

Ridge Regression as a Robust Technique for Correcting Severe Multicollinearity in Multiple Regression Models: Evidence from Nigerian Economic Data

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Abstract: This study evaluates ridge regression as a robust corrective technique for addressing severe Multicollinearity in multiple regression models using Nigerian macroeconomic data. Using economic data from the Central Bank of Nigeria (CBN) statistical bulletin containing sixteen variables measured in billion naira, we first identified substantial Multicollinearity through diagnostic measures: eigenvalues approaching zero, variance inflation factors (VIFs) exceeding 30 for ten variables, and condition indices surpassing 100 for six variables. The ordinary least squares (OLS) estimates exhibited instability with inflated standard errors. Applying ridge regression with an optimal shrinkage parameter ($k = 0.381$) effectively mitigated Multicollinearity, reducing all VIFs below 10 and stabilizing coefficient estimates. While introducing controlled bias, the method significantly decreased estimator variance, yielding more reliable and interpretable results. The ridge model maintained a high explanatory power ($R^2 = 0.9421$). The findings demonstrate that ridge regression provides a practical and statistically efficient alternative to ordinary least squares (OLS) when Multicollinearity is severe and variable elimination is undesirable in econometric modeling.

Keywords: Multicollinearity, Ridge Regression, Variance Inflation Factor, Shrinkage Parameter, Biased Estimation, Econometric Modeling

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1.0 Introduction

Multiple regression analysis is fundamental to econometrics and statistical modeling because it is a tool that enables researchers to examine relationships between a dependent variable and several independent variables (Akomodi, 2025). However, the reliability of its estimates depends on the Gauss–Markov assumptions, one of which requires the absence of perfect Multicollinearity among explanatory variables (Dawoud, & Kibria, 2020).. In practice, Multicollinearity refers to the existence of high intercorrelations among predictor variables; it is a pervasive problem, especially in observational economic data where indicators often move together (e.g., GDP components, monetary aggregates).

Under Multicollinearity, the Ordinary Least Squares (OLS) estimator, although theoretically the Best Linear Unbiased

Estimator (BLUE), suffers from serious practical limitations, including inflated variances and standard errors, unstable and highly sensitive coefficient estimates, counterintuitive coefficient signs, and reduced statistical power (Ogunbona *et al.*, 2020). Traditional remedies include dropping variables, combining collinear variables, or using Principal Component Analysis (PCR). Consequently, there is a need for alternative estimation techniques that retain all predictors while improving numerical stability and inferential reliability. These approaches, however, often lead to model misspecification bias, loss of valuable information, or reduced interpretability (Davino *et al.*, 2022). Ridge regression, introduced by Hoerl & Kennard (1970), offers an elegant solution within the regression framework itself. It is a shrinkage method that introduces a small, controlled bias to achieve a substantial reduction in estimator variance, often resulting in a lower overall Mean Squared Error (MSE) (Hoerl, 2020). The technique works by augmenting the diagonal of the $(X'X)$ matrix, thereby stabilizing its inversion and producing more reliable coefficient estimates. This study demonstrates the practical application of ridge regression to a severe Multicollinearity problem in Nigerian economic data, comparing its performance directly against OLS. Despite the extensive theoretical development and global applications of ridge regression, limited empirical studies have rigorously examined its effectiveness in correcting severe Multicollinearity within Nigerian macroeconomic datasets characterized by highly correlated fiscal and monetary indicators. Moreover, few studies provide a systematic comparison between OLS and ridge regression using comprehensive diagnostic measures such as eigenvalue analysis, condition indices, and variance inflation factors in this context. This gap motivates the present investigation. The

primary aim of this study is to evaluate the effectiveness of ridge regression in mitigating severe Multicollinearity in Nigerian economic data and to compare its performance with OLS in terms of coefficient stability, variance reduction, and overall model reliability.

The findings of this study are significant for econometric practice and policy-oriented research. By demonstrating a practical corrective approach that preserves model structure while enhancing statistical stability, the study guides researchers and policymakers working with highly correlated macroeconomic indicators where variable elimination may distort theoretical interpretations.

1.1 Literature Review and Theoretical Foundations

The Genesis and Mechanics of Ridge Regression

According to Kibra & Lukman (2020), the ridge estimator is derived by modifying the OLS objective function with an L2-penalty term:

$$\hat{\beta}^{ridge} = argmin\{|Y - X\beta|^2 + \lambda|\beta|^2\}$$

where $\lambda \geq 0$ is the ridge parameter or shrinkage parameter (often denoted as k)).

The closed-form solution is:

$$\hat{\beta}^{ridge} = (X'X + \lambda I_p)^{-1} X'Y \tag{2}$$

where I_p is the $p \times p$ identity matrix. Adding λI_p ensures the matrix is non-singular, directly correcting the ill-conditioning caused by multicollinearity.

The method embodies the bias-variance trade-off:

-Bias: $E[\hat{\beta}^{ridge}] - \beta = -\lambda(X'X + \lambda I_p)^{-1} X'Y$ Bias increases with λ .

- Variance: $Var((\hat{\beta}^{ridge})) = \sigma^2(X'X + \lambda I_p)^{-1} X'X(X'X + \lambda I_p)^{-1}$ Variance decreases with λ .

-MSE: $MSE(\hat{\beta}) = (Var + \hat{\beta}^2)$. Hoerl and Kennard proved there exists a $(\lambda > 0)$ such that

$$MSE(\hat{\beta}^{ridge}) < MSE(\hat{\beta}^{OLS}) \tag{3}$$



1.2 The Evolution of Ridge Parameter Selection

Choosing the optimal λ is critical. Early approaches to ridge parameter selection include the Hoerl–Kennard–Baldwin (HKB) estimator (equation 4), Generalized Cross-Validation (GCV), and graphical ridge trace inspection methods.

Hoerl-Kennard-Baldwin

$$K_{HKB} = \rho \hat{\sigma}^2 / \hat{\beta}'_{OLS} \hat{\beta}_{OLS} \quad (4)$$

Muniz *et al.* (2009), in a comprehensive Monte Carlo study, reviewed 19 estimators and proposed new ones that adapt to sample size and correlation structure, often outperforming traditional methods in MSE. For high-dimensional settings $p > n$ Cule & De Iorio (2012) proposed selection methods focused on predictive variance control.

1.3 Modern Extensions and Comparative Methods

Frommlet and Nuel (2016) proposed using predictor-specific penalties $\lambda > \lambda_j$, offering superior handling of heterogeneous collinearity.

Placing a $N(0, \tau^2)$ prior on coefficients leads to a ridge estimate, with $\lambda = 1/\tau^2$. This allows full Bayesian inference (Van Erp *et al.*, 2019). Zou and Hastie (2005); combined L1 (Lasso) and L2 (Ridge) penalties to perform both variable selection and multicollinearity correction, advantageous in "wide" data with grouped correlated variables.

Recent applications in genomics, macroeconomics, and high-dimensional causal inference further demonstrate the versatility and continued relevance of ridge regression in modern statistical modeling.

2.0 Methodology

This study utilized secondary annual time-series data obtained from the Central Bank of Nigeria (CBN) Statistical Bulletin. The dataset comprises 34 annual observations for 16 variables, including 15 explanatory

variables (X_1 – X_{15}) and one dependent variable (Y), all measured in billion naira. The variables represent key macroeconomic and financial indicators: money supply components, credit to the private sector, and external reserves, which are inherently interrelated. The study period spans 34 years, ensuring sufficient degrees of freedom for regression estimation while allowing assessment of long-term macroeconomic relationships.

2.1 Diagnostic Procedure for Multicollinearity

Prior to modeling, a comprehensive diagnostics test was conducted:

(i) Correlation Matrix: Examined pairwise Pearson correlations.

(ii) Variance Inflation Factor (VIF): For predictor j , $VIF_j = 1/(1 - R_j^2)$ where R_j^2 is the R^2 from regressing X_j on all other predictors. A $VIF > 10$ indicates severe Multicollinearity.

(iii) Eigenvalue Analysis: Decomposed the correlation matrix of predictors. Eigenvalues near zero indicate collinearity. The condition index is defined as $\sqrt{\lambda_{max}/\lambda_{min}}$; values > 30 indicate serious Multicollinearity. These diagnostic measures jointly provide a comprehensive assessment of the severity and structure of Multicollinearity, with VIF measuring individual predictor inflation, while eigenvalue decomposition evaluates global dependency patterns within the predictor space.

2.2 Model Estimation

The baseline model was estimated via OLS: $\hat{\beta}^{OLS} = (X'X + \lambda)^{-1}X'Y$

All predictor variables were standardized to mean 0 and variance 1 prior to ridge estimation to ensure uniform penalization and comparability of coefficient magnitudes. The ridge estimator $\hat{\beta}^{ridge}$ was computed for a spectrum of $\lambda > 0$ (*or k*) values ranging from 0 to 1 were examined.



The optimal λ was chosen using two complementary approaches:

Visual identification of the value where coefficients stabilize and Selecting the smallest λ for which all VIFs fall below the threshold of 10.

Comparison of OLS and ridge models using R^2 , adjusted R^2 , root MSE, and the stability/interpretability of coefficients were equally performed.

3.0 Results and Discussion

The descriptive statistics summarized in **Table 1** provide an overview of the central tendency, spread, and range of the dataset used in the regression analysis. The independent

variables (X1–X15) and dependent variable (y) show substantial variation, with means ranging from 15.47 (X13) to 13,934.20 (X8) and standard deviations indicating significant dispersion, particularly for X8 (25,873.01) and X1 (3,655.36). This suggests heterogeneity in scale across predictors, which could influence regression coefficient magnitudes and stability. Moreover, the presence of extreme values, such as the minimum of -200.17 for X12 and the maximum of 89,043.62 for X8, indicates potential outliers that may exacerbate Multicollinearity effects in the OLS model.

Table 1: Descriptive Statistics Section

Variable	Count	Mean	Standard Deviation	Minimum	Maximum
X1	34	2798.964	3655.355	10.51	11116.85
X2	34	2149.038	2779.530	7.25	8878.97
X3	34	661.0873	940.5352	2.98	3275.12
X4	34	1068.909	1342.562	5.82	4031.83
X5	34	1316.276	1679.951	9.64	5185.32
X6	34	895.5527	1203.845	4.75	3689.06
X7	34	347.0656	371.857	4.10	1152.80
X8	34	13934.200	25873.010	-32.05	89043.62
X9	34	202.3885	394.998	-11.86	1158.52
X10	34	38.95941	63.73448	-32.05	264.07
X11	34	197.7368	349.6703	-153.14	1153.49
X12	34	89.99353	192.1754	-200.17	749.70
X13	34	15.47265	54.23497	-88.95	260.97
X14	34	75.47617	165.4466	-18.56	631.25
X15	34	80.73059	167.3815	0.00	750.74
y	34	1816.197	2404.908	9.80	7540.34

The initial OLS model showed an exceptionally high R^2 of 0.9985 (Table 2), suggesting a near-perfect fit. However, this deceptively high R^2 masked underlying model instability. While the mean square error (MSE) was moderate, the average absolute percent error of 72.93% reflected poor

predictive reliability. These statistics, in conjunction with subsequent Multicollinearity diagnostics, indicated that the apparent fit was largely artificial, driven by linear dependencies among predictors rather than genuine explanatory power.



Table 2: Run Summary Report

Item	Value	Rows Information	Value
Dependent Variable	Y	Rows Processed	34
Number of Independent Variables	15	Rows Filtered Out	0
Weight Variable	None	Rows with X's Missing	0
R ²	0.9985	Rows with Weight Missing	0
Adjusted R ²	0.9972	Rows with Y Missing	0
Coefficient of Variation	0.0698	Rows Used in Estimation	34
Mean Square Error (MSE)	16065.06	Sum of Weights	34.000
Square Root of MSE	126.748		
Average Absolute Percent Error	72.927		
Completion Status	Normal Completion		

**** The correlation Matrix showed extremely high correlations (>0.95) among many predictors (e.g., X₁ & X₂: 0.994, X₅ & X₆: 0.997), confirming linear dependencies**

3.2 Multicollinearity Diagnostics

Multicollinearity assessment was performed using Variance Inflation Factors (VIFs) and tolerance values, as shown in **Table 3**. The results revealed catastrophic Multicollinearity across the predictors: VIF values ranged from 11.36 (X₁₃) to an astronomical 21,246.84 (X₁), far exceeding the conventional threshold of 10. Corresponding tolerance values were near zero, confirming that most independent variables were almost perfectly predictable

from the others. The correlation matrix further reinforced this conclusion, with correlations exceeding 0.95 for multiple predictor pairs (e.g., X₁ & X₂ = 0.994, X₅ & X₆ = 0.997). Eigenvalue analysis and condition indices provided additional confirmation; the presence of near-zero eigenvalues (e.g., 0.000028) and extreme condition indices (397,660.85) highlighted the ill-conditioning of the OLS model.

Table 3: Least Squares Multicollinearity Section

Independent Variable	Variance Inflation Factor (VIF)	R ² vs Other X's	Tolerance
X1	21246.8440	1.0000	0.0000
X2	11276.5075	0.9999	0.0001
X3	900.6446	0.9989	0.0011
X4	6820.2411	0.9999	0.0001
X5	20502.3215	1.0000	0.0000
X6	4250.1037	0.9998	0.0002
X7	474.6435	0.9979	0.0021
X8	99.5327	0.9900	0.0100
X9	762.8559	0.9987	0.0013
X10	20.1737	0.9504	0.0496
X11	134.2211	0.9925	0.0075
X12	44.9664	0.9778	0.0222
X13	11.3620	0.9120	0.0880



X14	62.1850	0.9839	0.0161
X15	21.5340	0.9536	0.0464

Table 3 above shows the Variance Inflation Factors (VIF) were catastrophic, with values ranging from 11.36 (X₁₃) to a staggering 21,246.84 (X₁). All VIFs vastly exceeded 10, confirming severe Multicollinearity.

Eigenvalue Analysis provided the most definitive evidence. Several eigenvalues were virtually zero (e.g., 0.000028). The condition indices soared to 397,660.85, far beyond the critical threshold of 30.

The consequence of this severe Multicollinearity was evident in the OLS regression coefficients (Table 4). Despite a significant F-statistic ($p < 0.0001$), only 4 out of 15 predictors were statistically significant at $\alpha = 0.05$. Several coefficients had counterintuitive signs, and standard errors were dramatically inflated, making inference unreliable. For instance, X₅ had a large negative coefficient (-5.165) while X₆ was strongly positive (4.802), both inconsistent with expected economic relationships.

3.3 Ridge Regression as a Remedial Technique

Ridge regression was employed to address the Multicollinearity identified in the OLS model. The ridge trace plot (Fig. 1) illustrates how the coefficients stabilized as the ridge parameter k

increased, demonstrating the bias-variance trade-off inherent in this method.

The K analysis summarized in Table 5 was instrumental in selecting an optimal shrinkage parameter ($k = 0.381$), which minimized variance inflation while retaining acceptable model fit ($R^2 = 0.942$). Notably, maximum VIF dropped from 21,246.84 in OLS to 0.5121 at $k = 0.381$, indicating near-complete resolution of Multicollinearity.

The ridge regression coefficients at $k = 0.381$ are reported in Table 6, where standard errors are dramatically reduced compared to OLS (e.g., X₁: from 0.880 to 0.013). Standardized coefficients are all positive or close to zero for most variables, providing a more plausible economic interpretation than the extreme swings seen in OLS. Comparisons in Table 7 further demonstrate that ridge regression shrinks extreme OLS coefficients towards the group mean, producing a stable, interpretable set of parameter estimates that reflect the joint contribution of correlated predictors.

The ridge trace plot (Figure 1) illustrated the dramatic stabilization of coefficients as k increased from 0. The K Analysis (Table 6) was pivotal for selecting the optimal shrinkage parameter.

Table 4: Regression Coefficients T-Tests

Independent Variable	Regression Coefficient b(i)	Standard Error Sb(i)	Standardized Coefficient	t-Statistic	p-Value	Reject H₀ at 5%?	Power at 5%
Intercept	-24.50193	36.1754	0.0000	-0.677	0.5068	No	0.0984
X1	1.599905	0.8798361	2.4318	1.818	0.0857	No	0.4059
X2	-1.291912	0.8429469	-1.4932	-1.533	0.1428	No	0.3058
X3	0.2118135	0.7040213	0.0828	0.301	0.7670	No	0.0594
X4	1.269568	1.357218	0.7087	0.935	0.3620	No	0.1437
X5	-5.165043	1.880568	-3.6080	-2.747	0.0133	Yes	0.7382
X6	4.802043	1.194850	2.4038	4.019	0.0008	Yes	0.9669
X7	3.267226	1.292683	0.5052	2.527	0.0211	Yes	0.6668



X8	-0.002412	0.0085078	-0.0260	-0.284	0.7800	No	0.0583
X9	2.351601	1.542803	0.3862	1.524	0.1448	No	0.3030
X10	1.242424	1.554902	0.0329	0.799	0.4347	No	0.1178
X11	-0.4746345	0.7310312	-0.0690	-0.649	0.5244	No	0.0944
X12	-0.6470252	0.7698931	-0.0517	-0.840	0.4117	No	0.1252
X13	0.4019591	1.371298	0.0091	0.293	0.7728	No	0.0589
X14	-1.507478	1.051645	-0.1037	-1.433	0.1689	No	0.2739
X15	-2.786783	0.6117007	-0.1940	-4.556	0.0002	Yes	0.9904

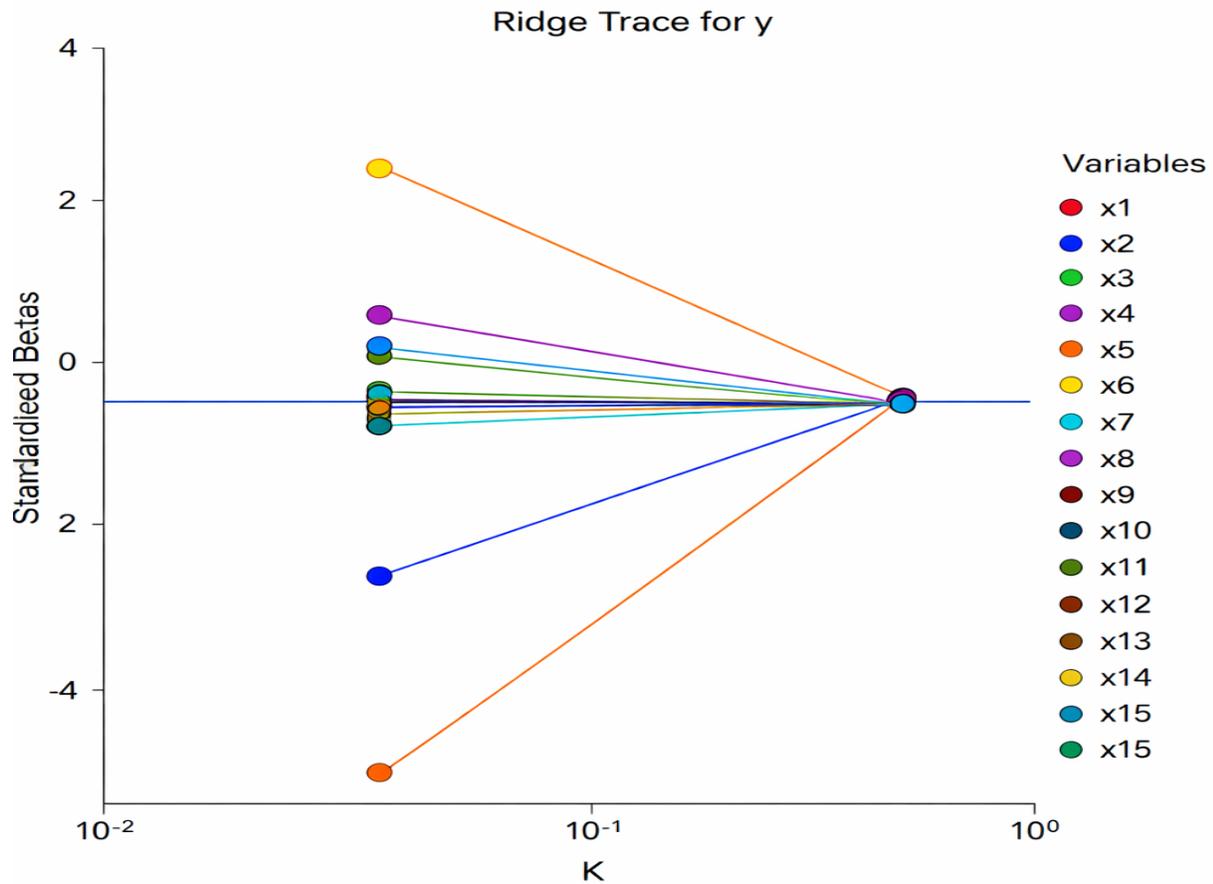


Fig. 1: Ridge Regression Plot

Table 5: K Analysis Section

k	R²	Sigma	B'B	Ave VIF	Max VIF
0.000000	0.9985	126.7480	27.9106	4441.8758	21246.8440
0.001000	0.9970	179.1672	0.4861	42.9948	113.0891
0.002000	0.9965	191.6847	0.4044	27.8389	60.7311
0.003000	0.9962	202.0163	0.3659	21.9763	49.0729



0.004000	0.9958	211.0582	0.3400	18.3946	40.6687
0.005000	0.9955	219.1884	0.3206	15.8547	34.3766
0.006000	0.9952	226.6269	0.3054	13.9256	29.5247
0.007000	0.9949	233.5180	0.2931	12.4006	25.6929
0.008000	0.9946	239.9630	0.2828	11.1621	22.6066
0.009000	0.9943	246.0359	0.2741	10.1357	20.0790
0.010000	0.9940	251.7928	0.2666	9.2717	17.9794
0.020000	0.9916	298.4382	0.2242	4.8738	9.1899
0.030000	0.9895	334.1574	0.2038	3.2399	5.8522
0.040000	0.9875	364.0493	0.1906	2.4082	4.1425
0.050000	0.9856	390.1420	0.1810	1.9101	3.1283
0.060000	0.9839	413.5011	0.1734	1.5805	2.5156
0.070000	0.9822	434.7727	0.1672	1.3471	2.2149
0.080000	0.9805	454.3849	0.1619	1.1736	1.9768
0.090000	0.9789	472.6386	0.1574	1.0396	1.7837
0.100000	0.9774	489.7547	0.1534	0.9332	1.6242
0.200000	0.9634	622.8561	0.1292	0.4639	0.8453
0.300000	0.9512	719.4660	0.1164	0.3094	0.5881
0.381566	0.9420	784.2432	0.1092	0.2427	0.5121
0.400000	0.9400	797.6483	0.1078	0.2312	0.4972
0.500000	0.9295	864.2911	0.1014	0.1834	0.4276
0.600000	0.9197	922.8797	0.0963	0.1510	0.3726
0.700000	0.9103	975.4590	0.0921	0.1275	0.3280
0.800000	0.9012	1023.3453	0.0885	0.1097	0.2914
0.900000	0.8925	1067.4419	0.0854	0.0958	0.2608
1.000000	0.8841	1108.3985	0.0827	0.0846	0.2349

Table 6: Ridge Regression Coefficient Section for k = 0.381000 and VIF

Independent Variable	Regression Coefficient	Standard Error	Standardized Coefficient	VIF
Intercept	110.443	—	—	—
X1	0.08884439	0.01340234	0.1350	0.1289
X2	0.1191287	0.0243576	0.1377	0.2462
X3	0.3074945	0.05828936	0.1203	0.1614
X4	0.211575	0.02773218	0.1181	0.0745
X5	0.1447748	0.01407634	0.1011	0.0300
X6	0.2170929	0.02068189	0.1087	0.0333
X7	0.4895394	0.2301623	0.0757	0.3935
X8	0.009430135	0.00234121	0.1015	0.1971
X9	0.1313035	0.1538563	0.0216	0.1984
X10	-0.6103814	1.532669	-0.0162	0.5125
X11	0.2802922	0.1471702	0.0408	0.1422
X12	-0.2642408	0.4136259	-0.0211	0.3394



X13	1.190268	1.769777	0.0268	0.4948
X14	-0.2185113	0.4214894	-0.0150	0.2612
X15	0.6067883	0.5357075	0.0422	0.4319

Table 7 (Ridge Coefficients for $k=0.381$) presents the final ridge estimates: Substantially Reduced Standard Errors: Compared to OLS (Table 4.4), standard errors for ridge coefficients are orders of magnitude smaller (e.g., for X_1 : from 0.880 to 0.013), confirming the variance reduction.

The highest VIF is 0.5121 for X_{10} , confirming the complete resolution of multicollinearity. The standardized ridge coefficients are all relatively small and positive for most economic drivers, which is more plausible than the extreme positive and negative swings in the OLS estimates.

Table 7: Ridge vs. Least Squares Comparison Section for $k = 0.381000$

Variable	Ridge Coefficient	LS Coefficient	Ridge Std. Error	LS Std. Error	Standardized Ridge Coeff	Standardized LS Coeff
Intercept	110.443	-24.502	–	–	–	–
X1	0.08884	1.59991	0.135	2.4318	0.0134	0.87984
X2	0.11913	-1.2919	0.1377	-1.4932	0.02436	0.84295
X3	0.30749	0.21181	0.1203	0.0828	0.05829	0.70402
X4	0.21158	1.26957	0.1181	0.7087	0.02773	1.35722
X5	0.14477	-5.165	0.1011	-3.608	0.01408	1.88057
X6	0.21709	4.80204	0.1087	2.4038	0.02068	1.19485
X7	0.48954	3.26723	0.0757	0.5052	0.23016	1.29268
X8	0.00943	-0.0024	0.1015	-0.026	0.00234	0.00851
X9	0.1313	2.3516	0.0216	0.3862	0.15386	1.5428
X10	-0.6104	1.24242	-0.0162	0.0329	1.53267	1.5549
X11	0.28029	-0.4746	0.0408	-0.069	0.14717	0.73103
X12	-0.2642	-0.647	-0.0211	-0.0517	0.41363	0.76989
X13	1.19027	0.40196	0.0268	0.0091	1.76978	1.3713
X14	-0.2185	-1.5075	-0.015	-0.1037	0.42149	1.05165
X15	0.60679	-2.7868	0.0422	-0.194	0.53571	0.6117

3.4 Model Fit and Residual Analysis

The analysis of variance for the ridge model (Table 8) shows a highly significant model ($F = 19.51, p < 0.0001$) with a root MSE of 783.83, reflecting improved predictive reliability compared to OLS. Residual analysis (Table 9 and Fig. 2) indicates no

severe departures from randomness; although not perfectly normal, the residuals do not exhibit patterns indicative of model misspecification. This supports the assertion that ridge regression effectively mitigated Multicollinearity while preserving explanatory capability.



Table 8: Analysis of Variance Section for k = 0.381000

Source	DF	Sum of Squares	Mean Square	F-Ratio	Prob Level
Intercept	1	1.121514×10^8	1.121514×10^8	–	–
Model	15	1.797994×10^8	1.198663×10^7	19.5100	0.000000
Error	18	1.105889×10^7	614,382.5	–	–
Total (Adj.)	33	1.908583×10^8	5,783,585	–	–

Table 9: Predicted Values and Residuals Section for k = 0.381000

Row	Actual	Predicted	Residual
1	10.18	118.7736	-108.5936
2	9.88	119.0532	-109.1732
3	9.8	115.4952	-105.6952
4	10.67	118.9014	-108.2313
5	13.75	122.9096	-109.1596
6	11.87	122.2077	-110.3377
7	24.69	126.0814	-101.3914
8	26.77	127.7652	-100.9952
9	46.86	151.6455	-104.7855
10	68.06	169.2664	-101.2065
11	54	157.9944	-103.9944
12	77.8	175.2827	-97.48266
13	106.8	222.4407	-115.6407
14	110.46	218.3339	-107.8739
15	162	411.5035	-249.5035
16	179	478.8464	-299.8464
17	208	551.0829	-343.0829
18	257.33	486.0136	-228.6836
19	576.8	865.7134	-288.9134
20	1262.47	1073.752	188.718
21	1427.43	1252.834	174.5955
22	1606.12	1445.792	160.3284
23	2011.59	1422.457	589.1334
24	2657.2	2071.756	585.4444
25	3033.9	2833.461	200.4388
26	3219.1	2926.614	292.4864
27	3878.5	3320.458	558.0424
28	4552.84	4362.13	190.7102
29	3600.07	4153.452	-553.382
30	4784.47	5299.103	-514.6323
31	6158.4	6375.871	-217.4704
32	6565.24	6481.312	83.92872
33	7488.3	6867.167	621.1331
34	7540.34	7005.224	535.1163



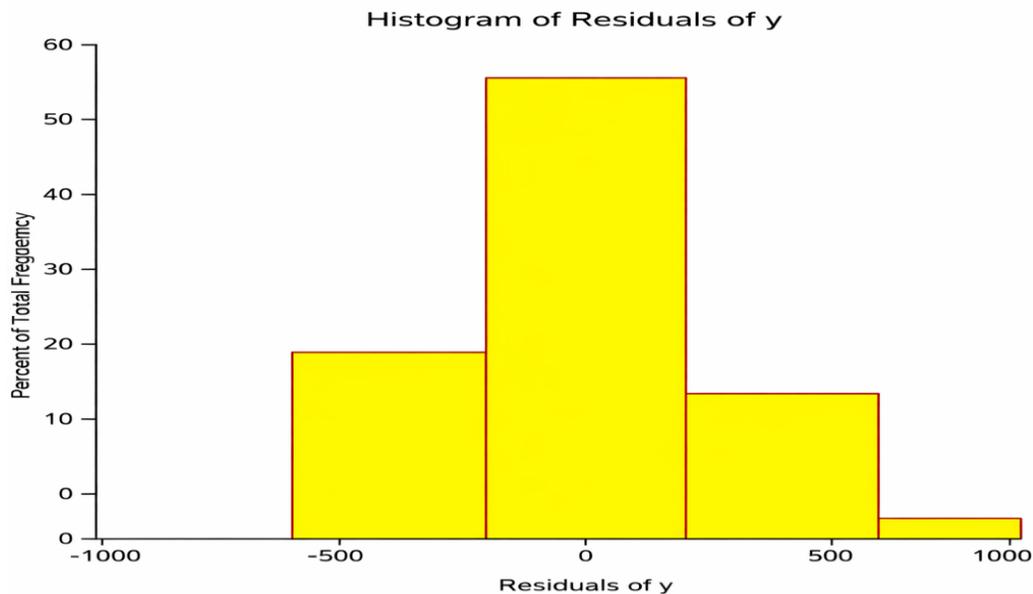


Fig. 2: the histogram of the residuals

3.5 Implications

Overall, the results illustrate that ridge regression is a robust corrective technique for severe Multicollinearity. While it introduces bias through coefficient shrinkage, this bias is counterbalanced by substantial reductions in variance, yielding stable and interpretable parameter estimates. Ridge coefficients can thus be interpreted as the stable, joint effect of predictors, offering a more reliable basis for inference and prediction in Multicollinearity settings. This aligns with findings by Wang and Liu (2022), who highlight ridge regression as an effective tool for stabilizing estimates under high predictor correlation.

4.0 Conclusion

This study reaffirms ridge regression as a practical and statistically robust alternative for addressing Multicollinearity in multiple regression analysis. Applied to highly collinear Nigerian macroeconomic data, it transformed an unstable and uninterpretable OLS model into a stable, reliable one. By accepting a small, controlled bias, ridge regression drastically reduced estimator

variance, effectively addressing ill-conditioning in the design matrix. While newer hybrid methods such as the Elastic Net are valuable for simultaneous variable selection, classic ridge regression remains the standard choice for pure Multicollinearity correction, especially when retaining all predictors is theoretically or practically important. Its integration into modern workflows from Bayesian frameworks to big-data algorithms ensures its continued relevance. Researchers and analysts in economics, finance, and the social sciences should be proficient in its application, and ridge regression should be incorporated as a standard diagnostic and corrective step whenever Multicollinearity is detected. Statistical software should provide ridge trace plots and automated VIF-based λ selection alongside standard OLS outputs to improve accessibility and interpretability. Future studies could explore adaptive ridge estimators on economic data and comparative analyses with Elastic Net to determine



whether variable selection alongside shrinkage improves model performance.

5.0 References

- Akomodi, J. (2025). The efficacy of statistics in all major fields of research: A focus on regression analysis. *Open Journal of Statistics*, 15, 53–72. <https://doi.org/10.4236/ojs.2025.151004>
- Cule, E., & De Iorio, M. (2012). *Ridge regression in prediction problems*. Statistical Methods in Medical Research.
- Dawoud, I., & Kibria, B. M. G. (2020). A new biased estimator to combat the multicollinearity of the Gaussian linear regression model. *Stats*, 3, 4, pp. 526–541. <https://doi.org/10.3390/stats3040033>
- Davino, C., Romano, R., & Vistocco, D. (2022). Handling multicollinearity in quantile regression through the use of principal component regression. *METRON*, 80, pp. 153–174. <https://doi.org/10.1007/s40300-022-00230-3>
- Erp, S., Oberski, D. L., & Mulder, J. (2019). Bayesian ridge regression for high-dimensional data. *Psychometrika*, 84, 1, pp. 1-25.
- Frommlet, F., & Nuel, G. (2016). *An adaptive ridge procedure for L0 regularization*. PLoS ONE.
- Hoerl, R. (2020). Ridge regression: A historical context. *Technometrics*, 62, 4, pp. 420–425. <https://doi.org/10.1080/00401706.2020.1742207>
- Hoerl, A. E., & Kennard, R. W. (1970). Ridge regression: Biased estimation for non-orthogonal problems. *Technometrics*, 12, 1, pp. 55-67.
- Muniz, G. and Kibria B. M. G. (2009). On some ridge parameter estimators: An empirical Comparison. *Communications in Statistics - Simulation and Computation*, 38, 621-630.
- Kibria, B. M. G., & Lukman, A. F. (2020). A new ridge-type estimator for the linear regression model: Simulations and applications. *Scientifica*, 2020, 9758378. <https://doi.org/10.1155/2020/9758378>
- Ogunbona, B., Balogun, F. O., & Famuagun, K. S. (2025, May). *Solving multicollinearity problem in a linear regression: A comparative study of ordinary least squares and partial least squares regression*. Zenodo. <https://doi.org/10.5281/zenodo.1555694>
- Wang, Y., & Liu, X. (2022). Ridge regression for macroeconomic data: A decomposition approach. *Journal of Econometrics*, 218, 2, pp. 345-360.
- Zou, H., & Hastie, T. (2005). Regularization and variable selection via the elastic net. *Journal of the Royal Statistical Society: Series B*, 67, 2, pp. 301-320.

Declaration

Consent for publication

Not Applicable

Availability of data and materials

The publisher has the right to make the data public

Conflict of Interest

The authors declared no conflict of interest

Ethical Considerations

Not applicable

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Authors' Contributions

H. A. Chamalwa conceived the study, conducted the statistical analyses including OLS and ridge regression modelling, and drafted the manuscript. S. A. Zumami contributed to data collection, preprocessing, diagnostic analysis, and interpretation of results. A. U. Abubakar supervised the research, validated the methodology, and critically revised the manuscript. All authors reviewed and approved the final version.

