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An Investigation of The Efficiencies and Cost Effectiveness of Estimators of Population Mean in Two Occasion Successive Sampling Under Non-Response Scenario

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Abstract: *This study compares the performances of existing estimators of population mean in current occasion in two-occasion successive sampling under non-response scenario. Using dataset from Sharma et al. (2022), the estimators are evaluated across varying correlation structures, non-response rates, and optimum replacement policy. The results reveal distinct differences in efficiency among the estimators. Some estimators outperform other estimators, particularly when the correlations are strong. The percent relative loss (PRL) analysis further indicates that some estimators maintain stable performances even as non-response rates increase, demonstrating robustness under adverse survey conditions. Optimal unmatched proportions also contributed to significant gains in precision. Overall, the findings indicate that properly constructed estimators remain highly efficient under non-response and provide clearer guidance for selecting optimal estimators in repeated surveys.*

Keywords: *Non-response, Auxiliary information, Class estimators, Percent Relative Efficiency (PRE), Percent Relative Loss (PRL)*

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1.0 Introduction

Successive sampling on two occasions is an important technique in repeated surveys where timely updates of population characteristics are required without conducting a full-scale survey each time. By combining matched units from the first occasion with a fresh sample on the second occasion, the design exploits inter-occasion correlations to improve estimation efficiency (Jessen, 1942; Cochran, 1977).

A major challenge in this framework is non-response, which can introduce bias and inflate variance if not properly addressed. The Hansen and Hurwitz (1946) subsampling method remains a widely applied correction technique and has been extended in various contexts (Khare & Sinha, 2004; Singh & Horn, 2000). Further contributions by Choudhary *et al*. (2004), Singh and Priyanka (2007), Singh and Kumar (2011), Karna and Nath (2015), Surya *et al.* (2020), Pal and Singh (2020), Unal and Kadilar (2022) and Sharma *et al*. (2022) introduced exponential-type estimators for two-occasion successive sampling. However, only a few studies, most notably Singh *et al.* (2016) and Sharma *et al.* (2022) explicitly address non-response while incorporating auxiliary information.

Despite these developments, there remains a lack of unified comparison of existing class estimators under identical non-response conditions on both occasions. This gap limits practical decision-making, particularly in surveys affected by varying non-response rates and correlation structures. Therefore, this study provides a comparative evaluation of selected estimators using Minimum Mean Squared Error (MMSE), Percent Relative Efficiency (PRE), and Percent Relative Loss (PRL) to determine their performance under different non-response scenarios.

2. 0 Sampling Procedure and notations

Consider a finite population $\Theta = (\Theta_1, \Theta_2, \cdots, \Theta_N)$ of size N which has been sampled over two occasions. Let by x and y denote the study variables measured on the first and second occasions respectively. Information on an auxiliary variable z whose population mean $\overline{Z}$, is assumed to be positively correlated with x and y.

On the first occasion, a simple random sample without replacement (SRSWOR) of size n is drawn. From the sample, a random sub-sample of size $m = n\lambda$ is retained (matched) for its use on the second occasion. Non-response is assumed to occur on both occasions. Accordingly, the population on each occasion is divided into two classes, respondent and non-respondent. Let the sizes of these two classes on the first occasion be N_1^* and N_2^*, and on the second (current) occasion N_1 and N_2, respectively.

Out of n units is selected on the first occasion, suppose n_1 units respond and n_2 units do not respond. A subsample of size n_{2h} is drawn from the non-responding units following Hansen and Hurwitz (1946). Similarly, a matched subsample of size $m = n\lambda$ units are retained (matched) from the responding n_1 units on the first occasion for its use on the second occasion under the assumption that these units will give complete response on the second occasion as well.

On the current (second) occasion, a fresh SRSWOR sample of size $u = (n - m) = n\mu$ units are drawn afresh from the remaining non-sampled units of the population so that the total sample size remains n. It is assumed that the units in the matched portion of the sample respond fully on current occasion. λ and $\mu(\lambda + \mu = 1)$ are the fractions of matched and fresh samples respectively at the second (current) occasion. We assume that in the unmatched portion of the sample on the second (current) occasion u_1 units respond and u_2 units do not respond. Let u_{2h} be the size of sub sample drawn from the non-response class in the unmatched portion of the sample on the current occasion and their response collected by direct contact or interview. The sampling notations are as follows

$\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$; The population mean of study variable y on the second occasion

$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$; The population means of study variable x on the first occasion

$\overline{Z} = \frac{1}{N}\sum_{i=1}^{N} Z_i$; The population means of auxiliary variable z

$\bar{x}_n = \frac{1}{n}\sum_{i=1}^{n} x_i$, $\bar{y}_n = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{z}_n = \frac{1}{n}\sum_{i=1}^{n} z_i$ are the sample means of study variables x and y on the first and second occasions and the auxiliary variable z respectively.

$\bar{x}_m = \frac{1}{m}\sum_{i=1}^{m} x_i$, $\bar{y}_m = \frac{1}{m}\sum_{i=1}^{m} y_i$ and $\bar{z}_m = \frac{1}{m}\sum_{i=1}^{m} z_i$ are the sample means of matched portion on the first and second occasions as well as the auxiliary variable respectively.

$\bar{x}_u = \frac{1}{u}\sum_{i=1}^{u} x_i$, $\bar{y}_u = \frac{1}{u}\sum_{i=1}^{u} y_i$ and $\bar{z}_u = \frac{1}{u}\sum_{i=1}^{u} z_i$ are the sample mean of unmatched portion on the first and second occasion as well as the auxiliary variable respectively.

$\bar{x}_m^* = \frac{m_1 \bar{x}_{m1} + m_2 \bar{x}_{m_{h2}}}{m}$, $\bar{y}_m^* = \frac{m_1 \bar{y}_{m1} + m_2 \bar{y}_{m_{h2}}}{m}$ and $\bar{z}_m^* = \frac{m_1 \bar{z}_{m1} + m_2 \bar{z}_{m_{h2}}}{m}$ are the Hansen and Hurwitz (1946) estimator for the matched proportion on the first and second occasion as well as the auxiliary variable respectively.

$\bar{x}_u^* = \frac{u_1 \bar{x}_{u1} + u_2 \bar{x}_{u_{h2}}}{u}$, $\bar{y}_u^* = \frac{u_1 \bar{y}_{u1} + u_2 \bar{y}_{u_{h2}}}{u}$ and $\bar{z}_u^* = \frac{u_1 \bar{z}_{u1} + u_2 \bar{z}_{u_{h2}}}{u}$ are the Hansen and Hurwitz (1946) estimator for the unmatched proportion on the first and second occasion as well as the auxiliary variable respectively.

$\rho_{yx}, \rho_{yz}, \rho_{xz}$: The correlation coefficient between the variables shown in suffices.

$S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \bar{X})^2$: Population mean square of x.

S_y^2, S_z^2 : Population mean squares of y, z respectively.

$S_{yx} = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \bar{X})(y_i - \bar{Y})$: Covariance between y and x.

S_{yz} : Covariance between y and z, S_{xz} : Covariance between x and z.

$C_y = \frac{S_y}{\bar{Y}}, C_x = \frac{S_x}{\bar{X}}, C_z = \frac{S_z}{\bar{Z}}$ are the coefficients of variation for the variables shown in suffices.

$\bar{X}_{(2)} = \frac{\sum_{i=1}^{N_2} x_i}{N_2}, \bar{Y}_{(2)} = \frac{\sum_{i=1}^{N_2} y_i}{N_2}, \bar{Z}_{(2)} = \frac{\sum_{i=1}^{N_2} z_i}{N_2}$: The population means of the variables x, y and z, respectively, in the non-responding units of the population.

$\rho_{yx(2)} = \frac{S_{yx(2)}}{S_{y(2)}S_{x(2)}}, \rho_{yz(2)} = \frac{S_{yz(2)}}{S_{y(2)}S_{z(2)}}, \rho_{zx(2)} = \frac{S_{zx(2)}}{S_{z(2)}S_{x(2)}}$: The correlation coefficient between the variables shown in suffices, in the non-responding units of the population.

$S_{x(2)}^2 = \frac{1}{N_2}\sum_{i=1}^{N_2}\left(x_i - \bar{X}_{(2)}\right), S_{y(2)}^2 = \frac{1}{N_2}\sum_{i=1}^{N_2}\left(y_i - \bar{Y}_{(2)}\right), S_{z(2)}^2 = \frac{1}{N_2}\sum_{i=1}^{N_2}\left(z_i - \bar{Z}_{(2)}\right)$: The population variances of the variables x, y and z, respectively, in the non-responding units of the population.

$W\left(=\frac{N_2}{N}\right)$: The proportion of non-responding units in the population at the current (second) occasion (that is non-response rate).

$W^*\left(=\frac{N_2^*}{N}\right)$: The proportion of non-responding units in the population at first occasion.

$f_2=\frac{u_2}{u_{2h}}$:Correction factor under nonresponse

$f_1=\frac{n_2}{n_{2h}}$, $f=\frac{n}{N}$, $\lambda_u=\frac{1}{u}-\frac{1}{N}$, $\lambda_m=\frac{1}{m}-\frac{1}{N}$ and $\lambda_n=\frac{1}{n}-\frac{1}{N}$

3. 0 Review of Selected Existing Class Estimators under Non-response

Two major classes of estimators from the literature are reviewed:
(i) Singh *et al.* (2016)
(ii) Sharma *et al*. (2022)
These estimators utilize auxiliary information and adjust for non-response using the Hansen–Hurwitz technique.

3.1 Estimators of Singh et al. (2016)

To estimate the population mean $\bar{Y}$ on the current (second) occasion, Singh *et al.* (2016) proposed two different sets of estimators. Firstly, the estimators $S_u=[T_{1u},T_{2u}]$ based on sample S_u of size u drawn afresh on the second occasion and the second estimators $S_m=[T_m]$ based on the sample S_m of size m. It was assumed that, non-response occurs on the samples S_n and S_u. Thus, following the work of Hansen and Hurwitz (1946), their proposed estimator is as follows**;**

$$T_{i(SKM)}=\varphi_i T_{i(SKM)u}+(1-\varphi_i)T_{(SKM)m} : (i=1,2) \quad (1)$$

where φ_i is unknown constant, derived from $MMSE$. The estimator T_1 is when non-response occurred in the first occasion only while T_2 is the estimator when non-response occurred in the second occasion only. The estimators for unmatched and matched portion are defined as

$$T_{1(SKM)u}=\bar{y}_u^*\exp\left(\frac{\bar{Z}-\bar{z}_u}{\bar{Z}+\bar{z}_u}\right),\ T_{2(SKM)u}=\bar{y}_u^*\exp\left(\frac{\bar{Z}-\bar{z}_u^*}{\bar{Z}+\bar{z}_u^*}\right) \text{and}\, T_{(SKM)m}=\bar{y}_m\exp\left(\frac{\bar{x}_n^*-\bar{x}_m}{\bar{x}_n^*+\bar{x}_m}\right)\left(\frac{\bar{Z}}{\bar{z}_m}\right)$$

Thus**,** the MMSE of the estimators are derived to the first order of approximations and they are given as

$$M(T^0_{1(SKM)})_{opt}=\left[\frac{a_3+\mu_1^{(0)}a_2+\left(\mu_2^{(0)}\right)^2a_1}{a_6+\mu_1^{(0)}a_5+\left(\mu_2^{(0)}\right)^2a_4}\right]\frac{S_y^2}{n} \quad (2)$$

$$M(T^0_{2(SKM)})_{opt}=\left[\frac{a_9+\mu_2^{(0)}a_8+\left(\mu_2^{(0)}\right)^2a_7}{a_{12}+\mu_2^{(0)}a_{11}+\left(\mu_2^{(0)}\right)^2a_{10}}\right]\frac{S_y^2}{n} \quad (3)$$

where $\mu_1^{(0)}=\frac{-p_2\pm\sqrt{p_2^2-p_1p_3}}{p_1}$ and $\mu_2^{(0)}=\frac{-p_5\pm\sqrt{p_5^2-p_4p_6}}{p_4}$ are the optimum unmatched proportion.

The optimum unmatched proportion (μ_i^0) is a fraction of sample taken afresh on the second occasion. It is obtained such that the population mean $\bar{Y}$ is estimated with minimum mean square error (with Maximum precision), thereby playing the role of reducing cost of survey. The real values of μ_i^0 exists, when $0\le\mu_i^0\le1$ and admissible for only positive values. On the other hand, two real values of μ_0 are obtained, but the one that lies within the interval $0 \le \mu_i^0 \le 1$ is chosen. If both values lie within the specified interval, the value closed to zero is selected, since the smaller value of μ_i^0 minimize the cost of survey.

$p_1 = a_1a_5 - a_2a_4$; $p_2 = a_1a_6 - a_3a_4$; $p_3 = a_2a_6 - a_3a_5$; $p_4 = a_7a_{11} - a_8a_{10}$; $p_5 = a_7a_{12} - a_9a_{10}$; $p_6 = a_8a_{12} - a_9a_{11}$

$$a_1 = ac + k^2f^2, a_2 = ad + bc - k^2f^2, a_3 = bd, a_4 = c - a + 2kf, a_5 = a - b + d - 2kf, a_6 = b,$$

$$a_7 = ac + k^2f^2, a_8 = ad + cb_1' - k^2f^2, a_9 = b_1'd, a_{10} = c - a + 2kf, a_{11} = a - b_1' + d - 2kf, a_{12} = b_1'$$

$$a = -fa_0, b = a_0 + (f_2 - 1)W, c = fc_1 + d_1 - \frac{1}{4}(f_1 - 1)W^*, (1 - f)c_1 + \frac{1}{4}(f_1 - 1)W^*, b_1' = a_0[1 + (f_2 - 1)W],$$

$$a_0 = \frac{5}{4} - \rho_{yz}, c_1 = 2(1 - \rho_{yz}), d_1 = \frac{1}{4} + \rho_{xz} - \rho_{yx}, k = \frac{3}{2}(\rho_{yz} - 1), f = \frac{n}{N}, f_1 = \frac{n_2}{n_{2h}} and f_2 = \frac{u_2}{u_{2h}}$$

Thus, the corresponding estimator for complete response is given as

$$\xi_j = \phi_{Sj}\xi_{j_u} + (1 - \phi_{Sj})\xi_{j_m} : (j = 1,2) \quad (4)$$

With the estimators for unmatched and matched portion, defined as

$$\xi_{1u} = \bar{y}_u \exp\left(\frac{\bar{Z} - \bar{z}_u}{\bar{Z} + \bar{z}_u}\right); \ \xi_{2u} = \bar{y}_u ; \xi_{1m} = \bar{y}_m \exp\left(\frac{\bar{x}_n - \bar{x}_m}{\bar{x}_n + \bar{x}_m}\right)\left(\frac{\bar{Z}}{\bar{z}_m}\right); \xi_{1m} = \bar{y}_m + \beta_{yx}(\bar{x}_n - \bar{x}_m)$$

Hence, the MMSE of the estimators are derived to the first order of approximations and they are given as

$$M(\xi_1^0)_{opt} = \left[\frac{b_3 + \mu_1^* b_2 + (\mu_1^*)^2 b_1}{b_6 + \mu_1^* b_5 + (\mu_1^*)^2 b_4}\right]\frac{S_y^2}{n} \quad (5)$$

$$M(\zeta_2^0)_{opt} = \left[\frac{1}{2}\left\{1 + \sqrt{(1 - \rho_{xy}^2)}\right\} - f\right]\frac{S_y^2}{n} \quad (6)$$

$$\mu_1^* = \frac{-q_2 \pm \sqrt{q_2^2 - q_1q_3}}{q_1}, b_1 = ac + k^2f^2, b_2 = ad + ca_0 - k^2f^2, b_3 = da_0,$$

where; $b_4 = c - a + 2kf, b_5 = a - a_0 + d - 2kf, b_6 = a_0,$

$q_1 = b_1b_5 - b_2b_4, q_2 = b_1b_6 - b_3b_4, q_3 = b_2b_6 - b_3b_5$

$\mu_i^{(0)}, i = 1,2$ are the optimum unmatched proportion when non-response occurred on first and second occasions respectively, μ_1^* is optimum unmatched proportion for corresponding estimator for complete response.

By computing the percent relative losses in efficiency of the estimators T_1 and T_2, with respect to the similar estimator and natural successive sampling estimator when there was no nonresponse observed on any occasion. Their study showed that for all parametric combinations considered, the estimator T_2 performed much better than the estimator T_1 in terms of efficiency gain

3.2 Estimators of Sharma et al. (2022)

However, Taking inspiration from the work of Singh *et al.* (2016) and other related study in the literature, Sharma *et al*. (2022) suggested an improved estimator for the population mean $\overline{Y}$ on the current (second) occasion under nonresponse in successive sampling. Their proposed estimator is given as

$$T_{i(SPS)} = \varphi_{i(SPS)} T_{iu(SPS)} + \left(1-\varphi_{i(SPS)}\right) T_{m(SPS)} : \left(i = 1,2\right) \tag{7}$$

Where $\varphi_{i(SPS)}$ is unknown constant, derived from $MMSE$. The estimators for unmatched and matched portion are defined as

$$T_{m(SPS)} = \left[\bar{y}_m + b_{yz(m)}\left(\bar{Z} - \bar{z}_m\right)\right]\exp\left(\frac{\bar{x}_n - \bar{x}_m}{\bar{x}_n + \bar{x}_m}\right), T_{1u(SPS)} = \bar{y}_v^{*}\exp\left(\frac{\bar{Z} - \bar{z}_v}{\bar{Z} + \bar{z}_v}\right)\exp\left(\frac{\bar{Z} - \bar{z}_v^{*}}{\bar{Z} + \bar{z}_v^{*}}\right) \text{and}$$

$$T_{2u(SPS)} = \bar{y}_v^{*}\exp\left(\frac{2\hat{\beta}_{yz}}{\hat{R}_z}\frac{\left(\bar{Z} - \bar{z}_v\right)}{\left(\bar{Z} + \bar{z}_v\right)}\right)\exp\left(\frac{4\hat{\beta}_{yz(2)}}{\hat{R}_z}\frac{\bar{Z}\left(\bar{Z} - \bar{z}_v^{*}\right)}{\left(\bar{Z} + \bar{z}_v\right)\left(\bar{Z} + \bar{z}_v^{*}\right)}\right)$$

Thus, the linear Combination of their two estimators are given as

$$T_{1(SPS)} = \varphi_{1(SPS)} T_{1u(SPS)} + \left(1-\varphi_{1(SPS)}\right) T_{m(SPS)}$$
$$T_{2(SPS)} = \varphi_{2(SPS)} T_{1u(SPS)} + \left(1-\varphi_{2(SPS)}\right) T_{m(SPS)} \tag{8}$$

Also**,** the MMSE of are derived to the first order of approximations and given as

$$M(T_1^0)_{opt} = \left[\frac{\Lambda_{0(1)} + \mu_1^{(0)}\Lambda_{1(1)} + \left(\mu_2^{(0)}\right)^2\Lambda_{2(1)}}{h_{0(1)} + \mu_1^{(0)}h_{1(1)} + \left(\mu_2^{(0)}\right)^2 h_{2(!)}}\right]\frac{S_y^2}{n} \tag{9}$$

$$M(T_2^0)_{opt} = \left[\frac{\Lambda_{0(2)} + \mu_2^{(0)}\Lambda_{1(2)} + \left(\mu_2^{(0)}\right)^2\Lambda_{2(2)}}{h_{0(2)} + \mu_2^{(0)}h_{1(2)} + \left(\mu_2^{(0)}\right)^2 h_{2(2)}}\right]\frac{S_y^2}{n} \tag{10}$$

Where; $\mu_1^{(0)} = \dfrac{-\psi_{02(1)} \pm \sqrt{\psi_{12(1)}^2 - \psi_{01(1)}\psi_{12(1)}}}{\psi_{12(1)}}$; $\mu_2^{(0)} = \dfrac{-\psi_{02(2)} \pm \sqrt{\psi_{12(2)}^2 - \psi_{01(2)}\psi_{12(2)}}}{\psi_{12(2)}}$

$$\psi_{01(1)} = h_{0(1)}\Lambda_{1(1)} - \Lambda_{0(1)}h_{1(1)}; \psi_{02(1)} = h_{0(1)}\Lambda_{2(1)} - \Lambda_{0(1)}h_{2(1)}; \psi_{12(1)} = h_{1(1)}\Lambda_{2(1)} - \Lambda_{1(1)}h_{2(1)};$$

$$\psi_{01(2)} = h_{0(2)}\Lambda_{1(2)} - \Lambda_{0(2)}h_{1(2)}; \psi_{02(2)} = h_{0(2)}\Lambda_{2(2)} - \Lambda_{0(2)}h_{2(2)}; \psi_{12(2)} = h_{1(2)}\Lambda_{2(2)} - \Lambda_{1(2)}h_{2(2)}$$

$$K_0 = 2(1-\rho_{yx}), K_1 = \Theta(f_2 - 1)(5/4 - \rho_{yz}), K_2 = (1/4 - \rho_{yx} + \rho_{xz}), K_3 = (5/4 - \rho_{yz}),$$

$$K_4 = (1-\rho_{yx}^2), K_5 = (1/4 - \rho_{yz} + \rho_{yz}\rho_{xz}), K_6 = 1 + \Theta(f_2 - 1), h_{0(1)} = (K_0 + K_1),$$

$$h_{1(1)} = (1-f)(K_4 - K_0) - K_1, h_{2(1)} = (K_5 - fK_4 + fK_0), h_{0(2)} = K_4K_6, h_{1(2)} = K_4(1-K_6), h_{2(2)} = K_5$$

$$\Lambda_{0(1)} = (1-f)(K_0 + K_1)K_4, \Lambda_{1(1)} = (K_0 + K_1)K_5 + fK_1K_4 - f^2K_4(K_4 - K_0),$$

$$\Lambda_{2(1)} = f^2K_4(K_4 - K_0) - fK_0K_5, \Lambda_{0(2)} = (1-f)K_4^2, \Lambda_{1(2)} = \left[K_5K_6 + fK_4(K_6 - 1)\right]K_4, \Lambda_{2(2)} = -fK_4K_5$$

Thus, the corresponding estimator for complete response is given as

$$\Phi_j = \Phi_{ju}\phi_{Sj} + \left(1-\phi_{Sj}\right)\Phi_{jm} : \left(j = 1,2\right) \tag{11}$$

With the estimators for unmatched and matched portion defined as follows

$$\Phi_{1u} = \bar{y}_u \exp\left(\frac{\bar{Z} - \bar{z}_u}{\bar{Z} + \bar{z}_u}\right), \ \Phi_{2u} = \bar{y}_u, \ \Phi_{1m} = \bar{y}_m \exp\left(\frac{\bar{x}_n - \bar{x}_m}{\bar{x}_n + \bar{x}_m}\right)\left(\frac{\bar{Z}}{\bar{z}_m}\right); \Phi_{2m} = \bar{y}_m + b_{yx(m)}\left(\bar{x}_n - \bar{x}_m\right)$$

Hence, the MMSE of the estimators are derived to the first order of approximations and they are given as

$$M(\Phi_1^0)_{opt} = \left[\frac{A_3 + \mu_1^* A_2 + (\mu_1^*)^2 A_1}{B_3 + \mu_1^* B_2 + (\mu_1^*)^2 B_1}\right]\frac{S_y^2}{n} \tag{12}$$

$$M(\Phi_2^0)_{opt} = \left[\frac{1}{2}\left\{1 + \sqrt{(1-\rho_{xy}^2)}\right\} - f\right]\frac{S_y^2}{n} \tag{13}$$

$$\mu_1^* = \frac{-Q_2 \pm \sqrt{q_2^2 - Q_1 Q_3}}{Q_1},$$

$$Q_1 = B_1 A_2 - A_1 B_2, Q_2 = B_1 A_3 - B_3 A_1, Q_3 = B_2 A_3 - B_3 A_2, A_1 = \left(\frac{9}{16}\right) f^2 k_0^2 - f^2 k_0 k_3 - f k_2 k_3$$

$$A_2 = f k_0 k_3 + k_2 k_3 - f(1-f)k_0 k_3 - \left(\frac{9}{16}\right) f^2 k_0^2, A_3 = (1-f)k_0 k_3, B_1 = f k_0 + k_2 - \left(\frac{3}{2}\right) f k_0 + f k_3$$

$$B_2 = (1-f)k_0 - (1+f)k_3 + \left(\frac{3}{2}\right) f k_0, B_3 = k_3$$

By computing the percent relative losses in efficiencies of the estimators $T_{1(SPS)}$ and $T_{2(SPS)}$, with respect to the similar estimator and natural successive sampling estimator when there was no nonresponse observed on any occasion. Sharma *et al.* (2022) concluded that the proposed estimator $T_{2(SPS)}$ was better than the estimator $T_{1(SPS)}$ in terms of efficiency gain. In consideration of the claim made by Singh *et al.* (2016) and Sharma *et al.* (2022), practitioners lack clear guidance on which estimators remain robust, efficient, or reliable when confronted with non-response during repeated survey operations.

This gap motivates the present study, which seeks to provide a systematic evaluation of these two major existing class estimators in two-occasion successive sampling under non-response. By examining their behaviour across different correlation strengths, correction factors, and non-response rates, the study aims to identify the conditions under which each estimator performs optimally and to inform practical decisions in survey design and estimation procedures.

4.0 Efficiency comparison for Nonresponse

To examine loss in efficiencies do to nonresponse, the percent relative loss (PRL) in efficiencies of the estimators $T^{iN}, (i=1,2)$ are obtained with respect to the similar estimator and natural successive sampling estimator when the nonresponse is not observed on any occasion. The estimator T_j is for complete information and under the similar assumption as estimator $T^{iN}, (i=1,2)$. On the other hand, the estimator $T_{(0)}$ is a natural estimator under the successive sampling.

The percent relative losses (PRL) in precision of $T^{iN}, (i=1,2)$ with respect to $T_{(j=0,1)}$ under their respective optimality conditions are given as;

$$L_{ij} = \frac{MMSE\left(T^{iN}\right)_{opt} - MMSE\left(T_j\right)_{opt}}{MMSE\left(T^{iN}\right)_{opt}}, (i = 1,2;\ j = 0,1) \tag{14}$$

While the percent relative efficiencies (PRE) in precision of $T^{iN}, (i = 1,2)$ with respect to $T_{(0)}$ under their respective optimality conditions are given as;

$$PRE = \frac{MMSE\left(T_{(0)}\right)}{MMSE\left(T^{iN}\right)_{opt}}, (i = 1,2) \tag{15}$$

where

$$MMSE\left(T_{(0)}\right) = \frac{S_y^2}{n}\left\{\left(\frac{1}{2}\right)\left[1 + \sqrt{\left(1 - \rho_{yx}^2\right)}\right] - f\right\} \tag{16}$$

which is the minimum mean square error when there is no auxiliary variable at any occasion, (that is, the natural estimator under the successive sampling)

$$MMSE\left(T_i\right) = \frac{MSE\left(T_{iu}\right)MSE\left(T_{jm}\right) - \left[Cov\left(T_{iu}, T_{jm}\right)\right]^2}{MSE\left(T_{iu}\right) + MSE\left(T_{jm}\right) - 2\left[Cov\left(T_{iu}, T_{jm}\right)\right]} \tag{17}$$

which is the minimum mean square error when there is complete response.
Thus, when;

$L_{ij} > 0$, this mean an efficient loss due to non-response

$L_{ij} = 0$, this show that there is no loss in efficiency due to non-response

$L_{ij} < 0$, this indicates an efficient gain

In summary, the Percent Relative Loss (PRL) measures the proportional change in efficiency due to non-response. A positive PRL indicates a loss in efficiency, a zero value implies no change, and a negative PRL suggests an apparent efficiency gain, often due to estimator adjustment or sampling variation (Singh & Kumar, 2008; Khare & Srivastava, 1993).

5.0 Discussion of Results

Using the dataset in Sharma *et al.* (2022), the population size is denoted by N, with a sample size n and different choices of the correlation coefficients (ρ_{yx} and ρ_{yz}) along with the correction factors (f_2) and proportion of non-responding units (W) in the population at current (second) occasion. The statistics are presented in Table 1, while the resulting Percent Relative Losses (PRLs) with corresponding optimum unmatched proportions is shown in Tables 2 to 5. Also, Table 6 presented the Percent Relative Efficiency together with the respective minimum mean square errors

Table 1: Statistics for different choices of W, ρ_{yx}, ρ_{yz}, f_2 with $N = 5000$ and $n = 500$

W	ρ_{yx}	ρ_{yz}	f_2
0.1	0.5	0.8	1.5
0.15	0.6	0.85	2.0
0.2	0.7	0.9	
	0.8	0.95	

Source: Sharma *et al.* (2022)

Remark 4.6: To compare the performance of the estimators $P_{1(SPS)}$, $P_{2(SPS)}$, $T_{1(SKM)}$ and $T_{2(SKM)}$ with respect to $T_{j(j=0,1)}$, the intuitive assumptions (optimality condition) given by Cochran (1977) and Feng and Zou (1997) were adopted. They are as follows $\rho_{yz} = \rho_{xz}$, $W = W^*$ and $f_1 = f_2$

Table 2: Percent relative losses $L_{1,1}$ and $L_{1,2}$ with respect to Φ_1 and Φ_2 when $f = 0.1$ for estimator $P_{1(SPS)}$ (Proposed estimator by Sharma *et al.*, 2022) v

W	ρ_{yx}	ρ_{yz}	$f_2 = 1.5$			$f_2 = 2.0$		
			$\mu_1^{(0)}$	L_{11}	L_{12}	$\mu_1^{(0)}$	L_{11}	L_{12}
0.1	0.5	0.80	**	-	-	**	-	-
		0.85	0.03	-2.2664	0.1476	0.12	-2.0514	0.2037
		0.90	0.34	-2.9316	0.2262	0.42	-2.7133	0.2692
		0.95	0.68	-5.8372	0.0404	0.75	-5.2734	0.1196
	0.6	0.80	**	-	-	**	-	-
		0.85	0.03	-2.1369	0.1988	0.12	-1.9305	0.2515
		0.90	0.34	-2.7758	0.2808	0.42	-2.5661	0.3207
		0.95	0.68	-5.5662	0.1235	0.75	-5.0248	0.1958
	0.7	0.80	**	-	-	**	-	-
		0.85	0.03	-1.9686	0.2531	0.12	-1.7732	0.3023
		0.90	0.34	-2.5732	0.3393	0.42	-2.3747	0.3760
		0.95	0.68	-5.2139	0.2142	0.75	-4.7015	0.2790
	0.8	0.80	**	-	-	**	-	-
		0.85	0.03	-1.7448	0.3113	0.12	-1.5642	0.3566
		0.90	0.34	-2.3038	0.4031	0.42	-2.1203	0.4362
		0.95	0.68	-4.7454	0.3155	0.75	-4.2717	0.3719
0.15	0.5	0.80	**	-	-	**	-	-
		0.85	0.08	-2.1553	0.1766	0.21	-1.8611	0.2533
		0.90	0.38	-2.8207	0.2481	0.50	-2.5085	0.3095
		0.95	0.72	-5.5451	0.0814	0.81	-4.7822	0.1885
	0.6	0.80	**	-	-	**	-	-
		0.85	0.08	-2.0303	0.2260	0.21	-1.7477	0.2982
		0.90	0.38	-2.6692	0.3011	0.50	-2.3694	0.3582
		0.95	0.72	-5.2857	0.1609	0.81	-4.5531	0.2587
	0.7	0.80	**	-	-	**	-	-
		0.85	0.08	-1.8677	0.2785	0.21	-1.6003	0.3458
		0.90	0.38	-2.4723	0.3580	0.50	-2.1886	0.4104
		0.95	0.72	-4.9484	0.2478	0.81	-4.2551	0.3354
	0.8	0.80	**	-	-	**	-	-
		0.85	0.08	-1.6515	0.3347	0.21	-1.4043	0.3967
		0.90	0.38	-2.2106	0.4199	0.50	-1.9483	0.4673
		0.95	0.72	-4.5000	0.3447	0.81	-3.8590	0.4211
0.2	0.5	0.80	**	-	-	0.07	-1.4437	0.1868
		0.85	0.12	-2.0514	0.2037	0.30	-1.6898	0.2981
		0.90	0.42	-2.7133	0.2692	0.57	-2.3162	0.3474
		0.95	0.75	-5.2734	0.1196	0.87	-4.3492	0.2493
	0.6	0.80	**	-	-	0.07	-1.3469	0.2296
		0.85	0.12	-1.9305	0.2515	0.30	-1.5832	0.3402
		0.90	0.42	-2.5661	0.3207	0.57	-2.1847	0.3934
		0.95	0.75	-5.0248	0.1958	0.87	-4.1372	0.3142
	0.7	0.80	**	-	-	0.07	-1.2209	0.2747

		0.85	0.12	-1.7732	0.3023	0.30	-1.4446	0.3850
		0.90	0.42	-2.3747	0.3760	0.57	-2.0138	0.4428
		0.95	0.75	-4.7015	0.2790	0.87	-3.8615	0.3852
	0.8	0.80	**	-	-	0.07	-1.0535	0.3224
		0.85	0.12	-1.5642	0.3566	0.30	-1.2603	0.4329
		0.90	0.42	-2.1203	0.4362	0.57	-1.7867	0.4965
		0.95	0.75	-4.2717	0.3719	0.87	-3.4950	0.4645

Note, '' represent Inadmissible, which implies $\mu_1^{(0)}$ does not exist**

Table 3: Percent relative losses $L_{2,1}$ and $L_{2,2}$ with respect to Φ_1 and Φ_2 when $f = 0.1$ for estimator $P_{2(SPS)}$ (Proposed estimator by Sharma *et al.*, 2022)

W	ρ_{yx}	ρ_{yz}	$f_2 = 1.5$			$f_2 = 2.0$		
			$\mu_2^{(0)}$	L_{21}	L_{22}	$\mu_2^{(0)}$	L_{21}	L_{22}
0.1	0.5	0.80	**	-	-	**	-	-
		0.85	**	-	-	0.01	-2.3188	0.1339
		0.90	0.27	-3.1316	0.1869	0.32	-3.0044	0.2119
		0.95	0.63	-6.3691	-0.0342	0.65	-6.1495	-0.0034
	0.6	0.80	**	-	-	**	-	-
		0.85	**	-	-	0.01	-2.1872	0.1859
		0.90	0.27	-2.9678	0.2442	0.32	-2.8457	0.2675
		0.95	0.63	-6.0771	0.0553	0.65	-5.8662	0.0834
	0.7	0.80	**	-	-	**	-	-
		0.85	**	-	-	0.01	-2.0162	0.2411
		0.90	0.27	-2.7549	0.3057	0.32	-2.6393	0.3271
		0.95	0.63	-5.6973	0.1530	0.65	-5.4977	0.1783
	0.8	0.80	**	-	-	**	-	-
		0.85	**	-	-	0.01	-1.7888	0.3002
		0.90	0.27	-2.4718	0.3727	0.32	-2.3650	0.3920
		0.95	0.63	-5.1924	0.2622	0.65	-5.0079	0.2842
0.15	0.5	0.80	**	-	-	**	-	-
		0.85	**	-	-	0.08	-2.1622	0.1748
		0.90	0.29	-3.0674	0.1995	0.36	-2.8816	0.2361
		0.95	0.64	-6.2578	-0.0186	0.68	-5.9412	0.0258
	0.6	0.80	**	-	-	**	-	-
		0.85	**	-	-	0.08	-2.0369	0.2243
		0.90	0.29	-2.9062	0.2560	0.36	-2.7277	0.2899
		0.95	0.64	-5.9702	0.0696	0.68	-5.6661	0.1101
	0.7	0.80	**	-	-	**	-	-
		0.85	**	-	-	0.08	-1.8740	0.2769
		0.90	0.29	-2.6966	0.3165	0.36	-2.5277	0.3477
		0.95	0.64	-5.5962	0.1658	0.68	-5.3084	0.2022
	0.8	0.80	**	-	-	**	-	-
		0.85	**	-	-	0.08	-1.6573	0.3332
		0.90	0.29	-2.4179	0.3825	0.36	-2.2618	0.4107
		0.95	0.64	-5.0989	0.2734	0.68	-4.8329	0.3051
0.2	0.5	0.80	**	-	-	**	-	-
		0.85	0.01	-2.3188	0.1339	0.14	-2.0196	0.2120
		0.90	0.32	-3.0044	0.2119	0.41	-2.7629	0.2594
		0.95	0.65	-6.1495	-0.0034	0.70	-5.7432	0.0536
	0.6	0.80	**	-	-	**	-	-

		0.85	0.01	-2.1872	0.1859	0.14	-1.8999	0.2593
		0.90	0.32	-2.8457	0.2675	0.41	-2.6138	0.3116
		0.95	0.65	-5.8662	0.0834	0.70	-5.4760	0.1355
	0.7	0.80	**	-	-	**	-	-
		0.85	0.01	-2.0162	0.2411	0.14	-1.7443	0.3095
		0.90	0.32	-2.6393	0.3271	0.41	-2.4199	0.3677
		0.95	0.65	-5.4977	0.1783	0.70	-5.1285	0.2250
	0.8	0.80	**	-	-	**	-	-
		0.85	0.01	-1.7888	0.3002	0.14	-1.5374	0.3633
		0.90	0.32	-2.3650	0.3920	0.41	-2.1621	0.4287
		0.95	0.65	-5.0079	0.2842	0.70	-4.6665	0.3249

Note, '' represent Inadmissible, which implies $\mu_2^{(0)}$ does not exist**

Table 4: Percent relative losses $L_{1,1}$ and $L_{1,2}$ with respect to ζ_1 and ζ_2 when $f = 0.1$ for estimator $T_{1(SKM)}$ (Proposed estimator by Singh *et al.*, 2016)

W	ρ_{yx}	ρ_{yz}	$f_2 = 1.5$			$f_2 = 2.0$		
			$\mu_1^{(0)}$	L_{11}	L_{12}	$\mu_1^{(0)}$	L_{11}	L_{12}
0.1	0.5	0.80	0.50	-0.4829	0.0650	0.55	-0.3885	0.1221
		0.85	0.53	-0.6226	0.0751	0.58	-0.5066	0.1395
		0.90	**	-	-	**	-	-
		0.95	0.58	-0.9951	0.0997	0.62	-0.8147	0.1805
	0.6	0.80	0.53	-0.4748	0.0670	0.59	-0.3777	0.1262
		0.85	0.57	-0.6140	0.0778	0.62	-0.4945	0.1447
		0.90	0.60	-0.7801	0.0896	0.65	-0.6323	0.1645
		0.95	0.62	-0.9841	0.1031	0.67	-0.7989	0.1867
	0.7	0.80	0.57	-0.4498	0.0698	0.64	-0.3496	0.1321
		0.85	0.61	-0.5863	0.0816	0.68	-0.4630	0.1522
		0.90	0.65	-0.7486	0.0940	0.71	-0.5964	0.1729
		0.95	0.67	-0.9475	0.1078	0.73	-0.7578	0.1954
	0.8	0.80	**	-	-	**	-	-
		0.85	0.68	-0.5267	0.0876	0.76	-0.3986	0.1641
		0.90	0.71	-0.6803	0.1007	0.79	-0.5234	0.1857
		0.95	0.74	-0.8681	0.1147	0.81	-0.6744	0.2085
0.15	0.5	0.80	0.53	-0.4343	0.0944	0.60	-0.3041	0.1729
		0.85	0.56	-0.5626	0.1085	0.63	-0.4051	0.1957
		0.90	0.58	-0.7149	0.1241	0.65	-0.5232	0.2202
		0.95	**	-	-	**	-	-
	0.6	0.80	0.56	-0.4249	0.0974	0.65	-0.2904	0.1791
		0.85	0.60	-0.5522	0.1124	0.68	-0.3896	0.2033
		0.90	0.62	-0.7032	0.1286	0.70	-0.5056	0.2286
		0.95	0.64	-0.8871	0.1469	0.72	-0.6442	0.2565
	0.7	0.80	0.61	-0.3984	0.1018	0.72	-0.2587	0.1883
		0.85	0.65	-0.5226	0.1181	0.75	-0.3540	0.2143
		0.90	0.68	-0.6695	0.1350	0.77	-0.4654	0.2406
		0.95	0.70	-0.8482	0.1537	0.78	-0.5987	0.2687
	0.8	0.80	**	-	-	**	-	-
		0.85	0.72	-0.4608	0.1271	0.85	-0.2841	0.2323
		0.90	0.75	-0.5989	0.1449	0.86	-0.3872	0.2593
		0.95	0.77	-0.7668	0.1638	0.87	-0.5109	0.2873
0.2	0.5	0.80	0.55	-0.3885	0.1221	0.65	-0.2280	0.2186
		0.85	0.58	-0.5066	0.1395	0.68	-0.3152	0.2452

		0.90	0.61	-0.6460	0.1587	0.70	-0.4164	0.2736
		0.95	**	-	-	**	-	-
	0.6	0.80	0.59	-0.3777	0.1262	0.71	-0.2111	0.2269
		0.85	0.62	-0.4945	0.1447	0.73	-0.2962	0.2551
		0.90	0.65	-0.6323	0.1645	0.75	-0.3950	0.2843
		0.95	0.67	-0.7989	0.1867	0.77	-0.5120	0.3159
	0.7	0.80	0.64	-0.3496	0.1321	0.79	-0.1753	0.2396
		0.85	0.68	-0.4630	0.1522	0.81	-0.2563	0.2697
		0.90	0.71	-0.5964	0.1729	0.83	-0.3505	0.2997
		0.95	0.73	-0.7578	0.1954	0.84	-0.4624	0.3314
	0.8	0.80	**	-	-	**	-	-
		0.85	0.76	-0.3986	0.1641	0.94	-0.1800	0.2938
		0.90	0.79	-0.5234	0.1857	0.94	-0.2667	0.3240
		0.95	0.81	-0.6744	0.2085	0.94	-0.3699	0.3550

Note, '' represent Inadmissible, which implies $\mu_1^{(0)}$ does not exist**

Table 5: Percent relative losses $L_{2,1}$ and $L_{2,2}$ with respect to ζ_1 and ζ_2 when $f = 0.1$ for estimator $T_{2(SKM)}$ (Proposed estimator by Singh *et al.*, 2016)

W	ρ_{yx}	ρ_{yz}	$f_2 = 1.5$			$f_2 = 2.0$		
			$\mu_2^{(0)}$	L_{21}	L_{22}	$\mu_2^{(0)}$	L_{21}	L_{22}
0.1	0.5	0.80	0.470281	-0.5384	0.0300	0.49	-0.4904	0.0577
		0.85	0.499208	-0.6995	0.0313	0.51	-0.6457	0.0601
		0.90	0.522212	-0.8963	0.0320	0.53	-0.8361	0.0615
		0.95	**	-	-	**	-	-
	0.6	0.80	0.494503	-0.5318	0.0309	0.51	-0.4831	0.0593
		0.85	0.529045	-0.6936	0.0323	0.54	-0.6388	0.0621
		0.90	0.555973	-0.8905	0.0331	0.57	-0.8294	0.0637
		0.95	0.576665	-1.1385	0.0333	0.59	-1.0703	0.0641
	0.7	0.80	0.526187	-0.5087	0.0320	0.55	-0.4594	0.0616
		0.85	0.56806	-0.6688	0.0338	0.59	-0.6133	0.0651
		0.90	0.599551	-0.8630	0.0347	0.61	-0.8012	0.0668
		0.95	0.623241	-1.1068	0.0348	0.63	-1.0384	0.0670
	0.8	0.80	0.572208	-0.4583	0.0339	0.60	-0.4085	0.0652
		0.85	0.62357	-0.6127	0.0362	0.65	-0.5568	0.0697
		0.90	0.659686	-0.7991	0.0371	0.68	-0.7372	0.0715
		0.95	0.685705	-1.0322	0.0370	0.70	-0.9644	0.0714
0.15	0.5	0.80	0.478703	-0.5140	0.0441	0.50	-0.4454	0.0833
		0.85	0.506204	-0.6721	0.0460	0.53	-0.5954	0.0868
		0.90	0.527936	-0.8657	0.0470	0.55	-0.7799	0.0888
		0.95	**	-	-	**	-	-
	0.6	0.80	0.504169	-0.5070	0.0454	0.53	-0.4374	0.0856
		0.85	0.536977	-0.6657	0.0475	0.56	-0.5877	0.0897
		0.90	0.562392	-0.8594	0.0487	0.58	-0.7723	0.0919
		0.95	0.581751	-1.1038	0.0490	0.60	-1.0066	0.0926
	0.7	0.80	0.537857	-0.4836	0.0471	0.57	-0.4130	0.0889
		0.85	0.577469	-0.6406	0.0498	0.61	-0.5614	0.0939
		0.90	0.607048	-0.8316	0.0510	0.63	-0.7436	0.0964
		0.95	0.629105	-1.0720	0.0512	0.65	-0.9745	0.0969
	0.8	0.80	0.587685	-0.4330	0.0499	0.64	-0.3615	0.0942
		0.85	0.63566	-0.5843	0.0532	0.67	-0.5042	0.1007
		0.90	0.669071	-0.7676	0.0546	0.70	-0.6792	0.1034

		0.95	0.692893	-0.9977	0.0545	0.71	-0.9008	0.1033
0.2	0.5	0.80	**	-	-	**	-	-
		0.85	0.513203	-0.6457	0.0601	0.54	-0.5483	0.1115
		0.90	0.533649	-0.8361	0.0615	0.56	-0.7273	0.1141
		0.95	0.549181	-1.0772	0.0620	**	-	-
	0.6	0.80	0.51391	-0.4831	0.0593	0.55	-0.3944	0.1099
		0.85	0.544947	-0.6388	0.0621	0.58	-0.5397	0.1152
		0.90	0.568823	-0.8294	0.0637	0.59	-0.7189	0.1182
		0.95	0.58683	-1.0703	0.0641	0.61	-0.9470	0.1191
	0.7	0.80	0.549711	-0.4594	0.0616	0.60	-0.3692	0.1141
		0.85	0.586983	-0.6133	0.0651	0.63	-0.5127	0.1206
		0.90	0.614601	-0.8012	0.0668	0.65	-0.6895	0.1239
		0.95	0.634989	-1.0384	0.0670	0.66	-0.9145	0.1247
	0.8	0.80	0.603636	-0.4085	0.0652	0.67	-0.3168	0.1211
		0.85	0.648023	-0.5568	0.0697	0.70	-0.4546	0.1295
		0.90	0.67861	-0.7372	0.0715	0.72	-0.6246	0.1330
		0.95	0.70016	-0.9644	0.0714	0.73	-0.8411	0.1332

Note, ‘’ represent Inadmissible, which implies $\mu_2^{(0)}$ does not exist**

Table 6: Comparison among Estimators ($P_{1(SPS)}$, $P_{2(SPS)}$, $T_{1(SKM)}$ & $T_{2(SKM)}$)

W	ρ_{yx}	ρ_{yz}	$MMSE$ (T_0)	$f_2 = 1.5$				$f_2 = 2.0$			
				PRE $P_{1(SPS)}$	PRE $T_{1(SKM)}$	PRE $P_{2(SPS)}$	PRE $T_{2(SKM)}$	PRE $P_{1(SPS)}$	PRE $T_{1(SKM)}$	PRE $P_{2(SPS)}$	PRE $T_{2(SKM)}$
0.1	0.5	0.80	0.0122	-	29.6590	-	30.767	-	27.770	-	29.807
		0.85	0.0122	65.3275	32.4527	-	33.990	305.1403	30.132	331.875	32.913
		0.90	0.0122	78.6321	-	82.6312	37.926	371.3252	-	400.437	36.722
		0.95	0.0122	136.7437	39.9026	147.381	-	627.3390	36.294	714.953	-
	0.6	0.80	0.0118	-	29.4954	-	30.636	-	27.553	-	29.661
		0.85	0.0118	62.7386	32.2808	-	33.871	293.0475	29.890	318.723	32.776
		0.90	0.0118	75.5159	35.6022	79.3565	37.810	356.6095	32.645	384.568	36.587
		0.95	0.0118	131.3246	39.6815	141.541	42.769	602.4775	35.978	686.619	41.406
	0.7	0.80	0.0111	-	28.9959	-	30.174	-	26.992	-	29.187
		0.85	0.0111	59.3720	31.7267	-	33.376	277.3222	29.259	301.619	32.266
		0.90	0.0111	71.4636	34.9720	75.0981	37.259	337.4734	31.928	363.932	36.024
		0.95	0.0111	124.2775	38.9493	133.945	42.135	570.1478	35.155	649.774	40.768
	0.8	0.80	0.0103	-	-	-	29.165	-	-	-	28.170
		0.85	0.0103	54.8963	30.5344	-	32.254	256.4166	27.972	278.882	31.135
		0.90	0.0103	66.0764	33.6066	69.4369	35.981	312.0333	30.468	336.497	34.743
		0.95	0.0103	114.9090	37.3628	123.848	40.643	527.1677	33.487	600.792	39.287
0.15	0.5	0.80	0.0122	-	28.6868	-	30.279	-	26.082	-	28.908
		0.85	0.0122	63.1070	31.2516		33.442	286.1109	28.102	316.224	31.908
		0.90	0.0122	76.4130	34.2972	81.3482	37.313	350.8490	30.463	388.156	35.598
		0.95	0.0122	130.9020	-	145.156	-	578.2249	-	694.122	-
	0.6	0.80	0.0118	-	28.4971	-	30.141	-	25.807	-	28.747
		0.85	0.0118	60.6060	31.0446	-	33.314	274.7723	27.792	303.692	31.753
		0.90	0.0118	73.3848	34.0636	78.1244	37.187	336.9448	30.111	372.774	35.446
		0.95	0.0118	125.7143	37.7418	139.404	42.076	555.3097	32.883	666.614	40.132
	0.7	0.80	0.0111	-	27.9687	-	29.672	-	25.174	-	28.259
		0.85		57.3538	30.4524	-	32.812	260.0277	27.079	287.396	31.228
		0.90	0.0111	69.4469	33.3893	73.9321	36.631	318.8639	29.307	352.770	34.871
		0.95	0.0111	118.9683	36.9632	131.923	41.440	525.5111	31.974	630.843	39.489
	0.8	0.80	0.0111	-	-	-	28.660	-	-	-	27.230

		0.85	0.0103	53.0303	29.2151	-	31.685	240.4257	25.681	265.730	30.083
		0.90	0.0103	64.2117	31.9775	68.3588	35.352	294.8267	27.744	326.177	33.583
		0.95	0.0103	110.0000	35.3364	121.978	39.954	485.8960	30.219	583.287	38.016
0.2	0.5	0.80	0.0103	-	27.7703	-	-	244.3728	24.559	-	-
		0.85	0.0122	305.1403	30.1323	331.875	32.913	268.9774	26.304	301.956	30.966

Table 6: Continued

W	ρ_{yx}	ρ_{yz}	$MMSE$ (T_0)	$f_2 = 1.5$ PRE $P_{1(SPS)}$	PRE $T_{1(SKM)}$	PRE $P_{2(SPS)}$	PRE $T_{2(SKM)}$	$f_2 = 2.0$ PRE $P_{1(SPS)}$	PRE $T_{1(SKM)}$	PRE $P_{2(SPS)}$	PRE $T_{2(SKM)}$
		0.90	0.0122	371.3252	32.9204	400.437	36.722	331.6165	28.327	376.293	34.546
		0.95	0.0122	627.3390	-	714.953	41.544	534.9187	-	674.323	-
	0.6	0.80	0.0122	-	27.5531	-	29.661	234.6883	24.222	-	27.887
		0.85	0.0118	293.0475	29.8905	318.723	32.776	258.3178	25.924	289.990	30.793
		0.90	0.0118	356.6095	32.6458	384.568	36.587	318.4745	27.900	361.381	34.377
		0.95	0.0118	602.4775	35.9789	686.619	41.406	513.7198	30.240	647.599	38.939
	0.7	0.80	0.0118	-	26.9927	-	29.187	222.0946	23.505	-	27.384
		0.85	0.0111	277.3222	29.2592	301.619	32.266	244.4562	25.125	274.428	30.253
		0.90	0.0111	337.4734	31.9280	363.932	36.024	301.3847	27.010	341.989	33.789
		0.95	0.0111	570.1478	35.1557	649.774	40.768	486.1530	29.247	612.848	38.290
	0.8	0.80	0.0111	-	-	-	28.170	205.3522	-	-	26.336
		0.85	0.0103	256.4166	27.9727	278.882	31.135	226.0281	23.599	253.741	29.091
		0.90	0.0103	312.0333	30.4680	336.497	34.743	278.6651	25.333	316.208	32.492
		0.95	0.0103	527.1677	33.4873	600.792	39.287	449.5048	27.397	566.649	36.821

The primary objective of this study is to evaluate the performances of two prominent classes of estimators of population mean under current occasion in two-occasion successive sampling under non-response: the estimators proposed by Singh *et al*. (2016) and the improved estimators suggested by Sharma *et al.* (2022)**.** The evaluation metrics considered include Percent Relative Efficiency (PRE)**,** Percent Relative Loss (PRL)**,** Minimum Mean Square Error (MMSE)**,** and the optimum unmatched proportion

5.1. Performance Based on Percent Relative Loss (PRL)

The PRL provides a direct measure of the efficiency lost due to non-response relative to an ideal scenario without non-response. From Tables 2, 3,4, and 5, several trends are apparent:

(1) Sharma *et al.* (2022) Estimators $P_{1(SPS)}$ and $P_{2(SPS)}$

(i) The PRL values of $P_{2(SPS)}$ show both positive and negative values across different correlation levels (ρ_{yx} and ρ_{yz}) and correction factors (f_1 and f_2) while some shows inadmissible.

(ii) Negative PRL values indicate apparent efficiency gains due to the estimator adjustment, particularly under strong correlations and higher proportions of non-respondents.

(iii) For lower correlations and lower non-response, PRL remains small but positive, reflecting minimal efficiency loss.

(2) Singh *et al*. (2016) Estimators $T_{1(SKM)}$ and $T_{2(SKM)}$

(i) PRL values of $T_{2(SKM)}$ are consistently smaller in magnitude compared to Sharma et al.'s estimator, suggesting lower sensitivity to extreme non-response scenarios.

However, the estimator occasionally fails to exist for certain high correlation/non-response combinations, as indicated by 'Inadmissible' values in Tables 4 and 5.

5.2. Percent Relative Efficiency (PRE)

PRE measures the relative efficiency of the estimator compared to the natural successive sampling estimator without non-response:

(i) For Sharma *et al.* (2022), PRE values often exceed 100% in strong correlation scenarios, confirming the significant improvement in precision due to the incorporation of optimal unmatched proportions and adjusted estimator constants.

(ii) Singh *et al.* (2016) generally achieves PRE values slightly above 100% for moderate correlations but performance diminishes at very high non-response levels.

5.3. Minimum Mean Square Error (MMSE)

MMSE represents the expected variability of the estimator:

- Sharma *et al.* (2022) consistently produces lower MMSE values than Singh *et al.* (2016) under strong correlation conditions and higher non-response, reflecting greater precision**.**
- Singh *et al.* (2016) estimator's MMSE is comparable to Sharma *et al*. (2022) in low correlation scenarios but increases more rapidly with rising non-response, highlighting the estimator's sensitivity to extreme non-response.
- These findings suggest that the improved linear combination proposed by Sharma et al. successfully reduces estimation variance, especially when auxiliary information is strong.

5.4. Optimum Unmatched Proportion ($\mu_i^{(0)}$)

The optimum unmatched proportion $\mu_i^{(0)}$ plays a crucial role in balancing survey cost and estimator precision:

- Sharma *et al*.'s estimator generally recommends lower $\mu_i^{(0)}$ for weak and moderate correlation but slightly higher under strong correlations and high non-response, suggesting that small units should be sampled afresh on the second occasion to achieve minimum MSE. This reduced survey cost at a higher precision in high non-response situations.
- Singh *et al.'s* estimator often identifies moderate $\mu_i^{(0)}$ values, which reduce survey cost but sometimes at the expense of precision in high non-response situations.

In summary, Sharma *et al.* (2022) achieves higher efficiency (PRE), lower MMSE, moderate unmatched proportion ($\mu_i^{(0)}$), and it is favorable for high correlation and high non-response. On the other hand, Singh *et al.* (2016) records stable performance (moderate PRE), slightly higher MMSE, moderate unmatched proportion ($\mu_i^{(0)}$), and it is more robust for moderate correlation and lower non-response.

6. 0 Conclusion

This study evaluates the estimators proposed by Singh *et al.* (2016) and Sharma *et al*. (2022) under two-occasion successive sampling with non-response. The results show that Sharma *et al*. (2022) estimator consistently outperforms that of Singh *et al*. (2016), exhibiting higher Percent Relative Efficiency (PRE), lower Percent Relative Loss (PRL), and reduced MMSE across most scenarios. This improvement is largely due to its adaptive handling of unmatched and matched portions. Although Singh *et al.* (2016) estimator performs adequately under low non-response and moderate correlations, its efficiency declines as non-response rate increases and the correlation between the study and auxiliary variables becomes strongly positive. Overall, Sharma *et al.* (2022) estimator provides a more reliable and efficient approach for estimating the population mean, particularly when non-response is substantial, offering valuable guidance for practical survey design and cost-effective estimation.

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Declaration

Consent for publication

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Charles Kelechi Ekezie designed the study, developed the theoretical framework for successive sampling under non-response, derived efficiency measures, analyzed the Sharma et al. dataset, and drafted the manuscript. Emmanuel John Ekpenyong contributed to estimator formulation, MMSE and PRE derivations, numerical simulations, and manuscript revision. David Friday Adiele supported literature review, result validation, interpretation of robustness outcomes, and final editing.