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Alternative Ratio-Product Type Estimator in Simple Random Sampling

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Abstract *In this paper, we proposed a new alternative ratio-product estimator in simple random sampling without replacement by using information on an auxiliary variable. The proposed estimator is a mixture of some of the commonly known estimators. We have derived the minimum mean square errors up to the first order of approximation. Theoretically, we compare the mean square error (MSE) equation of the proposed estimator with the mean square error (MSE) equations of the existing estimators in literature. Numerical examples with four real data sets shows that the proposed estimator is more efficient than the existing other estimators considered. Therefore, the findings of this research are important in identifying alternative ratio-product exponential estimator, its properties, as well as relevant empirical applications.*

Keywords: *Ratio estimator, product estimator, bias, mean square error, efficiency, auxiliary variables, single-phase sampling*

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1.0 Introduction

In survey research, there are situations in which the information is available for every unit in the population. If a variable's value is known for every unit of the population, then it is not a variable of direct interest. Instead it may be employed to improve the sampling plan or to improve the estimation of another variable of interest. Such a variable is called an auxiliary variable. Ratio, product, and regression type estimators rely on the use of an auxiliary variable to estimate parameters of the study variable. Auxiliary variables have been used by various authors in various estimation situations. Cochran (1940) introduced the use of an auxiliary variable at the estimation stage and proposed the ratio estimator for the population mean. It is well known that the ratio type mean estimator ensures better efficiency than the sample mean estimator if the study variable and an auxiliary variable have strong positive correlations. For situations when they are negatively correlated, the product estimator was introduced by Robson (1957). The product estimator is also more efficient than the sample mean estimator. Modified ratio and product type estimators have been introduced by different authors. Bahl & Tuteja (1991) introduced exponential ratio and product type estimators which perform better than the ordinary ratio and product estimators, respectively. Subramani (2013) proposed a generalized modified ratio estimator for estimating the population mean using the known population parameters of an auxiliary variable such as coefficient of variation, coefficient of kurtosis, coefficient of skewness, the coefficient of correlation, and various quartiles.

In this paper we proposed a new alternative estimator of the population mean of the study variable in single-phase sampling by combining the ratio, product estimator. The main aim is to gain efficiency in comparisons to the existing ratio and product estimators.

2.0 Symbols, notations and some existing estimators

Let $U = \{U_1, , , , , , U_N\}$ be a finite population of size N and let (y_i, x_i) be the value of the study variable Y and the auxiliary variable X on ith unit $U_i, i = 1, \dots N$. Let $\overline{Y}$ and $\overline{X}$ be population means of the study variable Y and the auxiliary variable X respectively. We assume that the population mean $\overline{X}$ and the population variance S_x^2 of the auxiliary variable are known. Let S_y^2 be the population variance of the study variable Y. Let the correlation coefficient between the study variable and the auxiliary variable be ρ. Also, let $C_y = S_y/\overline{Y}$ and $C_x = S_x/\overline{X}$ be the coefficients of variation of the study variable Y and the auxiliary variable X, and $C_{yx} = S_{yx}/\overline{Y}\,\overline{X}$ be the coefficient of covariance between Y and X with $S_y^2 = \sum_{i=1}^{N}\left(y_i - \overline{Y}\right)^2/N-1$, $S_x^2 = \sum_{i=1}^{N}\left(x_i - \overline{X}\right)^2/N-1$.

To obtain the asymptotic properties of the estimator, we define the following error terms, as in Sukathme and Sukathme (1970).

Let $e_y = (\overline{y} - \overline{Y})/\overline{Y}$ and $e_x = (\overline{x} - \overline{X})/\overline{X}$, Such that $E(e_y) = E(e_x) = 0$, $E(e_y{}^2) = \lambda C_y^2$, $E(e_x{}^2) = \lambda C_x^2$, $E(e_y e_x) = \lambda\rho C_y C_x$, Where $\lambda = (1-f)/n$ and $f = n/N$

2.1 Sample mean

It is very well known that the sample mean y is an unbiased estimator of population mean Y, and under simple random sample without replacement (SRSWOR), its variance is given by

$$\frac{1-f}{n}S_y^2 = \lambda\overline{Y}^2 C_y^2 \qquad (1)$$

2.2 The Ratio Estimator

The ordinary ratio estimator for the population mean $\overline{Y}$ of the study variable is given by Cochran (1940) as:

$$\hat{t}_R = \overline{y}\frac{\overline{X}}{\overline{x}} \qquad (2)$$

The bias and the mean square error respectively of this estimator, up to the first order approximation, are given by:

$$Bias\left(\hat{t}_R\right) = \lambda\overline{Y}\left[C_x^2 - \rho_{yx}C_yC_x\right] \qquad (3)$$

$$MSE\left(\hat{t}_R\right) = \lambda\overline{Y}^2\left[C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right] \qquad (4)$$

If the sample size n is sufficiently large, then up to the first order of approximation, the ratio estimator will be more efficient than the ordinary sample mean estimator if

$$\rho_{yx} > \frac{C_x}{2C_y} \qquad (5)$$

For situations where $C_y = C_x$, condition (5) becomes $\rho_{yx} > \frac{1}{2}$

2.3 The product estimator

The product estimator is used when the study variable Y and the auxiliary variable X are negatively correlated. The estimator introduced by Robson (1957), and revised by Murthy (1964) is given by:

$$\hat{t}_P = \overline{y}\frac{\overline{x}}{\overline{X}} \qquad (6)$$

The exact bias of the product estimator is given by:

$$Bias\left(\hat{t}_P\right) = \frac{\lambda S_{yx}}{\overline{X}} \qquad (7)$$

The mean square error, up to the first order of approximation, is given by:

$$MSE\left(\hat{t}_P\right) = \lambda\overline{Y}^2\left[C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x\right] \qquad (8)$$

Up to the first order of approximation, the product estimator is more efficient than the ordinary sample mean if

$$MSE\left(\hat{t}_P\right) < V\left(\overline{y}\right) \text{ or if } \rho_{yx} < \frac{-C_x}{2C_y} \text{ or if}$$

$$\rho_{yx} < -\frac{1}{2} \text{ when } C_y = C_x \qquad (9)$$

2.3.3 The regression estimator

The ratio type estimators often result in increased precision if the line of best fit of Y on X is linear and passes through the origin. If the line does not pass through the origin, it is better to use the regression estimator given by:

$$\hat{t}_{Reg} = \overline{y} + \hat{\beta}_{yx}\left(\overline{X} - \overline{x}\right) \qquad (10)$$

Where $\hat{\beta}_{yx} = \frac{s_{yx}}{s_x^2}$ is the sample regression coefficient between Y and X. The bias of the regression estimator, up to the first order of approximation, is given by:

$$Bias\left(\hat{t}_{\text{Reg}}\right) = -\lambda\beta_{yx}\left\{\frac{\mu_{12}}{\mu_{11}} - \frac{\mu_{03}}{\mu_{02}}\right\} \qquad (11)$$

Where $\beta_{yx} = \frac{S_{yx}}{S_x^2}$ is the population regression coefficient between the study variable Y and the auxiliary variable X, and $\mu_{rs} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^r\left(x_i - \bar{X}\right)^s$

Also $s_x{}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$ is the sample variance of X and $s_{yx} = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)\left(x_i - \bar{x}\right)$ is the sample covariance between X and Y.

The mean square error, up to the first order of approximation, is given by:

$$MSE\left(\hat{t}_{\text{Reg}}\right) = \lambda\bar{Y}^2 C_y^2\left[1 - \rho_{yx}^2\right] \qquad (12)$$

The conditions under which the regression estimator is more efficient than the ordinary sample mean and ratio estimator are given below:

1. The regression estimator is more efficient than the ordinary sample mean $\bar{y}$ if $MSE\left(\hat{t}_{\text{Reg}}\right) < V\left(\bar{y}\right)$, or if $C_y^2 - C_y^2\left[1 - \rho_{yx}^2\right] > 0$, or if $\rho_{yx} \neq 0$ and
2. The regression estimator is more efficient than the ratio estimator if $MSE\left(\hat{t}_{\text{Reg}}\right) < MSE\left(\hat{t}_{\text{R}}\right)$ i.e, if $C_x^2 - 2\rho_{yx}C_yC_x + \rho_{yx}^2C_y^2 > 0$; or if $\left[C_x - \rho_{yx}C_y\right]^2 > 0$

If the relationship between Y and X is linear, and passes through the origin, then the two estimators are equally efficient.

2.4 Bahl and Tuteja exponential estimators

The exponential type estimators are often used to improve efficiencies of the ratio and product type estimators and were introduced by Bahl and Tuteja (1991) as:

$$\hat{t}_{ER} = \bar{y}exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \qquad (13)$$

$$\hat{t}_{EP} = \bar{y}exp\left(\frac{\bar{x}-\bar{X}}{\bar{X}+\bar{x}}\right) \qquad (14)$$

The exponential part helps, since it captures the auxiliary variable effect for a longer duration. The bias of the exponential estimators, up to the first order of approximation, are given by:

$$Bias\left(\hat{t}_{ER}\right) = \lambda\bar{Y}\left[\frac{3}{8}C_x^2 - \frac{1}{2}\rho_{yx}C_yC_x\right] \qquad (15)$$

$$Bias\left(\hat{t}_{EP}\right) = \lambda\bar{Y}\left[\frac{1}{2}\rho_{yx}C_yC_x - \frac{1}{8}C_x^2\right] \qquad (16)$$

The mean square error of the exponential ratio and product type estimators, up to the first order of approximation, are given by:

$$MSE\left(\hat{t}_{ER}\right) = \lambda\bar{Y}^2\left[C_y^2 + \frac{1}{4}C_x^2 - \rho_{yx}C_yC_x\right] \qquad (17)$$

$$MSE\left(\hat{t}_{EP}\right) = \lambda\bar{Y}^2\left[C_y^2 + \frac{1}{4}C_x^2 + \rho_{yx}C_yC_x\right] \qquad (18)$$

2.6 *Rao (1991) estimator*

Rao (1991) introduced the generalized regression type estimator to improve efficiency of the ordinary regression estimator. The estimator is given by:

$$\hat{\mu}_{R,Reg} = k_1\bar{y} + k_2\left(\bar{X} - \bar{x}\right) \qquad (19)$$

Where k_1 and k_2 are suitably chosen constants. The minimum mean square error of this estimator, up to the first order of approximation, with optimum values of k_1 and k_2 i.e.,

$$k_{1(opt)} = \frac{1}{1 + \lambda\left(1 - \rho_{yx}^2\right)C_y{}^2}$$

$$k_{2(opt)} = k_{1(opt)}\frac{\bar{Y}}{\bar{X}}\frac{\rho_{yx}C_y}{C_x}$$

is given by:

$$MSE_{\min}\left(\hat{\mu}_{R,\text{Reg}}\right) = \bar{Y}^2\left[1 - \frac{1}{1 + \lambda\left(1 - \rho_{yx}^2\right)C_y^2}\right] \qquad (20)$$

2.7 Grover and Kaur (2011) estimator

Following Rao (1991) and Bahl & Tuteja (1991), Grover & Kaur (2011) suggested a regression exponential type estimator given by:

$$\hat{\mu}_{GK} = \left[l_1\bar{y} + l_2\left(\bar{X} - \bar{x}\right)\right]exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \qquad (21)$$

Where l_1 and l_2 are suitably chosen constants.

The minimum mean square error of this estimator, at the optimum values of l_1 and l_2, i.e,

$$l_{1(opt)} = \frac{-1+\frac{1}{8}\lambda C_x^2}{\left[-1+\lambda\left(1-\rho_{yx}^2\right)C_y^2\right]}$$

$$l_{2(opt)} = \frac{\overline{Y}}{\overline{X}}\left[\frac{1}{2} - l_{1(opt)}\left(1-\rho_{yx}\frac{C_y}{C_x}\right)\right]$$

is given by:

$$MSE_{\min}\left(\hat{\mu}_{GK}\right) = \frac{\lambda\overline{Y}^2\left[\lambda C_x^4 - 16\left(\rho_{yx}^2-1\right)\left(-4+\lambda C_x^2\right)C_y^2\right]}{64\left[-1+\lambda\left(\rho_{yx}^2-1\right)C_y^2\right]} \quad (22)$$

3.0 The Proposed Estimator

In this section, we propose a new alternative estimator by combining the ratio, product, and exponential ratio type estimators using linear combination. The estimator is given as:

$$\hat{t}_{GR} = \bar{y}\left[k\frac{\overline{X}}{\bar{x}} + (1-k)\frac{\bar{x}}{\overline{X}}\right] exp\left[\frac{\overline{X}-\bar{x}}{\overline{X}+\bar{x}}\right] \quad (23)$$

Where $k = (0, 1)$ is a suitable chosen constant to be determined.

Remarks;

Such that, when $k = 1$ then the alternative estimator reduces to ratio-type exponential estimator and when $k = 0$ then the alternative estimator reduces to product-type exponential estimator. The two classes estimators are as follows;

$$\hat{t}_{RP} = \bar{y}\left(\frac{\overline{X}}{\bar{x}}\right) exp\left[\frac{\overline{X}-\bar{x}}{\overline{X}+\bar{x}}\right]$$

$$\hat{t}_{RP} = \bar{y}\left(\frac{\bar{x}}{\overline{X}}\right) exp\left[\frac{\overline{X}-\bar{x}}{\overline{X}+\bar{x}}\right]$$

Expressing the estimator $\hat{t}_{GR}$ in terms of e_i $(i = y,\ x)$ we can write (3.1) as

$$\hat{t}_{GR} = \overline{Y}\left(1+e_y\right)\left[k\left(1+e_x\right)^{-1} + \left(1-k\right)\left(1+e_x\right)\right]\exp\left\{\frac{-\overline{X}e_x}{2\overline{X}+\overline{X}e_x}\right\}$$

$$= \overline{Y}\left(1+e_y\right)\left[k\left(1-e_x+e_x^{\ 2}\right)+\left(1+e_x-k-ke_x\right)\right]\exp\left\{\left(\frac{-e_x}{2}\right)\left(1+\frac{e_x}{2}\right)^{-1}\right\} \quad (24)$$

Expanding the RHS of (24) to the first order of approximation, multiplying out and neglecting the terms of e's greater than two, we get

$$\hat{t}_{GR} = \overline{Y}\left(1+e_y\right)\left[1+e_x-2ke_x+ke_x^{\ 2}\right]\exp\left\{\left(\frac{-e_x}{2}\right)\left(1-\frac{e_x}{2}+\frac{e_x^{\ 2}}{4}\right)\right\} \quad (25)$$

Thus, it follows

$$\hat{t}_{GR} = \overline{Y}\left[1+e_x-2ke_x+ke_x^{\ 2}+e_y+e_ye_x-2ke_ye_x\right]\exp\left\{\frac{-e_x}{2}+\frac{e_x^{\ 2}}{4}\right\}$$

$$= \overline{Y}\left[1+e_y+\left(1-2k\right)e_x-\left(1-2k\right)e_ye_x+ke_x^{\ 2}\right]*\left\{1-\frac{e_x}{2}+\frac{3e_x^{\ 2}}{8}\right\}$$

$$= \overline{Y}\left[1+e_y+\left(1-2k\right)e_x-\left(1-2k\right)e_ye_x+ke_x^{\ 2}-\frac{e_x}{2}-\frac{e_ye_x}{2}-\frac{\left(1-2k\right)e_x^{\ 2}}{2}+\frac{3e_x^{\ 2}}{8}\right]$$

$$= \overline{Y}\left[1+e_y+\frac{(1-4k)e_x}{2}+\frac{(16k-1)e_x^{\ 2}}{8}+\frac{(1-4k)e_ye_x}{2}\right] \quad (26)$$

Subtracting $\overline{Y}$ and taking expectation to both sides of (26), we obtained the bias of the estimator $\hat{t}_{GR}$ as

$$E\left(\hat{t}_{GR}\right)-\bar{Y}=\bar{Y}E\left[e_y+\frac{(1-4k)e_x}{2}+\frac{(16k-1)e_x^{\,2}}{8}+\frac{(1-4k)e_ye_x}{2}\right]$$

$$Bias\left(\hat{t}_{GR}\right)=\bar{Y}\left[\frac{(16k-1)}{8}\lambda C_x^2+\frac{(1-4k)}{2}\lambda\rho_{(y,x)}C_yC_x\right] \tag{27}$$

Similarly, subtracting $\overline{Y}$, taking expectation, and squaring both sides of (3.4), we obtained the mean square error of the estimator $\hat{t}_{GR}$ as

$$E\left[\hat{t}_{GR}-\bar{Y}\right]^2=\bar{Y}^2E\left[e_y+\frac{(1-4k)e_x}{2}+\frac{(16k-1)e_x^{\,2}}{8}+\frac{(1-4k)e_ye_x}{2}\right]^2$$

$$=\bar{Y}^2E\left[e_y^{\,2}+\frac{(1-4k)^2e_x^{\,2}}{4}+(1-4k)e_ye_x\right]$$

$$MSE\left(\hat{t}_{GR}\right)=\bar{Y}^2\left[\lambda C_y^2+\frac{(1-4k)^2}{4}\lambda C_x^2+(1-4k)\lambda\rho_{(y,x)}C_yC_x\right] \tag{28}$$

Differentiating (3.6) partially with respect to k, we have

$$\frac{\partial MSE\left(\hat{t}_{GR}\right)}{\partial k}=0\Rightarrow\bar{Y}^2\left[-2(1-4k)\lambda C_x^2-4\lambda\rho_{(y,x)}C_yC_x\right]=0 \tag{29}$$

Solving (3.7) linearly, we obtained the optimum value of k as

$$k_{(opt)}=\frac{2\rho_{(y,x)}C_y+C_x}{4C_x}$$

Substituting the value of k into (3.6) we obtained the minimum mean square error as

$$MSE_{\min}\left(\hat{t}_{GR}\right)=\bar{Y}^2\left[\lambda C_y^2+\frac{1}{4}\left[1-4\left(\frac{2\rho_{(y,x)}C_y+C_x}{4C_x}\right)\right]^2\lambda C_x^2+\left[1-4\left(\frac{2\rho_{(y,x)}C_y+C_x}{4C_x}\right)\right]\lambda\rho_{(y,x)}C_yC_x\right]$$

$$=\bar{Y}^2\left[\lambda C_y^2+\lambda C_y^{\,2}\rho_{(y,x)}^2-2\lambda C_y^{\,2}\rho_{(y,x)}^2\right]$$

$$=\bar{Y}^2\left[\lambda C_y^2-\lambda C_y^{\,2}\rho_{(y,x)}^2\right]$$

$$MSE_{\min}\left(\hat{t}_{GR}\right)=\lambda\bar{Y}^2C_y^2\left[1-\rho_{(y,x)}^2\right]=MSE\left(\hat{t}_R\right) \tag{30}$$

4.0 Theoretical Efficiency Comparisons

In this section, efficiency of the proposed estimator is compared with some of the commonly used estimators. Conditions under which the proposed estimator is more efficient are given below:

i. Comparing the MSE of the proposed estimator, given in (30), with the variance of the sample mean in (2.1), we have the following conditions:

$$MSE(\hat{t}_{RP})<V(\bar{y});$$

$$\bar{Y}^2\left[\lambda C_y^2-\lambda\rho^2C_y^{\,2}\right]<\bar{Y}^2\lambda C_y^2$$

$$\lambda C_y^2-\lambda\rho^2C_y^{\,2}<\lambda C_y^2$$

$$-\lambda\rho^2C_y^{\,2}<0$$

$$\lambda\rho^2C_y^{\,2}>0 \tag{31}$$

When the condition (3.9) is satisfied, the proposed estimator is more efficient than sample mean.

ii. Comparing the MSE of the proposed estimator with the MSE of the usual ratio estimator in (2.4), we have the following conditions:

$$MSE(\hat{t}_{RP}) < MSE(\hat{t}_{R});$$

$$\bar{Y}^2[\lambda C_y^2 - \lambda\rho^2 {C_y}^2] < \bar{Y}^2[\lambda C_y^2 + \lambda C_x^2 - 2\lambda\rho_{yx}C_yC_x]$$

$$\lambda C_x^2 + \lambda\rho^2 {C_y}^2 - 2\lambda\rho_{yx}C_yC_x > 0$$

$$C_x^2 + \rho^2 {C_y}^2 - 2\rho_{yx}C_yC_x > 0 \qquad (32)$$

When the condition (32) is satisfied, the proposed estimator is more efficient than the usual ratio estimator, given in (4).

iii. Comparing the MSE of the proposed estimator with the MSE of the usual product estimator in (8), we have the following conditions:

$$MSE(\hat{t}_{RP}) < MSE(\hat{t}_{P});$$

$$\bar{Y}^2[\lambda C_y^2 - \lambda\rho^2 {C_y}^2] < \bar{Y}^2[\lambda C_y^2 + \lambda C_x^2 + 2\lambda\rho_{yx}C_yC_x]$$

$$\lambda C_x^2 + \lambda\rho^2 {C_y}^2 + 2\lambda\rho_{yx}C_yC_x > 0$$

$$C_x^2 + \rho^2 {C_y}^2 + 2\rho_{yx}C_yC_x > 0 \qquad (33)$$

When the condition (3.11) is satisfied, the proposed estimator is more efficient than the product estimator, given in (2.8).

iv. Comparing the MSE of the proposed estimator with the MSE of Bahl and Tuteja (1991) in (17) and (18), we have the following conditions: For the exponential ratio, we have;

$$MSE(\hat{t}_{RP}) < MSE(\hat{t}_{ER});$$

$$\bar{Y}^2\left[\lambda C_y^2 - \lambda\rho^2 {C_y}^2\right] < \bar{Y}^2\left[\lambda C_y^2 + \frac{1}{4}\lambda C_x^2 - \lambda\rho_{yx}C_yC_x\right]$$

$$\frac{1}{4}\lambda C_x^2 + \lambda\rho^2 {C_y}^2 - \lambda\rho_{yx}C_yC_x > 0$$

$$\frac{1}{4}C_x^2 + \rho^2 {C_y}^2 - \rho_{yx}C_yC_x > 0 \qquad (34)$$

And for the exponential product, it is given by

$$MSE(\hat{t}_{RP}) < MSE(\hat{t}_{EP});$$

$$\bar{Y}^2\left[\lambda C_y^2 - \lambda\rho^2 {C_y}^2\right] < \bar{Y}^2\left[\lambda C_y^2 + \frac{1}{4}\lambda C_x^2 + \lambda\rho_{yx}C_yC_x\right]$$

$$\frac{1}{4}\lambda C_x^2 + \lambda\rho^2 {C_y}^2 + \lambda\rho_{yx}C_yC_x > 0$$

$$\frac{1}{4}C_x^2 + \rho^2 {C_y}^2 + \rho_{yx}C_yC_x > 0 \qquad (35)$$

When the conditions (34) and (35) are satisfied, the proposed estimator is more efficient than the exponential ratio and exponential product estimators, given in (17) and (18) respectively.

v. Comparing the MSE of the proposed estimator with the MSE of the Rao (1991) generalized regression type estimator in (20), we have the following conditions:

$$MSE(\hat{t}_{RP}) < MSE_{\min}(\hat{\mu}_{R,\mathrm{Reg}});$$

$$\bar{Y}^2\left[\lambda C_y^2 - \lambda\rho^2 {C_y}^2\right] < \bar{Y}^2\left[1 - \frac{1}{1+\lambda\left(1-\rho_{yx}^2\right)C_y^2}\right]$$

$$\left[\lambda C_y^2 - \lambda\rho^2 {C_y}^2\right] < \left[1 - \frac{1}{1+\lambda\left(1-\rho_{yx}^2\right)C_y^2}\right] \qquad (36)$$

When the condition (36) is satisfied, the proposed estimator is more efficient than the Rao (1991) generalized regression type estimator in (20).

vi. Comparing the MSE of the proposed estimator with the MSE of the Grover and Kaur (2011) regression exponential type estimator in 22), we have the following conditions:

$$MSE(\hat{t}_{RP}) < MSE_{\min}(\hat{\mu}_{GK});$$

$$\bar{Y}^2\left[\lambda C_y^2 - \lambda\rho^2 C_y{}^2\right] < \frac{\lambda\bar{Y}^2\left[\lambda C_x^4 - 16\left(\rho_{yx}^2 - 1\right)\left(-4+\lambda C_x^2\right)C_y^2\right]}{64\left[-1+\lambda\left(\rho_{yx}^2-1\right)C_y^2\right]}$$

$$\left[\lambda C_y^2 - \lambda\rho^2 C_y{}^2\right] < \frac{\lambda\left[\lambda C_x^4 - 16\left(\rho_{yx}^2 - 1\right)\left(-4+\lambda C_x^2\right)C_y^2\right]}{64\left[-1+\lambda\left(\rho_{yx}^2-1\right)C_y^2\right]} \quad (37)$$

When the condition (37) is satisfied, the proposed estimator is more efficient than the Grover & Kaur (2011) regression exponential type estimator in (22).

5.0 Empirical Study

In this section, we compare the efficiency of proposed estimator with other existing mean estimators using real data sets. The Population Statistics for the real data sets are given in Table 1. Table 4.2 gives mean square error of the estimators based on the first order of approximation. While Table 4.3 gives Percent Relative Efficiency of the proposed estimator as compared to the existing estimators based on the first order of approximation.

Table 1 Summary Statistics for the Real Populations Used

Parameters	Population I	Population II	Population III	Population IV
N	70	34	256	204
n	25	20	100	50
ρ_{yx}	0.7293	0.4491	0.887	0.71
$\bar{Y}$	96.7	856.4118	56.47	966
$\bar{X}$	175.2671	208.8824	44.45	26441
C_y	0.6254	0.8561	1.42	2.4739
C_x	0.8037	0.7205	1.40	1.7171
f	0.3571	0.5882	0.3906	0.2451

Population I [Source: Singh and Chaudhary (1986), pp.108]
Population II [Source: Singh and Chaudhary (1986), pp. 177]
Population III [Source: Cochran (1977), pp. 196]
Population IV [Source: Kadilar & Cingi (2006)]
We use the following equation to obtain the percent relative efficiency (PRE) of different estimators,

$$PRE = \frac{Var(\bar{y})}{MSE(\hat{t}) \quad or \quad MSE_{min}(\hat{t})} * 100$$

where $Var(\bar{y})$ is the variance of sample mean, $MSE_{min}(\hat{t})$ is the mean square error values of the proposed estimator in section 3 ($\hat{t}_{RP}$) and $MSE(\hat{t})$ is the mean square error values of the existing estimators mentioned in Section 2

Table 2: MSE Values of the Classical and Proposed Estimators

Estimators	Population I	Population II	Population III	Population IV
$\bar{y}$	94.04661	75.29876	39.18293	86226.17
$\hat{t}_R$	73.08217	112.7374	16.60388	8276509
$\hat{t}_P$	425.676	286.5683	76.12899	10843343
$\hat{t}_{ER}$	44.73302	62.92955	14.43892	54118.79
$\hat{t}_{EP}$	221.0181	149.845	82.97042	139103.5
$\hat{\mu}_{R,Reg}$	44.81894	60.72774	9.333182	42886.05
$\hat{\mu}_{GK}$	44.2117	60.17537	8.3901	42802.06
$\hat{t}_{RP}$	**44.02524**	**60.11169**	**8.355015**	**42759.56**

Table 3: Percent Relative Efficiency of Different Estimators with Respect to $\bar{y}$

Estimators	Population I	Population II	Population III	Population IV
$\bar{y}$	100	100	100	100
$\hat{t}_R$	128.6861	64.13024	235.9866	10.4182
$\hat{t}_P$	22.0935	26.2760	51.4691	0.7952
$\hat{t}_{ER}$	210.2398	119.6556	271.3702	159.3276
$\hat{t}_{EP}$	42.5515	50.2511	47.2252	61.9871
$\hat{\mu}_{R,Reg}$	209.8368	123.9940	419.8239	201.0588
$\hat{\mu}_{GK}$	212.7188	125.1322	467.0147	201.4533
$\hat{t}_{RP}$	**213.6198**	**125.2648**	**468.9750**	**201.6535**

Here, we obtained the efficiency of the proposed and existing estimators with respect to the sample mean using mean square error (MSE) values for the four data sets. The percent relative efficiency values of the usual ratio, usual product, usual regression, existing different ratio estimators, Bahl & Tuteja (1991) dual ratio-product exponential estimators, Rao , Grover & Kaur (2011) suggested regression exponential type estimator, and the proposed mixture of ratio-product exponential type estimator are presented in Table 4.3. Thus, the result shows that as inferred by the theoretical comparisons, the proposed mixture of ratio-product exponential type estimator performs better than the existing estimators for the entire four populations. This is due to the fact that the proposed estimator is equally as efficient as the regression estimator and confirms Cochran (1942), Robson (1957), Murthy (1967) and Perri (2005) assertion that the regression estimator is generally more efficient than ratio and product estimators.

6.0 Conclusion

We proposed a new generalized ratio-product estimator in simple random sampling without replacement by using information on an auxiliary variable. The proposed estimator is a mixture of some of the commonly known estimators. The derived mean square error (MSE) expression of the proposed estimator is derived to be equal to the mean square error (MSE) expression of the usual regression estimator. Thus, the proposed estimator happens to an alternative to the regression estimator and gives a better replacement to some existing ratio and product estimators.

7.0 References

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