension perpendicular to the layer surface, whereas the movement in the other dimensions is not restricted.

The confinement is a quantum effect. It has profound effects on the density of states for the confined particles. For a quantum well with a rectangular profile, the density of states is constant within certain energy intervals.

A quantum well is often realized with a thin layer of a semiconductor medium, embedded between other semiconductor layers of wider band gap (examples: GaAs quantum well embedded in AlGaAs, or InGaAs in GaAs). The thickness of such a quantum well is typically ≈ 5–20 nm. Such thin layers can be fabricated with molecular beam epitaxy (MBE) or metal-organic chemical vapor deposition (MOCVD).

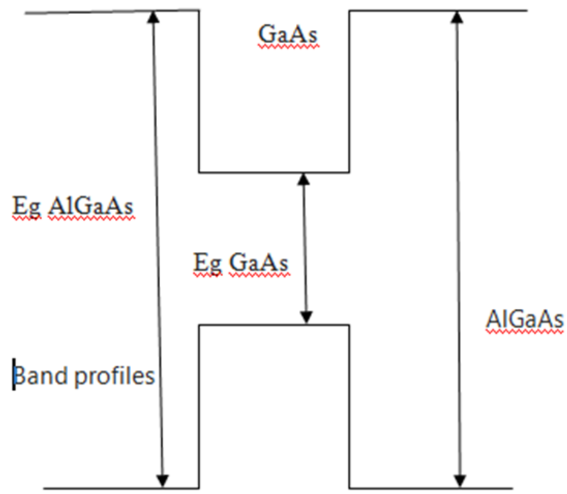

Fig.1: diagram of quantum well using semiconductor

A potential well in which electrons in a three-dimensional system are confined to a plane, i.e. to two dimensions. Quantum wells can be made with a semiconductor that is sandwiched between layers of materials with larger energy band gaps. These wells are used to study two-dimensional systems and also have technological applications including a type of laser (a quantum well laser).

Erwin Schrödinger derived the wave equation associated with such a microscopic particle called the Schrödinger wave equation (S.W.E). (Erwin,2014)

Schrödingerwave Equation is a mathematical expression describing the energy and position of the electron in space and time, taking into account the matter wave nature of the electron inside an atom.

$$\frac{-\hbar^2\partial^2\Psi}{2m\partial x^2} + V(x)\Psi = E\Psi \qquad (1)$$

where,$\hbar = \frac{h}{2\pi}$ is the reduced Planck's constant. h is Planck's constant. m is the mass of the particle, ψ is the wave function, V[x] is the function describing the potential energy at each point x, E is the energy, a real number, sometimes called eigenenergy.The Schrödinger wave equation is of two types

(i) Time-dependent.

(ii) Time independent

We consider a particle of mass m, moving with velocity V along the x- x-direction (One dimension time-dependent). The associated displacement is given by the wave function is a complex function of displacement x and time t.We have

$$\Psi(x,t) = Ae^{i(kx-\omega t)} \qquad (2)$$

Where A is the wave amplitude

Taking the derivation of equation (1) w.r.t x, we obtain

$$\Psi(\mathrm{x},t) = \frac{\partial\Psi(x,t)}{\partial x} = \mathrm{ik}e^{i(kx-\omega t)}$$

$$\Psi''(x,t) = \frac{\partial^2\Psi(x,t)}{\partial x^2} = -k^2Ae^{i(kx-\omega t)}$$

$$= -k^2\Psi(x,t) \qquad (3)$$

where $k=\frac{2\pi}{\lambda}$

$$\Psi'' = \frac{\partial^2\Psi}{\partial x^2} = -\frac{4\pi^2}{\lambda^2}\Psi \qquad (4)$$

But the de Broglie wavelength associated with the particle is

$\boldsymbol{\lambda}=\frac{h}{p}=\frac{h}{mv}$ i.e p= mv

$$\frac{1}{\lambda^2} = \frac{m^2v^2}{\hbar^2} = \frac{2m}{\hbar^2}\left(\frac{1}{2}mv^2\right) \qquad (5)$$

$\mathrm{k.E}=\frac{1}{2}mv^2$

but we define the total energy of the particle E be related to the kinetic energy K.E and potential energy V as

E =k.E + V

K.E = E- V (6)

From equation (5) and (6) obtain that

$$\frac{1}{\lambda^2} = \frac{2m}{\hbar^2}(E-V) \qquad (7)$$

Putting the value in equation (4)

$$\frac{\partial^2\Psi}{\partial x^2} = -\frac{8\pi^2}{h^2}m(E-V)\Psi \qquad \text{where } \hbar=\frac{h}{2\pi}$$

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\Psi = 0 \qquad (8)$$

Equation 8 therefore represents the time-independent Schrödinger wave equation

Recall that the wave equation is given by

$$\Psi(x,t) = Ae^{i(kx-\omega t)} \qquad (9)$$

Differentiating (1.8) w.r.t t

$$\frac{\partial\Psi}{\partial t} = -\mathrm{i}\omega Ae^{i(kx-\omega t)}\omega = 2\pi\varphi$$

$$= -i2\pi\varphi Ae^{i(kx-\omega t)} = -2\pi i\varphi\Psi$$

But, $E = \mathrm{h}\varphi$ and $\varphi = \frac{E}{h}$

$$\frac{\partial\Psi}{\partial t} = -\frac{2\pi iE\Psi}{h} = -\frac{2\pi i}{2\pi\hbar}\mathrm{E}\Psi$$

$$\frac{d\Psi}{dx} = \frac{i}{i\hbar}\mathrm{E}\Psi$$

$$\mathrm{E}\Psi = \mathrm{i}\hbar\frac{d\Psi}{dt}$$

Equation (7) is the Schrödinger time-dependent wave equation.

To obtain the Schrödinger time-dependent wave equation 3-D, we use the result of equation (7) in equation (8)

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}(\mathrm{E-V})\Psi = 0$$

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}\left[i\hbar\frac{d\Psi}{dt} - V\Psi\right] = 0$$

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}\left[i\hbar\frac{d\Psi}{dt} - V\Psi\right] =$$

Multiply through by $\frac{\hbar^2}{2m}$

$$\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} = \mathrm{i}\hbar\frac{d\Psi}{dt} + \mathrm{V}\Psi$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + \mathrm{V}\Psi = \mathrm{i}\hbar\frac{d\Psi}{dt} \qquad (10)$$

Equation (10) represents the Schrödinger wave equation in 1-D. To obtain the 3-D case, we follow the steps as equation (9)

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = \mathrm{i}\hbar\frac{d\Psi}{dt} \qquad (11)$$

Which represents the Schrödinger time-dependent equation in 3-D?

Particle in a 1-D infinite potential well, consider a particle of mass m in a 1-D box, operated by a distance and as depicted in the diagrams.

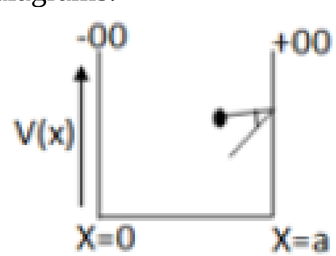

Fig. 2: particle of mass in a 1-D box

The particles inside the box do not lose energy as theycollide with the walls and its total energy remains constant. The potential energy of the particle is thus, infinitely high on both sides of the box.

Let's assume the potential V on both sides V=0 inside the box.
Then,
V(x) = 0 $\forall\ 0 < x < a$
V(x) = 00 $\forall x \leq 0\ \forall x \geq a$
In the region inside box where 0<x<a
V=0 can be described by the Schrödinger equation.

$$\frac{d^2\Psi}{dx^2} + \left(\frac{2m}{\hbar^2}\right)E\Psi=0 \quad (12)$$

$$\frac{d^2\Psi}{dx^2} + k^2\Psi=0$$

where $K = \sqrt{\frac{2mE}{\hbar^2}}$

Equation 1 has a general solution given as
V(x) A Sin Kx + B Cos Kx (13)
where A and B are arbitrary constants.
Applying the boundary condition at x=0, Ψ(x=0)=0, Ψ(x=0) =B=0
At Ψ(x<a) = Ψ(a)=0= A Sin ka=0
But when A $\neq$ 0, sin ka =0 and Ka=nπ

$$K=\frac{n\pi}{a} \quad (14)$$

But n$\neq o$when K=0, E=0 and Ψ=0 , therefore,

$$\Psi n(a) = A \text{ Sin } Kx = A \text{ Sin } \frac{n\pi x}{a} \quad (15)$$

To obtain the eigenvalue, we first observed that $K^2 = \frac{2mE}{\hbar^2}$ indicating thar $E_n = \frac{\hbar^2 k^2}{2mE_n}$, hence,

$$E_n = \frac{n^2\pi^2\hbar^2}{2mEa^2} = \frac{n^2\pi^2}{a^2}\frac{\hbar^2}{8\pi^2 m} = \frac{n^2h^2}{8m} \quad (16)$$

$$E_n = \frac{n^2h^2}{8ma^2} \quad (17)$$

The energy levelshown in equation 17 is fulfilled under the following conditions, n=1, $E_1 = \frac{h^2}{8ma^2}$ such that $E_n = n^2E_1$. When n=2, $E_2 = 2^2E_1 = 4E_1$ and $3^2\ E_1 = 9E_1$
We can observe the spacing between the energy level and the next higher level increase as

$$(n+1)^2E_1 - n^2E_1 =$$
$$(2n+n^2+I)E_1 - n^2E_1 = (2n+1)\ E_1 \quad (18)$$

Also, the wave function for the wave-particle has the solution given as $\Psi_n(x) = A \text{ Sin } \frac{n\pi x}{a} \forall 0 \leq x < a$ and $\Psi_n(x) = 0\quad \forall x \leq 0 \forall x \geq a$
The total probability that the particle in the box is somewhere in the box must be unity

$$\int_o^a p_x d_x = \int_o^a /\Psi n/2_{dx}=1$$

$$\int_0^a A^2\ Sin2\ \frac{n\pi^x}{a}\ dx=1 \quad (19)$$

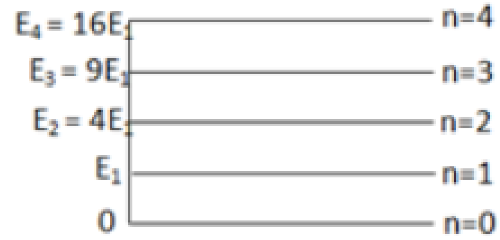

Fig. 3: Diagram of the energy level

But using the identity operation, $\sin^2 a = \frac{1}{2}$ (1-cos θ), the following expressions applies.

$$\frac{A^2}{2}\int_o^a \left[1 - Cos\frac{n\pi x}{a}\right] dx = 1$$

$$\frac{A^2}{2}\left[x - \frac{a}{n\pi} Sin\frac{n\pi x}{a}\right]_0^a = 1$$

$$\frac{A^2}{2} a = 1$$

Therefore,

$$A = \sqrt{\frac{2}{a}} \quad (20)$$

Therefore from the normalization condition

$$A = \sqrt{\frac{2}{a}}$$

Putting in equation (18)

$$\Psi n(x) = \sqrt{\frac{2}{a}} \text{ Sin } \frac{n\pi x}{a} \quad (21)$$

Equation (21) is the eigenfunction of the particle in a box. The plots of eigenfunctionare depicted below.

2.0 Materials and Methods

2.1 Schrödinger equation for a finite potential well

Solutions to the Schrödinger equation must be continuous, and continuously differentiable. These requirements are boundary conditions on the differential equations previously derived, that is, the matching conditions between the solutions inside and outside the well. (Chiani et Williams 2016)
This is the same potential as for the infinite square well, with ∞ replaced by V0; I've

shifted the well to center it at x = 0 because the resulting symmetry will slightly simplify the computer code and the description of the solutions. (This is actually an example that can be solved exactly, aside from the need to numerically solve a transcendental equation to match the wavefunction at the well boundary. But here I'll use it to illustrate the much more general method of numerically solving the TISE.(Schroeder, 2022)

Involving the Schrödinger equation for a finite potential well will produce values of the energy levels within the well.

Consider a potential well centered on the origin of width length $[L_W]$ and the barrier height V_0 $[H_b]$, inside the well the potential is zero.

The Schrödinger equation SE

$$\frac{ħ^2}{2m}\frac{\partial^2\Psi(x)}{\partial x^2} + V(x)\Psi = E\Psi \qquad (22)$$

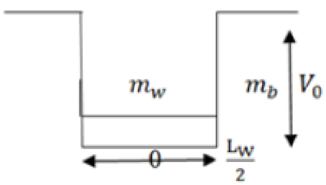

Fig. 4: Diagram Of Finite Potential Well

SE implies

$$\frac{ħ^2}{2m}\frac{\partial^2\Psi(x)}{\partial x^2} + Ķ\Psi(x) = 0 \qquad (23)$$

$X = \frac{L_W}{2}$

Inside the well the energy is larger than the potential

$$Ķ = \sqrt{\frac{2mE}{ħ^2}}$$

It implies that $\frac{\partial^2\Psi(x)}{\partial x^2} = -Ķ\Psi(x)$

SE $\longrightarrow$ $\frac{ħ^2}{2m}\frac{\partial^2\Psi(x)}{\partial x^2} - \alpha\Psi(x) = 0$

(24)

$$\boldsymbol{\alpha} = \sqrt{\frac{2m(V_0 - E)}{ħ^2}}$$

Outside the well, the energy is less than that of the edge of the well. This means it is less than the potential V_0

$\Longrightarrow \frac{d^2\Psi(x)}{\partial x^2} = \boldsymbol{\alpha}\Psi(\mathbf{x})$

The potential is an even function. If ψ'(x) is an even (odd) function, then ψ''(x) will be even (odd). So, every term in the Schrödinger equation is an even (odd) function. Therefore, the solutions are either even functions or odd functions.

2.2 Even wave function

The solution of the Schrödinger equation within the well is

$$\Psi(x) = \begin{cases} C_1 e^{+\alpha x} & |x| > -\frac{w}{2} \\ C_2\cos(Ķx) & |x| < -\frac{w}{2} \end{cases}$$

$$C_1 e^{+\alpha x} = C_2\cos(ĶX) \qquad (25)$$

$$C_1 e^{-\alpha\frac{L_W}{2}} = C_2\cos\left(Ķ\frac{L_W}{2}\right) \qquad (26)$$

Differentiate equation (25)

$$\alpha C_1 e^{\alpha X} = -ĶC_2\sin(ĶX)$$

$$\frac{\alpha}{m_b}C_1 e^{-\alpha\frac{L_W}{2}} = -\frac{Ķ}{m_w}C_2\sin(-Ķ\frac{L_W}{2})$$

$$\frac{\alpha}{m_b}C_1 e^{-\alpha\frac{L_W}{2}} = \frac{Ķ}{m_w}C_2\sin(Ķ\frac{L_W}{2}) \qquad (27)$$

Divide equation (27) by (25)

$$\boldsymbol{\alpha} = \frac{m_b}{m_w}Ķ\tan(Ķ\frac{L_W}{2}) \qquad (28)$$

3.3 *Odd wave function*

$$\Psi(x) = \begin{cases} C_1\exp[-\boldsymbol{\alpha}(x-\frac{L_W}{2})] & X > \frac{L_W}{2} \\ C_2\sin ĶX & |X| \le \frac{L_W}{2} \\ -C_1\exp\left[\boldsymbol{\alpha}\left(\boldsymbol{x}+\frac{L_W}{2}\right)\right] & X < -\frac{L_W}{2} \end{cases}$$

The boundary condition

$$C_1 e^{-\alpha x} = C_2\sin(ĶX) \qquad (29)$$

$$C_1 e^{-\alpha\frac{L_W}{2}} = C_2\sin\left(Ķ\frac{L_W}{2}\right) \qquad (30)$$

Differentiate equation (29)

$$-\alpha C_1 e^{-\alpha X} = ĶC_2\cos(ĶX)$$

$$-\boldsymbol{\alpha}\frac{\alpha}{m_b}C_1 e^{-\alpha\frac{L_W}{2}} = ĶC_2\cos Ķ\frac{L_W}{2} \qquad (31)$$

Divide equation (30) by (31)

$$-\boldsymbol{\alpha} = \frac{m_b}{m_w} Ϗ \cot\left(Ϗ \frac{LW}{2}\right) \quad (32)$$

The solution for the quantized eignenergies can be obtained by $Ϗ\frac{LW}{2}$ and $\boldsymbol{\alpha}\frac{LW}{2}$. Using a graphical approach since

$$[Ϗ\frac{LW}{2}]^2 + \frac{m_w}{m_b}[\alpha \frac{LW}{2}]^2 = \frac{2m_w V_0}{ħ^2}[\frac{LW}{2}]^2 \quad (33)$$

For the even solutions from equation (28)

$$\boldsymbol{\alpha}\sqrt{\frac{m_b}{m_w}}\frac{LW}{2} = -\frac{m_b Ϗ}{m_w}\frac{LW}{2}\tan Ϗ\frac{LW}{2} \quad (34)$$

For the odd solution from equation (32)

$$-\boldsymbol{\alpha}\sqrt{\frac{m_b}{m_w}}\frac{LW}{2} = -\frac{m_b Ϗ}{m_w}\frac{LW}{2}\cot Ϗ\frac{L_w}{2} \quad (35)$$

The allowed energies as functions of the barrier height can be found numerically by solving a transcendental equation (Murphy & Phillips, 1976)

$$Ϗ^2 + \alpha^2 = \left[\sqrt{\frac{2mE}{ħ^2}}\right]^2 + \left[\sqrt{\frac{2m(V_0-E)}{ħ^2}}\right]^2 = \frac{2mV_0}{ħ^2}$$

$$Ϗ^2 + \alpha^2 = \frac{2mV_0}{ħ^2}$$

3.0 Results and Discussion

In this study, we simulate the graphical solution of thestate of an electron in a finite quantum well using MATLAB programming. The result will be reported in this chapter.

The energy levels are found from a graphical solution of the two equations with the definition for $\boldsymbol{\alpha}$ and Ϗ given $\boldsymbol{\alpha} = +\left[\frac{2mE}{ħ^2}\right]^{\frac{1}{2}}$ and $Ϗ = +\left[\frac{2m(V_0-E)}{ħ^2}\right]^{\frac{1}{2}}$.

A simple graphical method for effecting this solution is described here since it shows quite clearly how the number of discrete levels depends on V_0 and $\frac{L_w}{2}$.

$$Ϗ^2 + \alpha^2 = \left[\sqrt{\frac{2mE}{ħ^2}}\right]^2 + \left[\sqrt{\frac{2m(V_0-E)}{ħ^2}}\right]^2 = \frac{2mLV_0}{4ħ^2}$$

$$Ϗ^2 + \alpha^2 = \frac{mLV_0}{2ħ^2} \quad (36)$$

$$\boldsymbol{\alpha} = \frac{m_b}{m_w} Ϗ \tan\left(Ϗ\frac{LW}{2}\right) \quad (37)$$

For $m_b \cong m_w$

Implies $m_b/m_w = 1$

$$\boldsymbol{\alpha} = Ϗ \tan\left(Ϗ\frac{LW}{2}\right) \quad (38)$$

$$-\boldsymbol{\alpha} = Ϗ \cot\left(Ϗ\frac{LW}{2}\right) \quad (39)$$

We can replace Ϗ and $\boldsymbol{\alpha}$ as Ϗ=ξand $\boldsymbol{\alpha} = \beta$

$$\beta^2 = \frac{2mV_0}{ħ^2}$$

For equation (38) we have

$$\beta^2 + \xi^2 = \beta^2$$

$$\beta = \xi\tan(\xi) \quad (40)$$

For equation (39) we have

$$-\beta = \xi\cot(\xi) \quad (41)$$

For equation (36) we have

$$\beta^2 + \xi^2 = \beta^2 \quad (42)$$

We substitute equations (40) into equation (42)

$$(\xi\tan(\xi))^2 + \xi^2 = \beta^2$$

$$\xi^2(\tan(\xi))^2 + \xi^2 = \beta^2$$

$$\xi^2(\tan(\xi))^2 = \beta^2 - \xi^2$$

$$(\tan(\xi)) = \sqrt{\frac{\beta^2}{\xi^2} - \frac{\xi^2}{\xi^2}}$$

$$\tan(\xi) = \sqrt{\frac{\beta^2}{\xi^2} - 1} \quad (43)$$

We substitute equation (41) into equation (42)

$$-(\xi\cot(\xi))^2 + \xi^2 = \beta^2$$

$$-\xi^2(\cot(\xi))^2 + \xi^2 = \beta^2$$

$$-\xi^2(\cot(\xi))^2 = \beta^2 - \xi^2$$

$$-\cot(\xi) = \sqrt{\frac{\beta^2}{\xi^2} - \frac{\xi^2}{\xi^2}}$$

$$-\cot(\xi) = \sqrt{\frac{\beta^2}{\xi^2} - 1} \quad (44)$$

where

$$\beta = \sqrt{\frac{2mV_0}{ħ^2}}$$

$$\xi = \frac{L_W}{2}\sqrt{\frac{2mE}{ħ^2}}$$

Table 1: Band parameters of binary semiconductors

Materials	Eg	$m_e(m_0)$
GaAs	1.424	0.067
AlAs	3.03	0.15

We consider the conduction band offset to the electron effective mass band transition only. From the Table 5 above the energy band gap Eg(eV) and electron effective mass $m_e(m_0)$

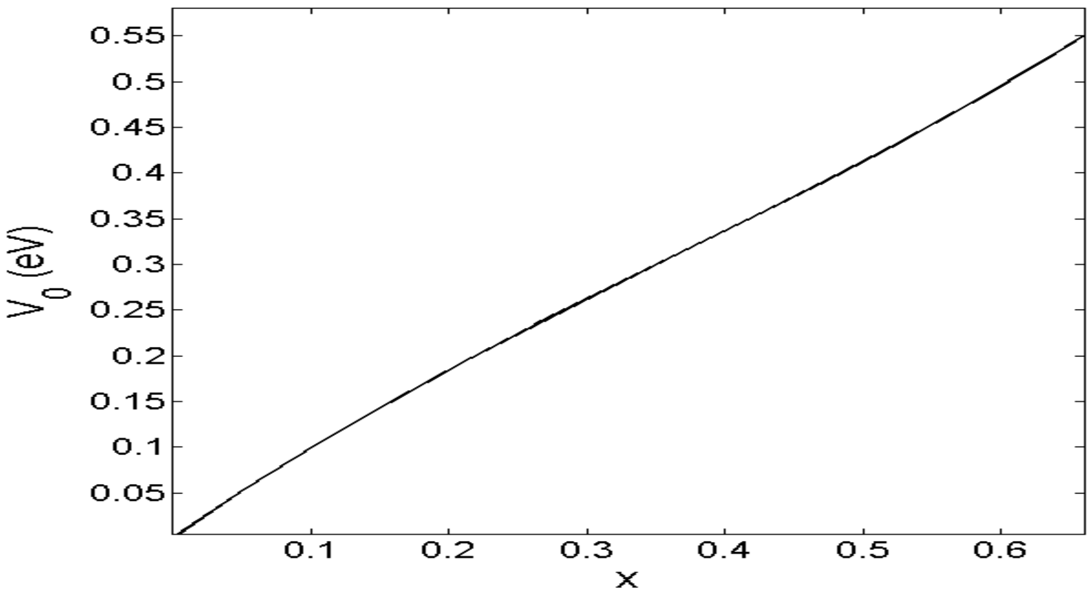

Fig. 5 : A Graph Of Band Offset Against X

From Fig.5 above we have that the band offset is plotted against x where x varies with

respect to band offset. The band offset is measured in electron volt, the graph start from the origin but its not a straight line graph because of the absorption of $Al_xGa_{1-x}As$.

V0 = e0*0.62*(1.594*xx + xx.*(1 - xx).*(0.127 - 1.310*xx));
Where x is 0.3

From the equation we can see where the band offset (conduction band offset) came about using MATLAB

Table2: Length of Width Saitta and The Energy State

x=0.3			*x=0.5*	
Lw (nm)	***§***	***En(eV)***	***Analytical E_G***	***Infinite Quantum Well***
10.0000	***0.3183***	***0.2805***	***0.3447***	***0.2183***
20.0000	***0.5526***	***0.1736***	***0.0862***	***0.0546***
40.0000	***0.8649***	***0.1063***	***0.0215***	***0.0136***
60.0000	***1.045***	***0.069***	***0.0096***	***0.0061***
80.0000	***1.129***	***0.0453***	***0.0054***	***0.0034***

From Table2, we have that x=0.5 is calculated both for analytical (graphical solution) and infinite quantum well then from our objective number 5 we have been able to compare the energies in electron volt and length of the well (nm) in both graphical method with the infinite quantum well method of the solution.

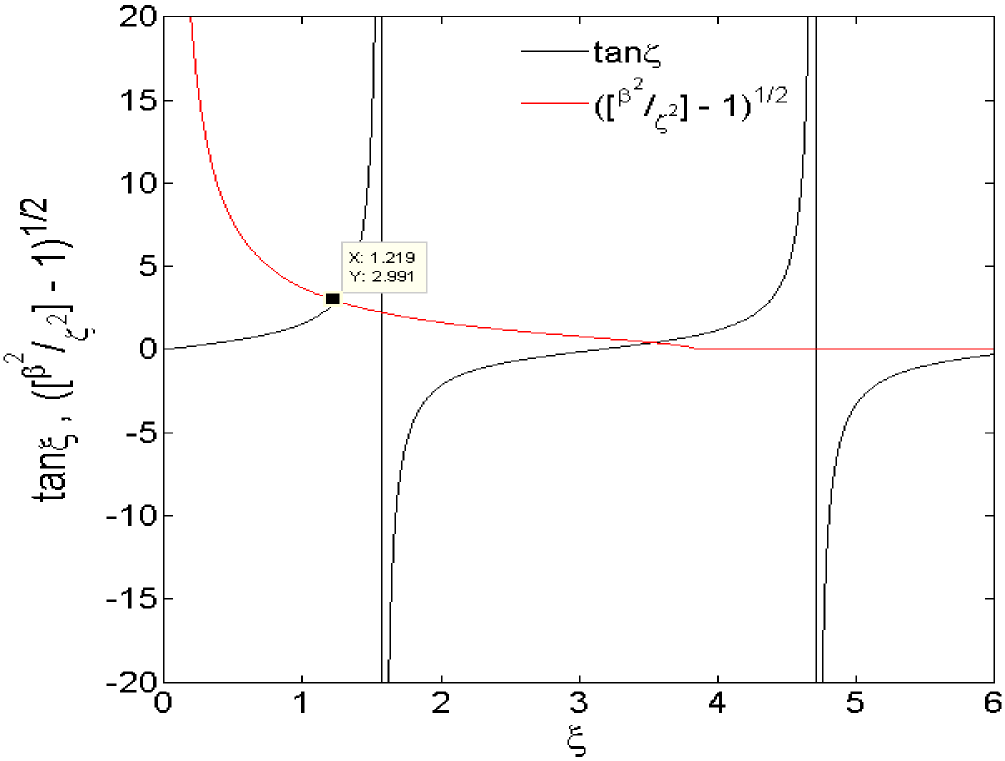

Fig. 6: A graph of tan(ξ), $\sqrt{\frac{\beta 2}{\xi 2}-1}$ against ξ

Fig. 6 is a graph which we use to get the energy of the eigenstates using the intersection of x and y axis, which leads us to the Table below.

Table3: Ground State Energy for GaAs/Al_xGa_{1-x}As Quantum Well

Length Of Width(Lw) In Quantum Well (nm)	Quantized Levels (eV)		
	0.2	0.3	0.5
10	0.1589	0.2305	0.3361
20	0.1379	0.1736	0.2437
40	0.0894	0.1063	0.1264
60	0.0591	0.069	0.0788
80	0.0402	0.0453	0.0518
100	0.0293	0.0335	0.0362

From Table3 we have that the length of width (nm) iscalculated at different quantized (energy) levels, which aided the development of the plots shown in . 7.

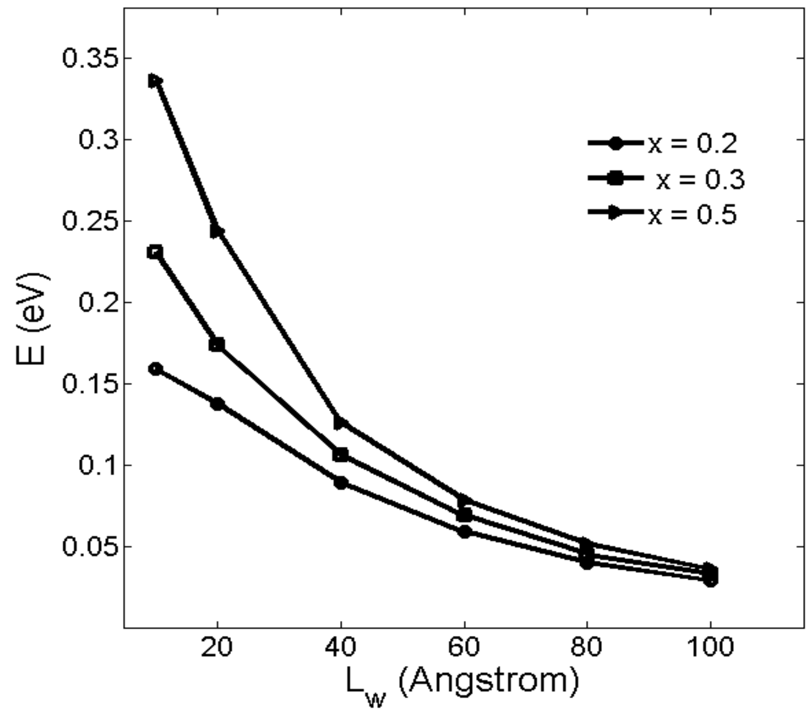

Fig. 7: A Graph Of Energy Measured In Electron Volt Against Length Of The Width Measured In Angstrom.

From Fig. 7 the energy band gap between the three quantized levels tells us that, at 0.5 and 0.3 there energy band gap is more when compared to that of 0.3and 0.2.Furthermore, Table 3 is the solution to the graph in Fig. 7.

4.0 Conclusion

The study of the graphical solution of the eigenstates of an electron in a finite quantum well has provided a deep insight into the complex behavior of quantum particles within confined potentials. By employing the Schrödinger wave equation, we derived and analyzed the eigenvalues and eigenfunctions that describe the energy states of electrons in a finite quantum well. The graphical methods used in this analysis, implemented through MATLAB, proved effective in solving for these eigenstates, particularly for illustrating how the number of discrete energy levels depends on the well parameters, such as barrier height and well width.

This study provides the following highlight

(i) Discrete Energy Levels: Unlike classical particles, electrons in a finite quantum well exhibit discrete energy levels due to quantum confinement. These energy levels are influenced by the well's dimensions and the potential barrier height, with fewer levels present as the well width decreases or the barrier height increases.

(ii) Probability Distribution: The wave functions derived from the Schrödinger equation illustrate the probability distributions of electrons within the well. The normalization of these wave functions ensures that the total probability of finding the electron within the well is unity, emphasizing the quantum mechanical principle of probability amplitudes.

(iii) Comparison with Infinite Well: The finite quantum well provides a more realistic model compared to the infinite potential well, where the probability of finding an electron outside the well is non-zero, even when the electron's energy is less than the potential barrier. This phenomenon is absent in the infinite well model, demonstrating the importance of considering finite potentials in practical applications.

(iv) Technological Relevance: The understanding of electron behavior in quantum wells has significant implications for the development of various semiconductor devices, including quantum well lasers, transistors, and LEDs. The precise control over electron states and energy levels in these devices underscores the importance of quantum mechanical principles in modern technology.

In conclusion, the graphical method used in this study provides a robust approach to solving for the eigenstates of electrons in a finite quantum well, offering a clear visualization of how well parameters affect the energy levels. This method, along with the detailed mathematical derivations, contributes to a deeper understanding of quantum confinement and its applications in semiconductor physics. As technology continues to evolve, such quantum mechanical analyses will remain pivotal in the innovation and enhancement of electronic and optoelectronic devices.

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Compliance with Ethical Standards Declarations

The authors declare that they have no conflict of interest.

Data availability

All data used in this study will be readily available to the public

Availability of data and materials

The publisher has the right to make the data public.

Competing interests

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Authors' contributions

Akwuegbu Ozochi Chinyere carried out sorting, review, Implementation and explanations. Oriaku Chijioke carried out analysis, review and implementation. Dinneya Obinna Chrisian, Nkpoku Emmanuel Chidiebere and Nwaehiodo Immaculate Ihechi carried out review, implementation and conclusion.