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A Mathematical Model of Fluid Dynamics in the Ear due to External Noise

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Abstract

A mathematical model of fluid dynamics in the Cochlea was formulated and solved using the potential flow theory. The obtained solutions of the model representing the flow in the Cochlea shows that the flow looks chaotic, but in reality, it is so because of the multi-dimensional nature of the variables associated with noise. Also determined is the nature of the flow at both chambers of the cochlea where we saw that the magnitudes of the velocity potentials are the same with only a difference in sign denoting their positions in relation to the basilar membrane. We observed that the velocity potential in the horizontal axis increases with distance from the base of the basilar membrane when the flexural rigidity varies as the distance increases. Other analyses were also carried out to confirm the experimental evidences about the effect of noise in the ear.

Key words: *Noise, Cochlea, Potential Theory, Fluid Motion, Hearing*

1. Introduction:

This paper presents a mathematical model on the mechanism of transmission of the noise in the ear through the fluid found in the inner ear. We define noise here as the sound that cause discomfort to the hearer but this definition is not scientifically enough as noise to one person may not be so for another, (Chalupnik ,1977). Therefore, put more scientifically, noise can be defined as sound that measures above 85db (db means decibel, the unit of measurement of sound). Most noise come as irregular vibrations. Every sound has two major aspects which are the frequency or pitch and the intensity or amplitude, (Cheremisionoff & Cheremisionoff,1978). The frequency is determined by how rapidly the generated sound waves vibrate. In terms of the sound intensity, the human ear has a wide range to which it can be exposed. The range is from one billionth (10^{-9}) Watt to 10^{7} Watt. Silence which we describe as the arbitrary threshold level of sound is represented by zero decibel while the faintest sound audible to the human ear is represented by 1db.

The greatest physiological effect of noise in a man is temporary deafness or permanent hearing loss, (Rau & Woolen, 1980) and increase in blood pressure. Other effects are discomfort, tiredness, stress and feeling of irritation, aplynopsys (inability to sleep), low blood resistance to diseases and many more others. Noise has been implicated as one of the causes of Ulcer and the allergies like hives. It can also lead to somatic manifestations such as gastric acid problems. In particular,, temporary exposure to noise can lead to impairment in hearing which is termed auditory fatigue while exposure to noise for a very long time without enough time for recovery usually leads to permanent hearing loss.

In general therefore, we see noise as a multidimensional problem because of multiple ways it can be interpreted and understood. Hence, a proper mathematical model and analysis of noise involves associating a random variable (as many variables are involved in propagating noise) with multidimensional physical processes causing the noise.

Mechanism of Hearing

The details of the structure of the ear can be found in the work of Adagba (2005), Burtons & Hopkin (1983). For a given sound to be heard there must be the presence of air medium. Thus when a sound is generated, it results in generation of waves as a result of the vibration of the object that produced the sound. These sound waves are collected by the pinnae and directed through the external auditory meateus to impinge on the tympanic membrane or the ear drum. This vibrates the eardrum and then passes the energy of vibration on to the middle ear. The middle ear is open to the throat through the Eustachian tube so that the air pressure through this tube equalizes the air pressure on both sides of the eardrum and then the transfer of the energy of vibration from the outside of the eardrum to the middle ear is possible. At the middle ear, we have a series of three bones which help in the transfer of the vibrations coming from the outer part of the eardrum to the inner part. These bones are the **malleaus, incus and stape**. These bones are connected such that they connect the tympanic membrane with the oval window of the cochlea which is the organ where the vibrations are again converted into energy impulse. In the course of conducting the vibrations to the cochlea by these bones, the vibration is increased in strength or rather amplified. It is known that sound pressure received at the tympanic membrane is relatively lost as most is transmitted at the cochlea where it is amplified to a 22-fold greater pressure, (Burtons and Hopkin, 1983; Wegel et al ,1932; Wever and Lawrence,1930b, 1950a). The mechanical forces that are transferred by the bones of the middle ear are transformed into the hydraulic pressure when the stape strikes the oval window. Since the oval window is filled with fluid, this pressure applied to the oval window by the striking of the stape is transmitted through this fluid which eventually causes a vibration of the **basilar membrane**, a slight structure which extends from the cochlea to the auditory nerves and totally lying in the fluid. This region of the ear is called the inner ear. The basilar membrane has **Hair cells** on its surface such that the vibration of the fluid and thus the basilar membrane produces shearing movement between the hair cells and the tectoral membrane of the organ of Corti . This initiates wave impulse in the fibres of the auditory nerve. It is generally agreed that sound waves are analysed at the cochlea and that each frequency has its own place in the basilar membrane. The auditory nerve will then send this received impulse to the brain for interpretation and subsequent response to it. Mathematically we shall not go into how we respond to this noise here but rather we shall look at how this noise is received and how the ear structures respond to it.

In this paper therefore, we wish to draw a mathematical model on the fluid motion due to pressure received at the cochlea as a result of a given noise that generated sound waves and picked up by the ear. Further work will show how this pressure deforms the basilar membrane which results in impulse generation and transfer to the brain through the auditory nerves.

The Fluid Mechanics of the Cochlea.

As stated earlier, the cochlea is the part of the inner ear which is a small fluid-filled chamber and contains the biological structures that convert mechanical signals into neural signals. In addition to the signal conversion, it does process signals. Thus, a clear understanding of the mechanism requires that we understand the cochlea fully as it relates to audition, (Lesser and Berkkley, 19972; Ranke, 1950b and Lamb, 1904).

To model the fluid dynamics in the cochlea, the following assumptions are made:

1. The model is a two dimensional model in an enclosed cavity containing a structure of spatial variable elastic properties.
2. The spiral cochlea is unwound
3. The central duct in the cochlea which contains the Corti and which is enclosed by Reisner's membrane and basilar membrane will be represented by a single elastic partition.
4. The mechanical properties of each partition are represented by the assumption that each point acts as a damped harmonic oscillator point to point, coupling being only through the surrounding fluid. This assumption leads to representing the partition by a mechanical impedance $z(x_1 ,t)$, x_1 being the distance from the oval window along the partition and t, the time.
5. The endolymph is considered incompressible for it has the same sound speed as water. The wavelength of an acoustic signal at 500Hz (at high frequency for hearing) is about 30 cm while the cochlea is only 35cm (Goldstein, 1967).
6. The fluid flow will be considered inviscid though we shall regard this as a first step in an expansion procedure.
7. The endolymph is considered inviscid
8. One point that is needed to be remembered is the fact that the motion of the basilar membrane is small, a displacement of about 10^{-6}cm corresponds to normal amplitude of sound.
9. Man can detect sound corresponding to the basilar membrane and eardrum

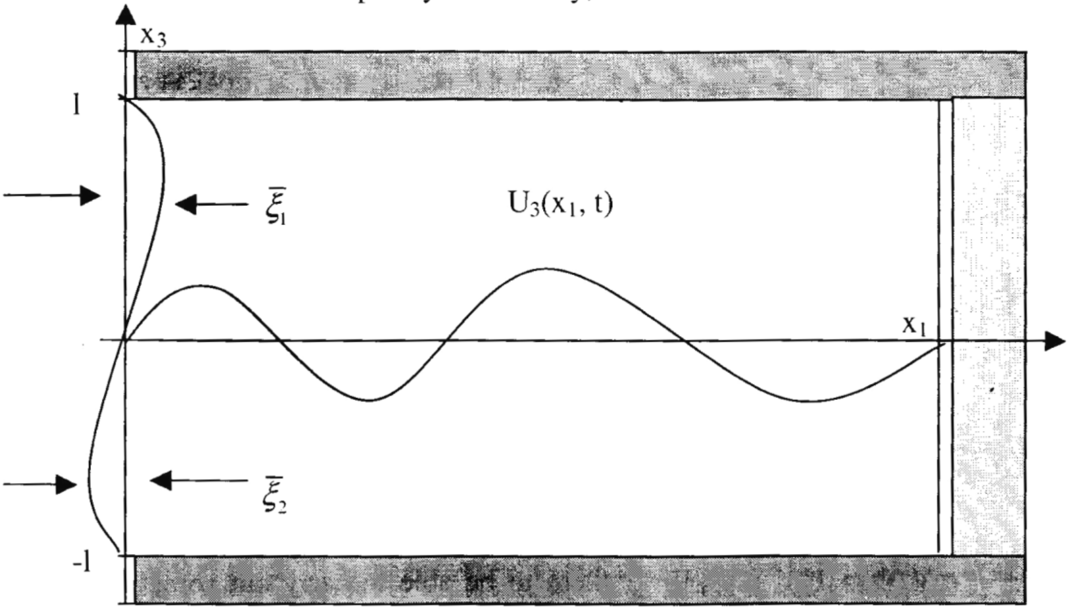
displacement of 10^{-10}cm. Also known is the fact that non-linearity does exist in mechanics, though many of these are to become noticeable over a long period of time. The works of Goldstein (1967), Goblick and Pfeiffer (1969) in electrophysiology include the presence of non-linearities in cochlea mechanics.

10. There exists non-linearity caused by eddies in the cochlea and this is different from the above linearity talked about. This eddy called the Bekesy eddy, is understood as resulting from the combination of viscosity and non-linear effects.

From all these, we have that the flow pattern in cochlea model excited by an oscillatory disturbance exhibits a steady streaming motion as well as motion typical of a fluid with a free surface. As the excitation is purely oscillatory, the steady motion must result from a non-linea interaction as can be seen from the works o Morse (1948), Pain (1976) and Rhode (1971).

The Mathematical Model.

To be able to get the required model that will describe the cochlea dynamics, we assumed that the basilar membrane motion is primarily controlled by potential flow. Physical measurements by Von Bekesy (1947) shows that the maximum basilar membrane slope is sufficiently small and this supports the linearization of the equations involved. Our model here is for an enclosed two-dimensional cavity and the basilar membrane appears in it as a thin plate immersed in the fluid in the cavity. We shall consider the linear short-time scale aspect of the cochlea behaviour. Thus, for the figure below, we assume a linearised two-dimensional potential flow, that is:



POTENTIAL FLOW MODEL OF THE COCHLEA

In this diagram, $\bar{\xi}_1$ and $\bar{\xi}_2$ are the displacements of the oval and round windows respectively and we equally denote the upper domain where $x_3 > 0$, by the subscript 1 and the lower domain $x_3 < 0$, we use the subscript 2. x_3 denotes the height of the wave in the cochlea while x_1 is the distance traveled by the wave.

Using the work of Green and Naghdi (1967), the equation of motion of such plate is given by:

$$\alpha_3 u_{3,\alpha\alpha} + F_3 = \rho \ddot{u}_3 \qquad (1)$$

where α_3 is the elastic constant or the flexural rigidity of the hair cells in the basilar membrane, ρ is the mass density of the basilar membrane, u_3 is the velocity of the vibrating membrane while F_3 is the load on the membrane which can be viewed as external forces acting on the membrane and α denotes the coordinate of interest. Lesser and Berkkley, (1972) gave the equations of motion characterizing the flow of a non-viscous incompressible fluid in such cavity as:

$$\frac{\partial U}{\partial x_1}+\frac{\partial U}{\partial x_3}=0 \tag{2}$$

$$\rho\left\{\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x_1}+V\frac{\partial U}{\partial x_3}\right\}=-\frac{\partial P_1}{\partial x_1} \tag{3}$$

$$\rho\left\{\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x_1}+V\frac{\partial V}{\partial x_3}\right\}=-\frac{\partial P_2}{\partial x_3} \tag{4}$$

where ρ is the fluid density, P_i is the fluid pressure, i = 0,1,2 and U and V are the velocities on both the x_3 and x_1 axes respectively.

Since the flow motion of interest is that motion with small amplitude, we neglect the product terms in equations (3) and (4) and we obtain

$$\rho\frac{\partial U}{\partial t}=-\frac{\partial P_1}{\partial x_1} \tag{5}$$

$$\rho\frac{\partial V}{\partial t}=-\frac{\partial P_{21}}{\partial x_3} \tag{6}$$

We shall assume a velocity potential Φ such that $\nabla\Phi=(\overline{U},\overline{V})$ where $\overline{U}$ *and* $\overline{V}$ are the x_1 and x_3 fluid velocity components and equation (2) is satisfied.
Hence,

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial t}\left(\frac{\partial\overline{\Phi}}{\partial x_1}\right)=\frac{\partial}{\partial x_1}\left(\frac{\partial\overline{\Phi}}{\partial t}\right) \tag{7}$$

$$\frac{\partial V}{\partial t}=\frac{\partial}{\partial t}\left(\frac{\partial\overline{\Phi}}{\partial x_3}\right)=\frac{\partial}{\partial x_3}\left(\frac{\partial\overline{\Phi}}{\partial t}\right) \tag{8}$$

If we substitute $U=\frac{\partial\overline{\Phi}}{\partial x_1}$ *and* $V=\frac{\partial\overline{\Phi}}{\partial x_3}$ into equation (2), we get

$$\frac{\partial U}{\partial x_1}+\frac{\partial V}{\partial x_3}=\frac{\partial^2\overline{\Phi}}{\partial x_1^2}+\frac{\partial^2\overline{\Phi}}{\partial x_3^2}=\nabla_i^2\overline{\Phi}=0 \tag{9}$$

This implies that in the upper and lower chambers,

$$\nabla^2_{(i)}\overline{\Phi}_1=\nabla^2_{(i)}\overline{\Phi}_2$$

where $\nabla^2_{(i)}=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}$ (10)

We note here that $\overline{\Phi}_1$ and $\overline{\Phi}_2$ are the potentials at the upper and lower chambers of the Cochlea respectively. Substituting equations (7) and (8) into equations (5) and (6) respectively, gives us:

$$\frac{\partial}{\partial x_1}\left(\frac{\partial\overline{\Phi}}{\partial t}\right)+\frac{1}{\rho}\frac{\partial\overline{P}_1}{\partial x_1}=\frac{\partial}{\partial x_1}\left(\frac{\partial\overline{\Phi}}{\partial t}+\frac{\overline{P}_1}{\rho}\right)=0 \tag{11}$$

and

$$\frac{\partial}{\partial x_3}\left(\frac{\partial\overline{\Phi}}{\partial t}\right)+\frac{1}{\rho}\frac{\partial\overline{P}_2}{\partial x_3}=\frac{\partial}{\partial x_3}\left(\frac{\partial\overline{\Phi}}{\partial t}+\frac{\overline{P}_2}{\rho}\right)=0 \tag{12}$$

Thus, we have the pressure equations in the upper and lower chambers as

$$\rho\frac{\partial\overline{\Phi}_1}{\partial t}+\overline{P}_1=0 \tag{13}$$

$$\rho\frac{\partial\overline{\Phi}_1}{\partial t}+\overline{P}_2=0 \tag{14}$$

Now the boundary conditions on $\overline{\Phi}$ at $x_1 = L$ and $x_3 = l$, are:

$$At\quad x_1=L,\qquad \frac{\partial\overline{\Phi}_1}{\partial x_1}=\frac{\partial\overline{\Phi}_2}{\partial x_1}=0$$

$$At\quad x_3=l,\qquad \frac{\partial\overline{\Phi}_1}{\partial x_3}=0$$

$$At\quad x_3=-l,\qquad \frac{\partial\overline{\Phi}_2}{\partial x_3}=0$$

L = end of the Basilar membrane and the beginning of the auditory nerve.

$\pm\, l$ = the walls of the Cochlea or the auditory channel

Having derived these equations let us then solve them to be able to carry out some analysis and discussions.

Solution of the Equation of Motion of the Fluid

The equations of the motion of the fluid in both chambers was given as

$$\nabla^2_{(i)}\overline{\Phi}=\frac{\partial^2\overline{\Phi}}{\partial x_1^2}+\frac{\partial^2\overline{\Phi}}{\partial x_3^2}=0 \tag{9}$$

as above

This equation is a Laplacian equation which is a second order partial differential equation with constant coefficients and of the type designated as elliptic equations in the Potential flow theory. Let us assume expressions for Φ_1 and Φ_2 and see whether they satisfy this differential equation. Thus, suppose

$$\overline{\Phi}_1(x_1,x_3)=(k_1\cos\lambda_1x_1+k_2\sin\lambda_1x_1)(A_1\cosh\lambda_1x_3+B_1\sinh\lambda_1x_3) \quad (15)$$

$$\overline{\Phi}_2(x_1,x_3)=(k_1\cos\lambda_2x_1+k_2\sin\lambda_2x_1)(A_2\cosh\lambda_2x_3+B_2\sinh\lambda_2x_3) \quad (16)$$

In these two equations above, k_i, A_i, and B_i are the amplifying factors of the noise while λ_i are measures of the flexural rigidity of the hair cells due to the generated noise.
We can see that they satisfied equation (9) respectively when substituted into it appropriately. Hence we call them possible solutions to the differential equation (9). These solutions and their derivatives with respect to x_1 are finite and continuous at all points except possibly at some points on the boundary of the field. Thus, the smoothness of the velocity distribution is ensured at all points of the fluid except at these points stated; Barbel et al (2002), Bell and Holmes (1986b), Gupta (1987), Harold (1982).

Using the boundary conditions as stated above, we then have that:

$$\frac{\partial\overline{\Phi}_1}{\partial x_1} = (-\lambda_1k_1\sin\lambda_1x_1+\lambda_1k_2\cos\lambda_1x_1)(A_1\cosh\lambda_1x_3 + B_1\sinh\lambda_1x_3)$$

so that at $x_1 = L$, and $x_3 = 0$, we have:

$$0 = -\lambda_1k_1\sinh\lambda_1L + \lambda_1k_2\cosh\lambda_1L$$

and for $\lambda_1 \neq 0$, we get that

$$k_2 = \frac{k_1\sin\lambda L}{\cos\lambda L} \quad (17)$$

Similarly, on $x_3 = l$, and $x_1 = 0$ we have that:

$$\frac{\partial\overline{\Phi}_1}{\partial x_3} = (\bar{k}_1\cos\lambda_1x_1+\bar{k}_2\sin\lambda_1x_1)(\lambda_1A_1\sinh\lambda_1x_3+\lambda_1B_1\cosh\lambda_1x_3)$$

$$\Rightarrow \quad 0 = A_1\sinh\lambda_1l + B_1\cosh\lambda_1l$$

$$i.e \quad B_1 = \frac{-A_1\sinh\lambda_1l}{\cosh\lambda_1l} \quad (18)$$

Substituting equations (17) and (18) into equation (15), we get

$$\overline{\Phi}_1(x_1,x_3) = \frac{\beta\cos\lambda_1(L-x_1)\cosh\lambda_1(l-x_3)}{\cos\lambda_1L\cosh\lambda_1l} \quad where\ \beta = k_1A_1 \quad (19)$$

In a similar way, equation (16) will give

$$\overline{\Phi}_2(x_1,x_3) = \frac{\gamma\cos\lambda_2(L-x_1)\cosh\lambda_2(l+x_3)}{\cos\lambda_2L\cosh\lambda_2l} \quad where\ \gamma=\bar{k}_1A_2 \quad (20)$$

To obtain the expression for λ, we solve the homogeneous part of the equation of the Basilar membrane, (Adagba, 2005), which is given as:

$$\frac{\partial^2\bar{u}_3}{\partial x^2_1} - \frac{\rho s^2\bar{u}_3}{\alpha_3} = \frac{\rho s(\beta-\gamma)\cos\lambda_i(L-x_1)}{\alpha_3\cos\lambda_iL}$$

to obtain $\bar{u}_3 = A\cosh\lambda_{io}x_1 + B\sinh\lambda_{io}x_1$ for i = 1, 2.

where $\lambda_{io} = i\omega\sqrt{\frac{\rho}{\alpha_3}}$

To be able to obtain the value of A and B in the solution above, we define λ_i , (Titchmarch, 1986 ; Barrett and Wylie, 1995) , as

$\lambda_i = \omega\sqrt{\frac{\rho}{\alpha_3}}$ where ω is the wave number

For convenience, let $\lambda_1 = \lambda_2$ although we know that α_3, the flexural rigidity of the hair cells on the basilar membrane, is not the same at x_1 and x_3.

Hence,

$$\overline{\Phi}_1(x_1,x_3,t) = \frac{\beta\cos wt\cos\omega\sqrt{\frac{\rho}{\alpha_3}}(L-x_1)\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(l-x_3)}{\cos\omega\sqrt{\frac{\rho}{\alpha_3}}L\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}l} \quad (21)$$

$where\ \beta = k_1A_1$

and

$$\overline{\Phi}_2(x_1,x_3,t) = \frac{-\beta\cos wt\cos\omega\sqrt{\frac{\rho}{\alpha_3}}(L-x_1)\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(l+x_3)}{\cos\omega\sqrt{\frac{\rho}{\alpha_3}}L\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}l} \quad (22)$$

$where\ \beta=\gamma$

Going by the expression for λ_i, we assume it to have the same value at both x_1 and x_3. Equations (21) and (22) are the solutions of the fluid equation in potential form where the velocities can be determined very easily. The term $\cos\omega t$ is to take care of the fluctuation in the flow with respect to time. If the noise is not a function of time in which case t can be assumed to varnish, then cos 0 = 1 and we have our equations (19) and (20) still describing our potentials at both sides or chambers with appropriate substitution for λ.

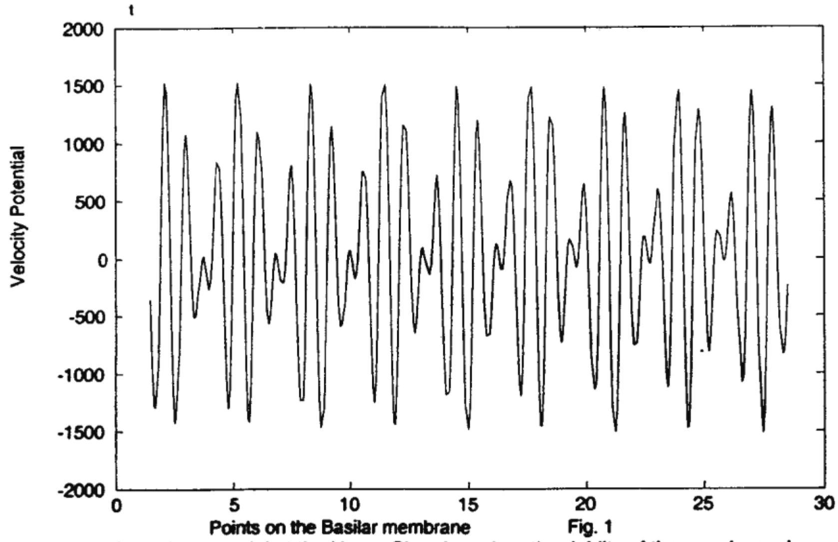


Fig. 1

Velocity potential at the Upper Chamber when the rigidity of the membrane is constant

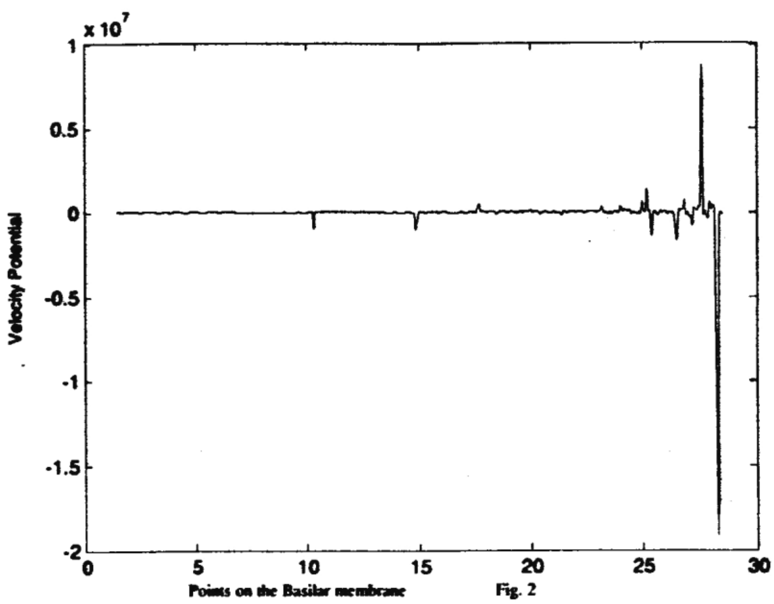


Fig. 2

Velocity Potential at the Upper Chamber with decrease in the membrane rigidity towards the end of the membrane

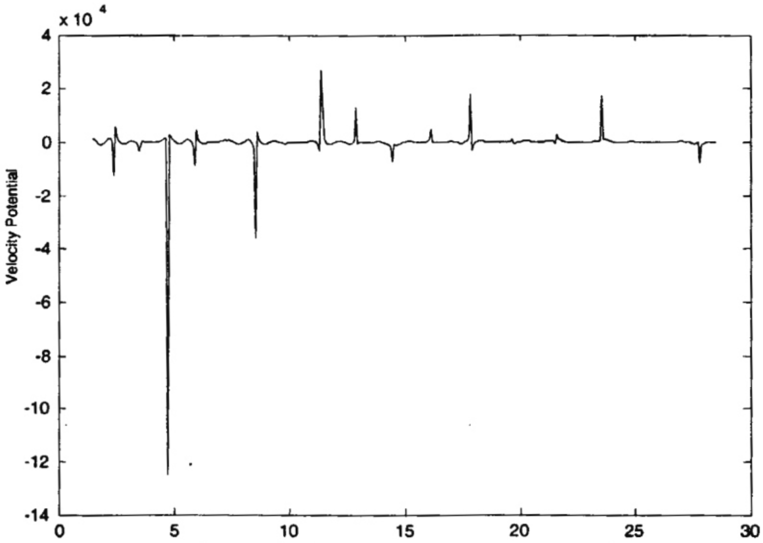


Fig. 3
Velocity Potential at the Upper Chamber with increase in the flexural rigidity towards the end of the membrane

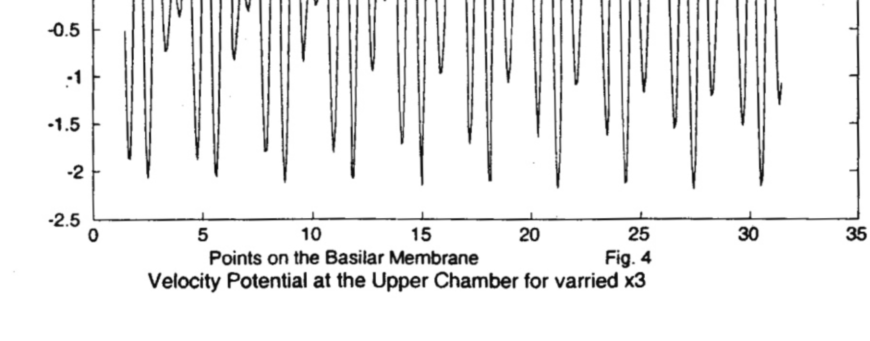
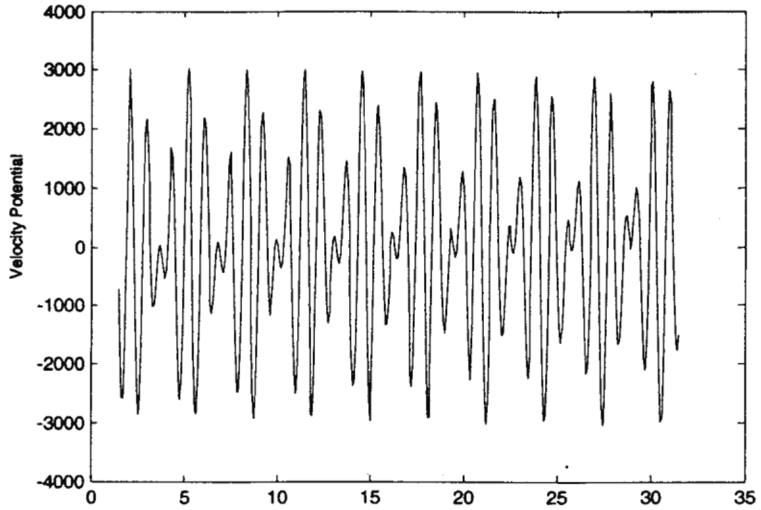
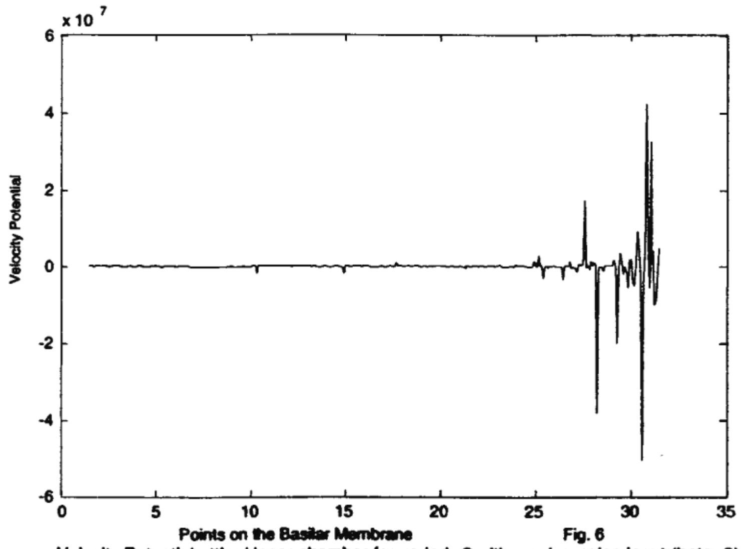


Fig. 4
Velocity Potential at the Upper Chamber for varried x3



Points on the Basilar Membrane Fig. 5
Velocity Potential at the Upper Chamber for change in Noise input (beta=2)



Points on the Basilar Membrane Fig. 6
Velocity Potential atthe Upper chamber for varied x3 with varying noise input (beta=2)

Analysis of Results

At the initial time of formulation of the model, we had denoted the fluid variables in the upper domain where $x_3 > 0$ by the subscript 1 and those at the lower domain where $x_3 < 0$ by the subscript 2 while the velocity components for the x_1 and x_3 directions or axis are U and V respectively. Thus, we can reduce the velocities obtained in equations (21) and (22) to the velocity components in the upper and lower domains to see whether the structures of the Cochlea in these domains in any way alter the flow motion and thus velocities there. Hence from equation (21), we get that:

$$U_1 = \frac{\partial\Phi_1}{\partial x_1} = \frac{\omega\sqrt{\frac{\rho}{\alpha_3}}\beta\cos\omega t\,\sin\omega\sqrt{\frac{\rho}{\alpha_3}}(L-x_1)\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(l-x_3)}{\cos\omega\sqrt{\frac{\rho}{\alpha_3}}L\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}l} \quad (23)$$

$$U_2 = \frac{\partial\Phi_2}{\partial x_1} = \frac{-\omega\sqrt{\frac{\rho}{\alpha_3}}\beta\cos\omega t\,\sin\omega\sqrt{\frac{\rho}{\alpha_3}}(L-x_1)\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(l+x_3)}{\cos\omega\sqrt{\frac{\rho}{\alpha_3}}L\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}l} \quad (24)$$

We also use equation (22) to get V_1 and V_2 as

$$V_1 = \frac{\partial\Phi_1}{\partial x_3} = \frac{-\omega\sqrt{\frac{\rho}{\alpha_3}}\beta\cos\omega t\,\cos\omega\sqrt{\frac{\rho}{\alpha_3}}(L-x_1)\sinh\omega\sqrt{\frac{\rho}{\alpha_3}}(l-x_3)}{\cos\omega\sqrt{\frac{\rho}{\alpha_3}}L\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}l} \quad (25)$$

$$V_2 = \frac{\partial\Phi_2}{\partial x_1} = \frac{-\omega\sqrt{\frac{\rho}{\alpha_3}}\beta\cos\omega t\,\cos\omega\sqrt{\frac{\rho}{\alpha_3}}(L-x_1)\sinh\omega\sqrt{\frac{\rho}{\alpha_3}}(l+x_3)}{\cos\omega\sqrt{\frac{\rho}{\alpha_3}}L\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}l} \quad (26)$$

From equations (21) and (22), we also obtain the complex Velocity potential field of flow in the upper chamber (thus on the surface of the basilar membrane) as:

$$U + iV = \frac{\partial\overline{\Phi}_1}{\partial x_1} - i\frac{\partial\overline{\Phi}_1}{\partial x_3}$$

$$= \frac{\omega\sqrt{\frac{\rho}{\alpha_3}}\cos\omega t\left\{\sin\omega\sqrt{\frac{\rho}{\alpha_3}}(L-x_1)\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(l-x_3) - \cos\omega\sqrt{\frac{\rho}{\alpha_3}}(L-x_1)\sinh\omega\sqrt{\frac{\rho}{\alpha_3}}(l+x_3)\right\}}{\cos\omega\sqrt{\frac{\rho}{\alpha_3}}L\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}l} \quad (26)$$

We can see from equations (23) to (26) above that these variables U_1, U_2, V_1 and V_2 will be unbounded if $\cos\omega\sqrt{\frac{\rho}{\alpha_3}}L = 0$. This implies that

$$\omega = \frac{(2n-1)\pi}{2L}\sqrt{\frac{\alpha_3}{\rho}}, \quad n = 1, 2,$$

…., where the velocities are unbounded.

Hence, the solutions obtained so far exist and have meaning only if

$$\omega \neq \frac{(2n-1)\pi}{2L}\sqrt{\frac{\alpha_3}{\rho}}.$$

which ensures that the velocities are bounded.

Equations (23) and (25) denote the velocity components in the horizontal (U) and vertical (V)- axes at the upper chamber while equations (24) and (26) are for the case of the lower chamber respectively.

From these equations therefore, we have been able to determine the velocity potentials for both chambers. We saw that they are of the same magnitude, although of opposite signs. We equally can see that as far as ω is as predefined, the velocity potentials and all their derivatives with respect to x_1 are finite and continuous at all points except possibly at some points on the boundary of the flow field where w fails. Hence, the smoothness of the velocity distribution is ensured at all points of the fluid.

To be able to see these conclusions or results more clearly, we plot the curves representing these velocity potential expressions. To do this, certain values have to be chosen for the constants. Thus, adapting values from the work of Lesser and Berkley ((1972), we have that $\alpha_3=2$ (Calculated), $|\omega| = 10^4$, $\rho = 1$, $L = 35mm$, $\beta=1$.Using these values, although with appropriate variations, we arrived at the following figures and then conclusions:

In Figure 1, we saw that when we assume that the basilar membrane has constant flexural rigidity, the wave generated by the fluid motion in the cochlea as a result of noise externally, has the same shape at all points of the basilar membrane. This means that there is equal transmission of the noise to all points of the basilar membrane. Experimentally, this is not true. Hence, we considered a case where the flexural rigidity decreases with x_1. This is shown in Figure 2. Here, the peaking of the wave increases with x_1 and this goes to confirm that the noise is not equally received at all points of the basilar membrane and therefore agrees with known experimental evidence,(Rhode, 1971). To still be sure of our conclusion here, we considered a case where the flexural rigidity increases with x_1 as shown in Figure 3. Here the wave peaks very close to the entrance of the inner ear, which is the beginning of the basilar membrane. This appears unrealistic for a normal person although this may be true for one who is hard-at-hearing. Thus, for one who is partially deaf, Figure 3 appears real for the reception of the noise in his/her ear.

In Figure 4, we assumed the same flexural rigidity but varied the x_3. The figure appears the same with Figure 1 with the difference only in the magnitude of the velocity potential. This is realistic enough as we do not expect a shift in the wave form but may be in terms of magnitude.

To test for the effect of the noise input in the nature of the generated wave in the ear, we varied the value of β. Thus, for $\beta=2$, the wave form becomes highly magnified for the case where the flexural rigidity of the basilar membrane was assumed constant. It actually doubled. In Figure 6, we considered the case where the flexural rigidity decreases with increase in x_1. We obtained the same result that the wave form will be highly magnified and in this case not necessarily doubled but even higher than that. From this two results therefore, it follows that since the flexural rigidity reduces with increase in x_1 and since the basilar membrane is elastic, then there must be certain level of β that must be dangerous to the hearing mechanism. This is because at such level, the elasticity of the basilar membrane will be broken and there will be no noise impulse transmission to the brain for interpretation thereby leading to deafness. This therefore corroborates the experimental evidence that intense noise destroys the mechanism of hearing or leads to temporary deafness, (Cheremisionoff & Cheremisionoff, 1978)

Concluding therefore, we may say that noise effect in the ear is transmitted effectively in the Cochlea via these fluid equations which we have solved to obtain the velocity potentials. As we have seen certain level of the noise may damage the hearing ability of an individual but will not distort the flow mechanism and the velocity of flow in the Cochlea as shown here. This is so because the basilar membrane is immersed in the fluid and the rocking of the foot of the **stapes** on the cochlea must transmit the noise to the fluid in the cavity in the form of vibration, (Bell & Holmes, 1986b; Lesser and Berkley, 1972; Rhode, 1971). It is this vibration that sets up this motion in the fluid. Therefore, whether the hearing ability in an individual is lost or not does not in any way affect the establishment of fluid motion in the Cochlea once there is noise or even sound entering the ear.

References

Adagba, O.H. : Mathematical model describing the effect of noise in the cochlea. Ph.D. Thesis at the University of Nigeria, Nsukka. 2005.

Barbel, Herrnberger; Stefan Kempf and Gunter Ehret: Basic Maps in the auditory Stefan Kempf Midbrain: Bio. Lynbern. 87, Gunter Ehret 231- 240, Springer-Verlag 2002.

Barrett, L. C. & Wylie, C. R. : Advanced Engineering Mathematics. McGraw Hill Book Company, New York. 1995

Bell, J & Holmes, M.H. (1986b): Modeling auditory nerve response due to strain activated transduction mechanism. In conf. Proc. of the twelfth Winter congress. Acoustics Toronto Can.

Burtons, G & Hopkin J.W: Understanding Biology. Harcourt Brace, Jovanovich Inc. New York, 1983

Chalupnik, J.D.: Transportation noise: In a symposium on acceptability criteria, Michigan. Ann Arbor Science publisher Inc. 8,1977

Cheremisionoff, P.N. & Cheremisionoff, P.P. : Industrial noise control book, Michigan, Ann Arbor Science publisher Inc. 9,1978

Goblick, J.J & Pfeiffer, R.R (1969): Time domain measurements of cochlea non-linearities using combination click stimuli. J Acoust. Soc. Am. 46, 124.

Golstein, J. L. (1967): Auditory non-linearity. J. Acoust. Soc. Am. 41, 76

Green, A.E. & Naghdi, P.M. (1967): Micro polar and director theories of plates. Quart. Mech. Appl. Math.26, 183-202.

Gupta, B. D.: Mathematical Physics, Viskas Pub. House, Prt, Ltd India, 1987.

Harold, T. D. : Introduction to Non-linear differential equations. Dover pub. Inc. NY. 1982

Lamb, H (1904): On deep-water waves. Proceeding of the London Math. Soc. Series 2, 2, 371-400

Lesser, M. B. and Berkley, D.A. (1972): Fluid Mechanic of the Cochlea Part I. J. Fluid Mech. 51, 3, 497-512

Lesser, M. B. and Berkley, D.A. (1976): A Simple Mathematical Model of the Cochlea, Proc. 7th Am. S. E. S. meeting (ed A.C. Eringen), Reprinted

Morse, P. M.: Vibration and sound, Second edition, New York, McGraw-Hill Ltd, 1948

Pain, H. S.: The Physics of vibrations and waves, 2nd edition, John Wiley and Sons Ltd, London, 1976.

Ranke, O. F. (1950b): Theory of Operation of the Cochlea. J. Acoust. Soc. Am. 22, 772-777

Rau, J.G. & Woolen, D. C. : Environmental impact analysis handbook, New York and London, McGraw-Hill, 1980

Rhode, W. S. (1971): Observations on the vibration of the basilar membrane in squirrel monkeys using the mosbaur technique. J. Acoust. Soc. Am. 49, 1218-1231

Titchmarch, E. C.: The theory of functions, Oxford University Press, Second Edition, New York. 1986

Von Bekesy (1947): The vibration phase along the basilar membrane with sinusoidal variation, J. Acoust. Soc. Am. 19, 452-460

Wegel, G.; Reisz, R. K. & Blackman, R. B. (1932): Low frequency thresholds of hearing and of feeling in the ear and ear mechanism J. Acoust. Soc. Am. 4, 6.

Wever, E. G. & Lawrence, M. (1930b); The transmission properties of the Stapes. Ann. Otol. Rhinol. Laryngol. 59, 322-330

Wever, E. G. & Lawrence, M. (1930b); The transmission properties of the middle ear. Ann. Otol. Rhinol. Laryngol. 59, 5-18.