Communication in Physical Sciences, 2025, 12(8):2528-2536

Mathematics in Technology and Engineering: Modeling, Optimization, Algorithms, and Emerging Industrial Applications

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Received: 17 July 2025/Accepted: 03 December 2025/Published: 13 December 2025
https://doi.org/10.4314/cps.v12i8.27

Abstract: Mathematics is still one of the most important bases of innovation in technology and engineering. From manufacturing to AI to cybersecurity to banking to healthcare to energy systems to supply chains, the ideas of mathematics can be used to model, algorithmize, simulate and make decisions about complex problems in the real world that can be evaluated and improved. This paper deals with the role of mathematical modeling, optimization, computation, machine learning, digital twins and risk analysis in the current industrial practice. Recent literature indicates that the importance of mathematics has increased with the implementation of data-driven automation, Industry 4.0 technologies, predictive analytics and cyber-physical systems in firms. The paper argues that mathematics is not just a background subject for design, prediction, control, security and decision-making, but an applied language. Limitations pertaining to data quality, model assumptions, ethical governance, and the necessity of interdisciplinary training are also covered. The study concludes that more integration of mathematical thinking with engineering judgment, computational tools, and appropriate innovation will be necessary for future technological advancement.

***Keywords**: Digital twins, machine learning, engineering, technology, optimisation, and Industry 4.0*

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1.0 Introduction

Technology has become integral to almost every facet of life today, from banking to public infrastructure, transportation, agriculture, health care, and communication. Many obvious technology products are based on mathematical concepts that enable them to be dependable, scalable, and effective. Engineers use mathematics as a language to explain physical systems, quantify uncertainty, create algorithms, examine data, and assess conflicting options. For this reason, it is important to learn the mathematics behind modern technology to understand its development.

The relationship between technology and mathematics is not new. Classical engineering relied on geometry, mechanics, algebra, and differential equations to design bridges, roads, machinery, aircraft, and power systems. In contemporary engineering, however, mathematical methods have assumed an increasingly central role alongside physical design, particularly in digital systems, automation, optimization, and data-driven decision-makine confirm that the fundamental framework ov for the representation of complicated real-world systems is rooted in mathematical mod 1 while optimization is effective in creating solutions with respect to technological, economical and operational constraints (Ali et al., 2015; Raghuvanshi et al., 2025). The contemporary developments show that mathematical modeling is being combined with computational engineering, artificial intelligence, machine learning, uncertainty quantification and simulation technologies (Talib, 2023). Thus, mathematics has transformed from an analytical tool to an integral part of modern engineering and technological innovations. There is no doubt about the importance of technology for mathematics. In order to develop bridges, roads, machines, airplanes and power systems, classical engineering uses geometry, mechanics, mathematics and differential equations. However, in the modern world mathematics became more significant than physical design. For the creation of digital systems and automation of decision-making the modern enterprises use linear algebra, statistics, optimization, probability, graph theory, stochastic processes, numerical analysis and discrete mathematics. There are some fields in which the application of mathematical reasoning is inherent to technological practice: artificial intelligence, cybersecurity, digital twins, smart manufacturing and predictive analytics (Bertsimas & Kallus; Bishop & Bishop, 2024; Deisenroth et al., 2020).

The present article is devoted to the direction of applications in industry and technology, and it considers the way mathematical reasoning becomes applied in practice with the help of mathematical modeling, optimization, computation, artificial intelligence, machine learning, digital twins and risk analysis. The previous literature showed the application of mathematical optimization for solving such complex problems as scheduling, logistics, inventory management, transportation, resource allocation, etc. (Ali et al., 2015; de Freitas & Fu, chigami, 2022). Also, resent studies have confirmed that computational tools, machine learning and hybrid optimization are useful options for the improvement of prediction and decision making (Raghuvanshi et al., 2025; Talib, 2023). Thus, it can be said that mathematical methods work as practical mechanisms that transform data, physical principles and operational constraints to technological decisions.

This article focuses on industrial applications and considers the role of mathematics in engineering and technology. Concepts covering mathematical modeling, optimization, machine learning, digital twins, cybersecurity, health care analytics, finance, energy systems and supply chain management are are presented (Javaid et al., 2023). The key message is that mathematics is not just a science for the engineer, but is an active tool for innovation. Algorithms enable machines to learn from patterns, simulation enables engineers to test systems prior to expensive real-world deployment, and mathematical models enable companies to question what might happen. Previous research works have claimed to explore mathematical modeling, optimization, computational engineering, etc., in certain fields. Nevertheless, most of the recently written works have been dedicated to particular techniques or applications of these techniques in particular industries. For instance, the previous literature has concentrated on the application of optimization in industries, engineering computation, scheduling, and modeling methodologies that use artificial intelligence (Ali et al., 2015; de Freitas & Fuchigami, 2022; Talib, 2023; Raghuvanshi et al., 2025). Nevertheless, there is a need for a comprehensive review of these methodologies and applications as far as the connections between the mathematics behind these methods and their applications in modern days of engineering and technology are concerned. That is to say, there are no integrative works that systematize the connections that exist between mathematical modeling, optimization, machine learning, digital twins, cybersecurity, health analytics, finance, energy, and supply chain management. Therefore, the main objective of the present review is to give an integrated evaluation of the importance of mathematics in modern engineering and technology, especially as regards mathematical modeling, optimization, computation, machine learning, digital twins, and cybersecurity..

The review also examine how these mathematical approaches are applied across manufacturing, supply chains, healthcare, finance, energy systems, and other technology-intensive sectors.The purpose and scope of the article is to provide a comprehensive overview of the study. This paper is not an experimental investigation, but a conceptual review. It provides an overview of the latest research on applied analytics, digital twins, artificial intelligence, industrial optimization, and mathematical modeling. The emphasis is on the transfer of mathematical ideas from theory to practice and how they promote creativity in practical systems.

This review is significance because it provides approach to interdisciplinary perspective, based on the evaluation of mathematics as foundational academic discipline and an applied language for the modelling, forecasting. prediction, optimization, control, risk assessment, and technological decision-making. By bringing together developments that are often discussed separately, the review provides a conceptual link between mathematical theory and practical engineering systems. This perspective may be useful to engineers, mathematicians, computer scientists, technology developers, educators, researchers, and industrial practitioners seeking to understand how mathematical methods can support innovation, efficiency, reliability, sustainability, and responsible technological development.

2.0 The Mathematical Basis of Technological Innovation

Mathematics is the foundation of technology through organized ways of representing reality. Real world systems are often too complex to understand directly, so engineers use equations, constraints, variables, graphs, or probability distributions to simplify the system. While complexity is not entirely eliminated by this method, it is made more manageable. Processing times, machine capacities, inventory levels, and delivery constraints, for instance, can all be used to illustrate a manufacturing line. Bandwidth, routing rules, nodes, and edges can all be used to depict a communication network. Predicted outcomes, event probabilities, and patient characteristics can be used to illustrate a medical risk model.

In technology and engineering, a number of mathematical fields are particularly crucial. Calculus is used to analyze rates of change and accumulation, and is therefore crucial to control systems, fluid mechanics, signal processing, and optimization. Linear algebra is used in machine learning, computer graphics, image processing, robotics, and equation systems. Uncertainty analysis, risk prediction, quality assurance, and data-driven inference are all supported by probability and statistics. Discrete mathematics and number theory are used in digital logic, networks, encryption and algorithms. Tools for selecting the optimal course of action under constraints are provided by optimization. These domains are now more and more overlapping in modern applications, especially when real-time data and mathematical models are combined (Brunton & Kutz, 2022).The conceptual framework underpinning this review is shown in Fig,1

2.1 Engineering Simulation and Mathematical Modeling

The most apparent application of mathematics in engineering and technology is mathematical modeling. An arrangement of a system that can be analyzed, experimented upon, simulated and improved is a model. The models can either be stochastic (with uncertainty being represented) or deterministic (having the same output for similar input). Models are empirical (based on the facts), physical (based on the physical laws), or both. Engineering models allow predicting performance, failure, cost estimates and comparing various design alternatives before their implementation.

The classical models consist of the differential equations, algebraic equations and numerical algorithms that describe physical processes, such as the transfer of heat, chemical reactions, flow of fluids, stress in structures and electric signals. Since most technological systems operate in the real world, where reliability and safety matter greatly, such models remain valuable. Quantitative models of the load, risk, efficiency and failure of a bridge, an airplane, medical instrument or energy system are necessary for the evaluation of the performance; intuition is not enough.With the advent of advanced technologies, the conventional modeling was supplemented with hybrid and data-based modeling. Data-driven approach combines the measurable data and computational methods to detect trends and predict system behavior (Brunton & Kutz, 2022). The hybrid modeling involves statistical inference or machine learning along with mechanical equations. This approach is especially valuable in industry, where the real-time information from sensors can increase the quality of the model or where the physical laws are known, but not entirely clear.

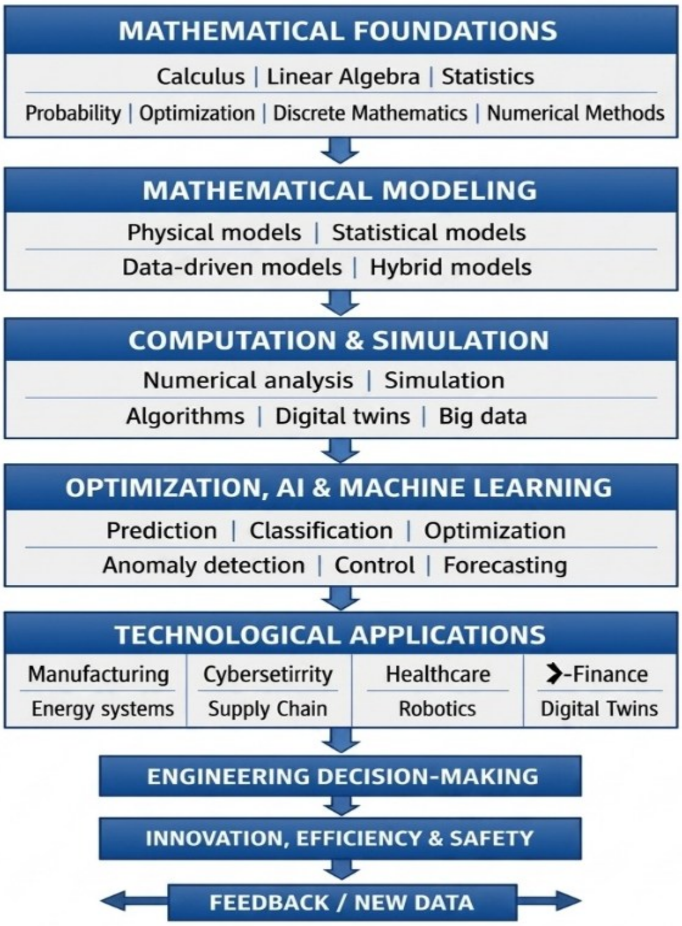

Fig. 1. Conceptual framework illustrated by a flowchart showing the role of mathematics in technological innovation and engineering practice

2.2 Optimization and Operations Research for Industrial Engineering

A key field of applied mathematics that aids businesses in making better choices under pressure is optimization. In other words, optimization is the question of how to maximize or minimize an objective subject to constraints. A business might want to reduce the cost of production, boost production, reduce waiting times, reduce waste or improve delivery reliability.These objectives are usually limited by the availability of manpower, machine resources, raw materials, energy consumption, budget constraints, safety regulations, and consumer demands. Linear programming, integer programming, nonlinear programming, dynamic programming, queuing theory, simulation optimization, and stochastic optimization are some tools applied in industrial engineering. If the limitations and the objectives can be represented in terms of linear relationships, then linear programming can come in handy. Integer programming is necessary in making discrete decisions on things like opening facilities or assigning shifts to individuals. Nonlinear programming helps in managing systems with complex or curved variables. In decision-making where there is uncertainty, stochastic optimization can be considered, and dynamic programming may prove useful in decision-making incrementally.

The topic of optimization has become increasingly important in the era of Industry 4.0 due to the massive amounts of real-time data generated by the modern manufacturing processes. Today, operations research is connected to cloud computing, automated control, sensors and cyber-physical systems. Xu et al. (2024) stated that Industry 4.0 technologies are gearing effectively in reshaping the landscape of supply-chain optimization, in addition to the provision of improved visibility, coordination, and support of decision-making. In this environment, companies can leverage mathematics to make dynamic changes to decisions as well as track operations.

2.3 Algorithms, Artificial Intelligence and Machine Learning

One of the most obvious examples of mathematics in use within modern technology is artificial intelligence and machine learning. ML algorithms employ mathematical structures to store patterns from data and make predictions or classifications following the learbed patterns. Linear algebra explain the structuring of data in vectors, matrices, and tensors. Gradient-based optimization is functional through the application of calculus. Probability facilitates systems to reason in the face of uncertainty. Generally, statistics are basic tools for the assessment of the performance of models and for the separation of signal from noise. Consequently, the listed mathematical tools are essential for the operation of modern AI, indicating they are most fundamental for their functionality (Bishop & Bishop, 2024; Deisenroth et al., 2020).

In-depth advantages of mathematical tools can be further illustrated by citing deep learning. Consequently, neural networks are positioned to carry out transformations on the input using layers of weighted operations, nonlinear activation functions, and optimization procedures. The training of a neural network requires the minimization of a loss function, which is typically achieved by some variant of gradient descent. The model facilitates learning by repeated adjustments to the functional parameters based on the errors. The final application can be speech recognition, image analysis, recommendation systems, or autonomous navigation, but it is all mathematical optimization on high dimensional data (Bishop & Bishop, 2024).

Machine learning has also revolutionized engineering practice. Predictive maintenance involves using information from sensors to forecast possible failure of machines. The computer vision technology is applied in quality control systems to detect flaws in products. Geometry, probability, control theory, and machine learning are used by autonomous vehicles to perceive and move around the environment. Kinematics, dynamics, optimization, and feedback control are applied to orchestrate the movements in robotics. In both cases, mathematical algorithms are employed to derive actionable conclusions from collected data. Regarding the application of AI in the realm of industrial material processing, the significance of mathematics in increasing productivity can be demonstrated through the following example. Ahmmed et al. (2024) explored the use of artificial intelligence and industry 4.0 technologies in the manufacturing industry. They also highlighted the importance of cyber-physical systems, IoT, and big data analysis for real-time monitoring and optimization of processes. All these technologies rely on mathematical modeling and algorithmic reasoning. 2.4 Cybersecurity, Cryptography and Digital Trust One other area where mathematics plays an important role is that of cybersecurity. The concepts of number theory, algebra, probability, and computational complexity are involved in the field of cryptography for securing information. Cryptography involves mathematical algorithms that are simple to perform in one way but difficult to undo without a key. Examples of such concepts include modular arithmetic, prime factorization, elliptic curves, lattice problems, and hash functions.

As societies rely more and more on digital infrastructure, the need for mathematical cybersecurity is growing. Sensitive information is sent across networks by banks, hospitals, governments, schools and industries, and networks need to be protected from unauthorized access. Confidentiality, authentication, integrity and nonrepudiation are formally based on mathematical cryptography. It also facilitates digital signatures, secure key exchange, and blockchain-based verification systems.

Digital Twins, Smart Manufacturing and Industry 4.0

Digital twins are one of the most significant recent advances in mathematics, engineering and technology. A digital twin represents a virtual model of a physical system, process or object that is consistently updated with data from the physical system. They are needed in several applications such as aerospace, manufacturing, energy, health care, transportation and other infrastructures. They bring together modeling, simulation, data analytics, control theory, and optimization for monitoring and decision-making (National Academies of Sciences, Engineering, and Medicine, 2024).

The true potential of a digital twin is in being able to connect models with real-time data. Whereas a conventional model can only be employed to represent a system at a particular point in time, a digital twin is able to be updated as more and more data is generated. It thus becomes highly sensitive to changing conditions. Kantaros et al. (2025) explain that digital twins are a new technological concept that combines mathematical modeling, simulation, artificial intelligence, and connected industrial systems. Similarly, Hartmann and Van der Auweraer (2025) assert that digital twins offer an important opportunity for industrial mathematics. Digital twins can be utilized in the field of smart manufacturing for monitoring machinery, simulating changes in processes, testing production problems, and forecasting maintenance needs. A manufacturing firm can use a digital twin to test whether a new approach to scheduling will increase production without overworking the machinery. An energy company can use the digital twin to model the impact of renewables on demand variability. A city can use a transportation digital twin to analyze traffic congestion and emergency response. In both, the technology relies on mathematical representation and computational updating.

2.5 Manufacturing and Supply Chains Applications

One of the best examples of mathematics in industrial technology is manufacturing. Mathematical reasoning is needed in production planning, quality control, inventory management, equipment maintenance, and facility layout. Probability distributions, control charts, sampling plans and process capability analysis are used to identify variation and enhance consistency in statistical quality control. Optimization models can be used to determine the optimal production quantity, material storage locations, and allocation of labor and machinery.

The essentiality of mathematics cannot be understated upon the consideration the operation of supply chains, which involve a large number of complex decisions under conditions of uncertainty. Firms need to take into account various parameters like forecasting of demands, dependability of suppliers, transportation costs, inventories, deadlines for deliveries, and risks of disruptions. Mathematics is applied for route optimization, network design, demand planning, and resilience analysis. Small adjustments to routing and inventory management strategies may lead to significant gains in global supply chains.

2.6 Applications in Health Care and Biomedical Technology

Mathematical and computational approaches are becoming more and more important in health care for prediction, diagnosis, monitoring, and resource allocation. Predictive analytics can be leveraged to predict patient risk, detect early warning signs, predict hospital demand and personalize treatment. The present-day healthcare analytics include statistical modeling, machine learning, survival analysis, time series models, and Bayesian techniques. These kinds of models can be applied to transform clinical data into evidence-based information for decision-making.

Badawy et al. (2023) have analyzed the use of machine learning and deep learning in health care predictive analytics and illustrated that such methods are utilized in predicting diseases, supporting clinical decisions, and managing health.. In addition to prediction accuracy, the mathematical challenge in health care is interpretability, fairness, and safety. In medicine, a model has to be carefully assessed as errors can have an impact on patient outcomes. Hence, in health care, mathematics needs to be connected to clinical expertise, ethical considerations, and clear validation.

2.6 Applications in Finance and Risk Management

Mathematics has been applied to modelling uncertainty, risk and return in finance for a long time. Portfolio management, derivative pricing, credit scoring, fraud detection, insurance modeling, and market-risk assessment are supported by stochastic processes, time-series analysis, Monte Carlo simulation, optimization, and statistical learning. Mathematical algorithms are used in financial technology to facilitate high-speed transactions, digital banking, automated lending, and anti-money laundering systems.

In recent years, quantitative finance has demonstrated the increasing importance of machine learning and statistical techniques. Pattnaik et al. (2024) reported the growing adoption of AI and machine learning in the financial sector. The developments demonstrate that finance is shifting from classical mathematical models to hybrid models that integrate financial theory, statistical learning, and computational optimization.

2.7 Applications in Energy Systems and Sustainability

Mathematical methods are needed for energy systems because energy production, storage, distribution and consumption need to be coordinated under uncertainty. The challenge is complicated by renewable energy, as solar and wind power are weather and time dependent. Mathematical models can be used to predict energy demand, optimize grid operations, schedule energy storage systems, plan for renewable energy integration, and assess sustainability targets.

Optimization modeling and methods for the renewable energy systems have been reviewed by Yu et al. (2023), and it was shown that optimization has been used in generation, distribution, storage, and planning of energy systems. Transition to clean energy sources therefore requires more than technological solutions; it requires mathematical modeling techniques that will be able to cope with variability and reliability problems.

2.8 Implications for Engineering Education and Practice

The increasing involvement of mathematics in technological development holds many important consequences for the educational process. The skills that an engineer or a technologist need are not limited only to knowing the formulas; they should be able to model the system, make assumptions, deal with uncertainty, and communicate the result. Calculus, linear algebra, statistics, numerical analysis, optimization, and discrete mathematics should be related to real-world problems for the students to realize their significance.

Continuous learning is also necessary for practicing in the field. Technologies such as digital twins, AI, robotics, and post-quantum cryptography develop very fast (National Institute of Standards and Technology, 2024). Math knowledge allows an engineer to recognize the structure of the new tool even if its concrete implementation changes. For example, the specific machine learning library can be modified but the idea of loss functions, regularization, bias, variance, and optimization will stay the same. A new cybersecurity standard can be implemented in software, but it relies on mathematical assumptions.

2.9 Ethical, Practical and Methodological Limitations

Mathematics is powerful, but mathematical models are not necessarily accurate or unbiased. A model is a simplified version of reality and all simplifications are based on assumptions. Results can be misleading if assumptions are unrealistic. No matter how advanced the algorithm is, decisions will be poor if the data is biased, limited, or inaccurately measured. Such issues are especially critical in AI, health care, finance, and infrastructure sectors, since decisions derived from modeling may affect people directly.

The emergence of AI necessitates the development of governance of models. Based on the NIST Artificial Intelligence Risk Management Framework, there are many issues related to the usage of AI technologies, such as trustworthiness, validity, reliability, safety, security, explainability, privacy, and fairness (Tabassi, 2023). Issues mentioned above are both ethical and mathematical.Thus, there is a need for technical competence and accountability in responsible technology.

3. 0 Conclusion

Mathematics is a key enabler of technology and engineering. It is used in the design of physical systems, prediction of uncertain events, optimization of industrial processes, security of digital communication, and intelligence of modern algorithms. Mathematics is increasingly relevant in the world of technology, which is increasingly connected, automated and data-driven, as it offers the formal structures to interpret complexity and make decisions.

As demonstrated in the examples in this paper, mathematics is not confined to the classroom. It is actively being used in manufacturing, supply chains, health care, finance, energy systems, cybersecurity, artificial intelligence and digital twins. The applications show that mathematics is a key enabler for innovation, efficiency, safety and competitiveness. They also demonstrate that mathematics is best used in conjunction with engineering judgment, domain expertise and responsible governance.

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Declarations

Conflict of Interest

The authors declared no conflict of interest

Funding

No funding was obtained for this work

Ethical consideration

Has been declared in section 2.7

Data Availability

Data shall be made available upon request

Author Contributions

RNN and JR jointly conceptualized the study, reviewed relevant literature, developed the analytical framework, analyzed and synthesized evidence, interpreted the findings, drafted and critically revised the manuscript, and approved the final version for publication.